Thesis for the Degree of Master of Engineering

Frequency-shaped Linear Quadratic Control Approach to Anti-sway System Design

by

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February 2009

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(주파수 성형 LQ 제어기법을 이용한 Anti-sway 시스템 설계)

Advisor : Prof. Young Bok Kim

by

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A thesis submitted in partial fulfillment of the requirements
for the degree of
Master of Engineering

in Department of Control and Mechanical Engineering
The Graduate School
Pukyong National University

February 2009

Luu Hoang Minh 의 공학석사 학위논문

2008 년 .. 월 .. 일

주	심	공학박사	양주호	(인)
위	원	공학박사	장지성	(인)
위	원	공학박사	김영복	(인)

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A dissertation

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ACKNOWLEGMENTS

The help and continuous support from professors, colleagues, friends, and family to whom I am most grateful, will never be forgotten. Without you, all of you, I would not be what I am today. I would like to thank each of you individually by word, but I do so in my heart.

I would like to express my deepest gratitude to my supervisor, Professor Kim Young Bok for his patient guidance, encouragement and advice in this study. I appreciate the time that we had spent in the numerous discussions and this research. He had provided me a lot of examples how to conduct a research and make real contribution. I have learned a lot from his thorough and insightful review of this research and his dedication to producing high quality and practical research. Without him, I wouldn't be graduation today.

I would like to recognize the contributions and helpful suggestions provided by my thesis advisory committee members Professor Joo Ho Yang and Professor Ji Seong Jang. These two Professors have given me their valuable comments, feedback and great suggestions based on their work, thus greatly contributing to the improvement, refinement and final completion of my thesis.

In addition, I would like to thank my managers also, Dr. Dang Van Uy, Rector of Vietnam Maritime University and Dr. Pham Ngoc Tiep, director of the research institute of Vietnam Maritime University and, as well as Mr. E. B. Kim, manager of CMR company who gave me the opportunity to have been studying here.

I show great appreciation to members of Marine Cybernetics Lab. for giving me a comfortable and active environment as well as enthusiastic and invaluable help during the periods of time spent working with them. They all, in this way or another, have helped me a lot from the time of my first step here to the time of my graduation.

CONTENTS

Frequency-shaped Linear Quadratic Control Approach to Anti-sway System Design

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Abstract

In this paper, a crane controller design approach for anti-sway system is studied. To reduce the swing motion of container in desired area, we use a small auxiliary mass which is installed on the spreader, such that the actuator reacting against the auxiliary mass applies inertial control forces. In this study, we apply the Frequency-shaped Linear Quadratic control approach to anti-sway control system design problem. The frequency-shaped approach can be carried a step further by augmenting the plant with frequency-shaped filters so as to penalize their outputs in addition to other cost terms in the performance index. A considerable magnitude reduction in the resonances frequency range is obtained by introducing frequency dependent weighting functions. By using MATLAB program, we calculate and design controller for anti-sway system and evaluate system performance through simulation.

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Introduction

1.1 Background of Container Crane System

Now, in all ports, the transporting container from the container ship onto trucks is undertaken by the container crane. But there must be decrease of swing motion in the all transportation process. This swing motion happens when trolley is at the end of acceleration process, deceleration process, stopping or in the case of that the unexpected disturbance input exist.

In fact, we have two control objects which are trolley and spreader. The trolley control method is used to decrease swing motion of container, but this method has several problems relating to working time of the crane drives such as increase of fatigue and discomfort.

In this paper, we study a method to decrease swing motion of container using a small auxiliary mass [1] [2] which is installed on the spreader of the container crane, and the actuator reacting against the auxiliary mass applies inertial control forces. Based on these facts, we have introduced many control techniques. And in this paper, Linear Quadratic control will approach to anti-sway system using auxiliary mass damper.

1.2 Outline of Dissertation and Summary of Contribution

This dissertation designs new controller for anti-sway system for container crane by Linear Quadratic (LQ) methods [7] [8] [9] [10] [11]. The content and summary of contribution in this dissertation is organized as follows:

In chapter 1:

This chapter introduces background of container crane system and the reason of swing motion in container crane. The outline of each chapter for this dissertation is given.

In chapter 2:

This chapter introduces some methods reduce swing motion of container crane. And give state space equation of anti-sway system using mass damper.

In chapter 3:

This chapter introduces Linear Quadratic and Linear Quadratic Gaussian methods with new design will be termed frequency-shaped designs [3] [4] [5] [6].

In chapter 4:

Design new controller for anti-sway system and evaluate system performance through simulation by using MATLAB program.

In chapter 5:

Conclusions.

Chapter 2

Anti-sway System of Container Crane

2.1 Trolley Control Method

The key point of this method is identifying the times, when the trolley in the end of acceleration, deceleration process or stopping.

Figure 2.1 shows movement of trolley and container, where, m_T is mass of trolley, M is mass of container, f_T is driving force for trolley, l is rope length.

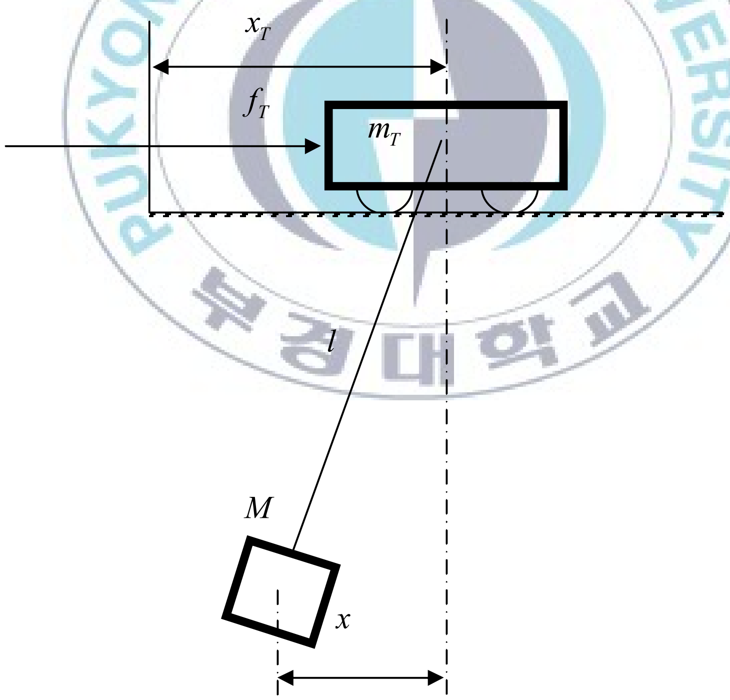

Fig. 2.1: Movement of trolley and container

Moving equations of container and trolley are described as following:

$$M\left(\frac{d^2x}{dt^2}-\frac{d^2x_T}{dt^2}\right)+\frac{Mg}{l}x=0 \tag{2.1}$$

$$m_T\frac{d^2x_T}{dt^2}+\frac{Mg}{l}x=f_T \tag{2.2}$$

From equation (2.1) we obtained:

$$\frac{d^2x_T}{dt^2}=\frac{d^2x}{dt^2}+\frac{g}{l}x \tag{2.3}$$

Substitute $\frac{d^2x_T}{dt^2}$ in equation (2.2) we get differential equation of movement of container:

$$m_T\frac{d^2x}{dt^2}+\frac{g}{l}(m_T+M)x=f_T \tag{2.4}$$

$$\Leftrightarrow\frac{d^2x}{dt^2}+\frac{g}{l}\left(1+\frac{M}{m_T}\right)x=\frac{f_T}{m_T} \tag{2.5}$$

General solution of equation (2.5) is:

$$x=A\sin\sqrt{\frac{g}{l}\left(1+\frac{M}{m_T}\right)}t+B\cos\sqrt{\frac{g}{l}\left(1+\frac{M}{m_T}\right)}t+\varphi(t) \tag{2.6}$$

Where, $\varphi(t)$ is partial solution of equation (2.5).

Define as f_T isn't changed, from equation (2.5) we get partial solution as following.

$$\varphi(t) = \frac{f_T l}{(m_T + M)g} \tag{2.7}$$

So equation (2.6) can be rewritten

$$x = A\sin\sqrt{\frac{g}{l}\left(1+\frac{M}{m_T}\right)}t + B\cos\sqrt{\frac{g}{l}\left(1+\frac{M}{m_T}\right)}t + \frac{f_T l}{(m_T + M)g} \tag{2.8}$$

With initial conditions $t = 0$ then $x = 0; \frac{dx}{dt} = 0$

Therefore: $A = 0; B = -\frac{f_T l}{(m_T + M)g}$

$$\Rightarrow x = \frac{f_T l}{(m_T + M)g}\left[1 - \cos\sqrt{\frac{g}{l}\left(1+\frac{M}{m_T}\right)}t\right] \tag{2.9}$$

We see a few solution to decrease sway after trolley post-acceleration or stop .Derivative equation (2.9) follow time, we acquire container's velocity equation:

$$\frac{dx}{dt} = \frac{f_T}{(m_T + M)}\sqrt{\frac{l}{g}\left(1+\frac{M}{m_T}\right)}\sin\sqrt{\frac{g}{l}\left(1+\frac{M}{m_T}\right)}t \tag{2.10}$$

When $x = 0; \frac{dx}{dt} = 0$ then $\sqrt{\frac{g}{l}\left(1+\frac{M}{m_T}\right)}t = 2n\pi$ Where, *n=1, 2, 3…*

$x = 0, \frac{dx}{dt} = 0$ and $f_T = 0$ then sway of container is zero. Deduce period of time accelerate and stop must following

$$t = 2n\pi\sqrt{\frac{m_T l}{(m_T + M)g}} \tag{2.11}$$

We selected t as equation (2.11), then sway of container is zero, when the trolley in acceleration, deceleration process or start, stop. But this method has several problems relating to working time of the crane drives such as increase of fatigue and discomfort. So in next part, we introduce other method of anti-sway for container crane.

2.2 Mass-damper type Anti-sway System

Here we introduce a solution to suppress swing motion by installing an auxiliary mass damper on the spreader. The auxiliary mass is showed in figure:

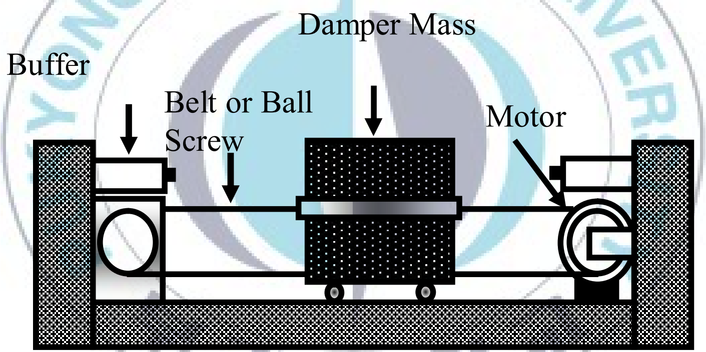

Fig. 2.2: Auxiliary mass damper

This damper-mass is installed on the spreader of container crane, the belt or ball-screw to transfer power to the moving mass and a motor to move a damper mass. In this system, the actuator reacting against the auxiliary mass applies inertial control force to the container to reduce the swing motion.

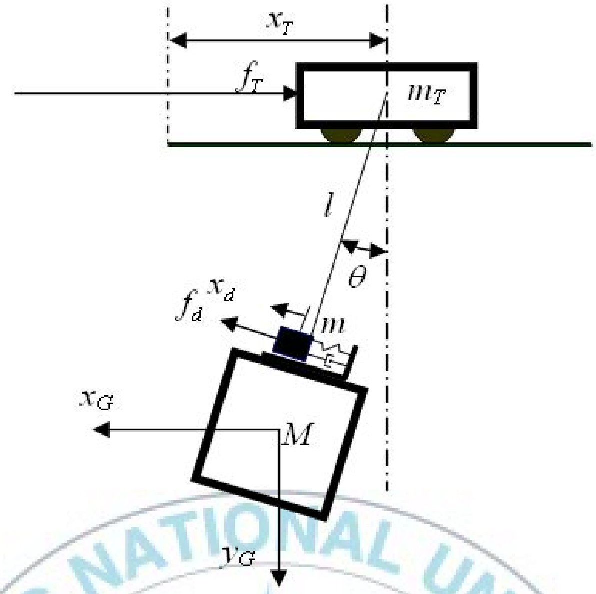

Fig. 2.3: Dynamic model for swing motion analysis

Where, the parameters are described as following.

m_T : mass of trolley

M : mass of container

m : mass of damper mass

f_T : driving force for trolley

f_d : horizontal force generated by actuator

l : rope length

x_G, y_G : gravity center

θ : sway angle

We define that the center of gravity of the spreader is equal to that of the mass-damper then x_G, y_G can be written as:

$$\begin{aligned} x_G &= l\sin\theta + x_T \\ y_G &= -l\cos\theta \end{aligned} \tag{2.12}$$

We define K as kinetic energy and V as potential energy then:

$$K = \frac{1}{2}m_T\dot{x}T^2 + \frac{1}{2}(M+m)(\dot{x}_G^2 + \dot{y}_G^2) \tag{2.13}$$

$$V = -(M+m)gl\cos\theta \tag{2.14}$$

Define $L = K - V$, we have the Lagrange equations as following:

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_T}\right) - \frac{\partial L}{\partial x_T} = f_T \\ &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = T - T_d \end{aligned} \tag{2.15}$$

where, T : moment generated by disturbance

T_d : moment generated by actuator

In this thesis we don't consider dynamic of trolley, because it can be regarded as a kind of disturbance input, so the kinetic energy and the kinetic energy produced by the mass m can be written:

$$(M+m)l\ddot{x} + \frac{C}{l}\dot{x} + (M+m)gx = T - mgx_d - f_d l \tag{2.16}$$

$$m\ddot{x}_d = -\frac{mg}{l}x + f_d - C_d\dot{x}_d - k_d x_d \tag{2.17}$$

where, C :damping constant

g :acceleration of gravity

C_d :damping constant of actuator

k_d :stiffness of actuator

In fact: C, C_d and k_d are given as follows: C=0.005324, C_d=1.5865 and k_d=0.00095

Define: $x_1 = x, \quad x_2 = \dot{x}, \quad x_3 = x_d, \quad x_4 = \dot{x}_d$

Then

$$\dot{x}_1 = x_2 \tag{2.18}$$

$$\dot{x}_3 = x_4 \tag{2.19}$$

Substitute $\ddot{x} = \dot{x}_2, \quad \dot{x} = x_2, \quad x = x_1, \quad x_d = x_3$ into equation (2.16) and $\ddot{x}_d = \dot{x}_4, \quad x = x_1, \quad \dot{x}_d = x_4, \quad x_d = x_3$ into equation (2.17), we obtain:

$$(M+m)l\dot{x}_2 + \frac{C}{l}x_2 + (M+m)gx_1 = T - mgx_3 - f_d l \tag{2.20}$$

$$m\dot{x}_4 = -\frac{mg}{l}x_1 + f_d - C_d x_4 - k_d x_3$$

(2.21)

Therefore:

$$\dot{x}_2 = -\frac{g}{l}x_1 - \frac{C}{(M+m)l^2}x_2 - \frac{mg}{(M+m)l}x_3 + \frac{T - f_d l}{(M+m)l} \tag{2.22}$$

$$\dot{x}_4 = -\frac{g}{l}x_1 - \frac{k_d}{m}x_3 - \frac{C_d}{m}x_4 + \frac{f_d}{m} \tag{2.23}$$

where $f_d = K_m v$, with K_m is motor torque coefficient and v is voltage to motor.

From equations (2.18), (2.19), (2.22), (2.23) we obtained:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0\\ -\frac{g}{l} & -\frac{C}{(M+m)l^2} & -\frac{mg}{(M+m)l} & 0\\ 0 & 0 & 0 & 1\\ -\frac{g}{l} & 0 & -\frac{k_d}{m} & -\frac{C_d}{m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{-K_m}{M+m}\\ 0\\ \frac{K_m}{m}\end{bmatrix}v + \begin{bmatrix}0\\ \frac{1}{(M+m)l}\\ 0\\ 0\end{bmatrix}T \tag{2.24}$$

$$\text{Define}\quad y = \begin{bmatrix}x\\ x_d\end{bmatrix} = \begin{bmatrix}x_1\\ x_3\end{bmatrix},\ \text{So}\ \ y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \tag{2.25}$$

The state equation of anti-sway system can be written:

$$\begin{aligned}\dot{x}_p &= Ax_p + Bu + Gw\\ y &= Cx_p\end{aligned} \tag{2.26}$$

where, the states $x_p = [x \quad \dot{x} \quad x_d \quad \dot{x}_d]^T$, control input $u = v$ (input voltage to motor), $w = T$ (disturbance input), and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{g}{l} & -\dfrac{C}{(M+m)l^2} & -\dfrac{mg}{(M+m)l} & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{g}{l} & 0 & -\dfrac{k_d}{m} & -\dfrac{C_d}{m} \end{bmatrix} \tag{2.27}$$

$$B = \begin{bmatrix} 0 & -\dfrac{K_m}{M+m} & 0 & \dfrac{K_m}{m} \end{bmatrix}^T \tag{2.28}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{2.29}$$

$$G = \begin{bmatrix} 0 \\ \dfrac{1}{(M+m)l} \\ 0 \\ 0 \end{bmatrix} \tag{2.30}$$

Table parameter values of the reduction model

Parameters	Values
Spreader specification:	
Length	0.70 [m]
Width	0.43 [m]
Height	0.42 [m]
Weight (M)	0.565 [kg]
Rope length (l)	0.40 [m]
Moving-mass weight (m)	0.095 [kg]
Motor torque coefficient (K_m)	150 [N/v]

Table 2.1 Parameter values of the reduction model

If we use the parameter values given in table 2.1, the system matrices are obtained as following:

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ -24.5 & -0.0504 & -3.5265 & 0\\ 0 & 0 & 0 & 1\\ -24.5 & 0 & -0.01 & -16.7\end{bmatrix} \quad (2.31), \quad B=\begin{bmatrix}0\\ -227.3\\ 0\\ 1578.9\end{bmatrix} \quad (2.32)$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix} \quad (2.33), \qquad G=\begin{bmatrix}0\\ 3.7879\\ 0\\ 0\end{bmatrix} \quad (2.34)$$

Chapter 3

Linear Quadratic Methods

3.1 Linear Quadratic Regulator (LQR)

3.1.1 The Linear Quadratic Regulator Problem

Consider the linear system and the quadratic cost function:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$J = \frac{1}{2}\int_0^T (x'Qx + u'Ru)dt$$

The problem is to minimize cost function J with respect to the control input *u(t)*, this problem is known as the linear quadratic regulator problem. We see that cost function J is the weighted sum of energy of state and energy of control input.

3.1.2 LQR Solution using the Minimum Principle

The linear quadratic regulator control problem can be solved using many techniques, for example Euler-Lagrange equation, Hamilton-Jacobi-Bellman theory, and Pontriagin's minimum principle…etc.

In this chapter we must first from the so-called Hamiltonian to arrive at the minimum principle.

$$H(x,\lambda,t) = \frac{1}{2}(x'Qx + u'Ru) + \lambda'(Ax + Bu) \qquad (3.1)$$

The minimum principle states must satisfy the following equations:

$$\dot{x} = \frac{\partial H}{\partial \lambda} \qquad x(0) = x_0 \qquad \text{State equation}$$

(3.2)

$$-\dot{\lambda} = \frac{\partial H}{\partial x} \qquad \lambda(T) = 0 \quad \text{Costate or adjoint equation} \tag{3.3}$$

$$\frac{\partial H}{\partial u} = 0 \tag{3.4}$$

Where, x_0 is initial state and T is final time.

From equation (3.1) we obtain:

$$\dot{x} = Ax + Bu \qquad x(0) = x_0$$

(3.5)

$$-\dot{\lambda} = Qx + A'\lambda \qquad \lambda(T) = 0 \tag{3.6}$$

$$\begin{aligned} & Ru + B'\lambda = 0 \\ & \Rightarrow u = -R^{-1}B'\lambda \\ & \Rightarrow \dot{x} = Ax - BR^{-1}B'\lambda \end{aligned} \tag{3.7}$$

The above coupled linear differential equation form a two point boundary value problem (TPBVP), which is difficult to solve numerically because of mixed boundary conditions. Note that matrix R must be positive definite for R^{-1} exist. Now we get state equation of optimal control:

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B' \\ -Q & -A' \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} \triangleq H \begin{bmatrix} x \\ \lambda \end{bmatrix} \tag{3.8}$$

Where, matrix H is called the Hamiltonian matrix.

However we can’t solve the two point boundary value problem after all. To solve it we must define the substitution:

$$\lambda = Px \tag{3.9}$$

Differentiating both sides with respect to time we get:

$$\begin{aligned}\frac{d\lambda}{dt} &= \frac{dP}{dt}x + P\frac{dx}{dt} \\ &= \frac{dP}{dt}x + PAx - PBR^{-1}B'\lambda \\ &= \frac{dP}{dt}x + PAx - PBR^{-1}B'Px\end{aligned} \tag{3.10}$$

With equation (3.6) we get:

$$\frac{dP}{dt}x + PAx - PBR^{-1}B'Px = -Qx - A'Px \tag{3.11}$$

This equation must hold for any x, so matrix P is solution of following equation:

$$-\frac{dP}{dt} = A'P + PA + Q - PBR^{-1}B'P, \qquad P(T) = 0 \tag{3.12}$$

The above equation is called Riccati differential equation. It is a nonlinear first order differential equation that has to be solved backwards in time. Recall that the TPBVP is a linear second order differential equation with mixed boundary conditions. It is usually the Riccati equation form of the LQR solution that is use.

The above formulation and solution of the LQR problem is know as the finite time problem. It results in a linear time varying controller of the feedback form:

$$u(t) = -K(t)x(t) \qquad \text{where} \quad K(t) = R^{-1}B'P(t) \tag{3.13}$$

For the infinite time LQR problem, we let T approach infinity. Of course, now one runs into the question of the convergence of the cost function and, hence, the existence of the optimal controller. Event if the optimal control exists, it doesn't necessarily result in a stable closed loop system. It turn out that under mild conditions, $P(t)$ is a constant matrix so dP/dt equal zero, and the positive definite solution of the algebraic Riccati equation result in an asymptotically stable closed loop system.

$$A'P + PA + Q - PBR^{-1}B'P = 0 \tag{3.14}$$

$$u = -Kx, \qquad K = R^{-1}B'P \tag{3.15}$$

Matrix R and matrix Q can be selected so that $R > 0$ and Q can be as $Q = C_q{}'C_q$, where C_q is any matrix such that $[C_q, A]$ is detectable. These conditions are necessary and sufficient for existence and uniqueness of an optimal controller that will asymptotically stabilize the system.

3.2 The Linear Quadratic Gaussian (LQG)

In this method, the frequency-shaped approach can be carried a step further by augmenting the plant with frequency-shaped filters so as to penalize their outputs in addition to other cost terms in the performance index. A considerable magnitude reduction in the resonances frequency range is obtained by introducing frequency dependent weighting functions.

3.2.1 The Linear Quadratic Gaussian Problem

Consider the problem of estimating the state of the stochastic system given by following figure:

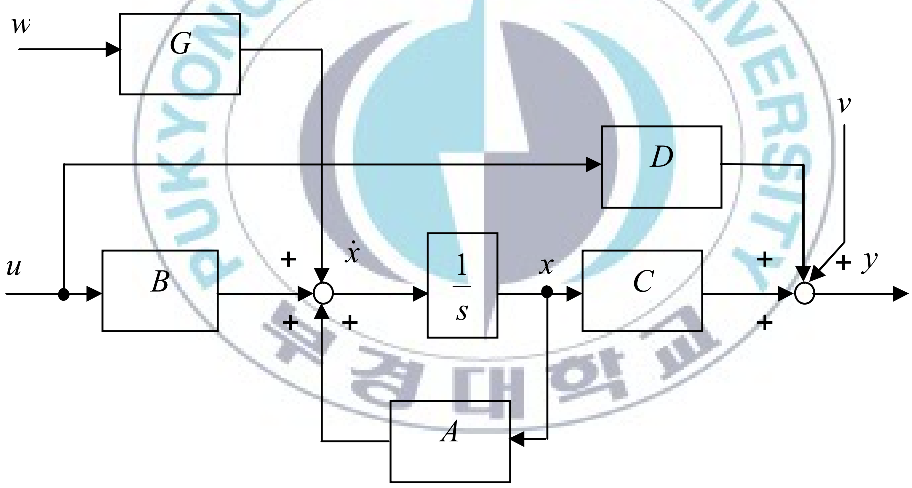

Fig. 3.1: Diagram of open-loop system

Then the state equation of this system is

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) + Gw(t) \\
y(t) &= Cx(t) + Du(t) + v(t)
\end{aligned}
\tag{3.16}
$$

Where the state is available only indirectly through the noisy-output measurement, $w(t)$ and $v(t)$ is uncorrelated zero-mean, Gaussian, white-noise, random vectors with correlation matrices. And known covariance given below.

$$E[w(t)w'(t)] = W(t)\delta(t-\tau) \tag{3.17}$$

$$E[(v(t)v'(t)] = V(t)\delta(t-\tau) \tag{3.18}$$

The problem is to find a dynamical system that optimally estimates of the system $x(t)$ given by measurements. This problem is minimizing the quadratic performance measure:

$$\begin{aligned} J &= \lim_{t_f \to \infty} \frac{1}{t_f} E\left[\int_{-t_f}^{t_f} [x'(t)Qx(t) + u'(t)Ru(t)]dt\right] \\ &= E[x'(t)Qx(t) + u'(t)Ru(t)] \end{aligned} \tag{3.19}$$

As we know the results in a linear time varying controller of the feedback form:

$$u(t) = -K(t)x(t) \tag{3.20}$$

Where $K(t) = R^{-1}B'P(t)$

And P is solution of Riccati differential equation:

$$A'P + PA + Q - PBR^{-1}B'P = 0 \tag{3.21}$$

3.2.2 LQG Solution: Frequency-shaped

Consider state available and control input in frequency-shaped, then the quadratic performance is given:

$$J = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E[X^*(j\omega)QX(j\omega) + U^*(j\omega)RU(j\omega)]d\omega \tag{3.22}$$

Consider matrices Q and R by substituting as weighting functions $Q(j\omega)$ and $R(j\omega)$, then we obtain

$$J = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E[X^*(j\omega)Q(j\omega)X(j\omega) + U^*(j\omega)R(j\omega)U(j\omega)]d\omega \tag{3.23}$$

Where, X^* and U^* are conjugate vectors of X and U.

Define $\overline{Q}(j\omega)$ and $\overline{R}(j\omega)$ satisfy following conditions.

$$Q(j\omega) = \overline{Q}^*(j\omega)\overline{Q}(j\omega) \tag{3.33}$$

$$R(j\omega) = \overline{R}^*(j\omega)\overline{R}(j\omega) \tag{3.34}$$

And define $\overline{X}(j\omega)$ and $\overline{U}(j\omega)$ satisfy:

$$\overline{X}(j\omega) = \overline{Q}(j\omega)X(j\omega) \tag{3.35}$$

$$\overline{U}(j\omega) = \overline{R}(j\omega)U(j\omega) \tag{3.36}$$

From equations (3.35) and (3.36), each state equation for $\overline{Q}(j\omega)$ and $\overline{R}(j\omega)$ is described as following respectively.

$$\begin{aligned} \dot{x}_q(t) &= A_q x_q(t) + B_q x(t) \\ z_q(t) &= C_q x_q(t) + D_q x(t) \end{aligned} \tag{3.36}$$

$$\begin{aligned} \dot{x}_r(t) &= A_r x_r(t) + B_r u(t) \\ z_r(t) &= C_r x_r(t) + D_r u(t) \end{aligned} \tag{3.37}$$

Synthesize 3 state equations (3.16), (3.36) and (3.37), we obtain

$$\dot{\tilde{z}}(t) = \tilde{A}\tilde{z}(t) + \tilde{B}u(t)$$

(3.38)

where,

$$\tilde{z}(t) = \begin{bmatrix} x(t) \\ x_q(t) \\ x_r(t) \end{bmatrix} \tag{3.39}$$

$$\tilde{A} = \begin{bmatrix} A & 0 & 0 \\ B_q & A_q & 0 \\ 0 & 0 & A_r \end{bmatrix} \tag{3.40}$$

$$\tilde{B} = \begin{bmatrix} B \\ 0 \\ B_r \end{bmatrix} \tag{3.41}$$

For the system described in (3.38), we introduce a quadratic performance criteria.

$$J = E[\tilde{z}(t)'\tilde{Q}\tilde{z}(t) + 2\tilde{z}(t)'\tilde{S}u(t) + u(t)'\tilde{R}u(t)] \tag{3.42}$$

Where,

$$\tilde{Q} = \begin{bmatrix} D_q'D_q & D_q'C_q & 0 \\ C_q'D_q & C_q'C_q & 0 \\ 0 & 0 & C_r'C_r \end{bmatrix} \tag{3.43}$$

$$\tilde{S} = \begin{bmatrix} 0 \\ 0 \\ C_r'D_r \end{bmatrix} \tag{3.44}$$

$$\tilde{R} = D_r'D_r \tag{3.45}$$

In (3.38) and (3.42), the control law is given

$$\begin{aligned} u(t) &= -F_b x(t) - F_q x_q(t) - F_r x_r(t) \\ &= -\tilde{R}^{-1}(\tilde{B}'\tilde{P} + \tilde{S})\tilde{Z}(t) \end{aligned} \tag{3.46}$$

where $\tilde{P}$ is a solution of following Riccati equation.

$$\tilde{P}(\tilde{A} - \tilde{B}\tilde{R}^{-1}\tilde{S}') + (\tilde{A} - \tilde{B}\tilde{R}^{-1}\tilde{S}')'\tilde{P} - \tilde{P}\tilde{B}\tilde{R}^{-1}\tilde{B}'\tilde{P} + \tilde{Q} - \tilde{S}\tilde{R}\tilde{S}' = 0 \tag{3.47}$$

Then the closed-loop system can be described as Fig. 3.2.

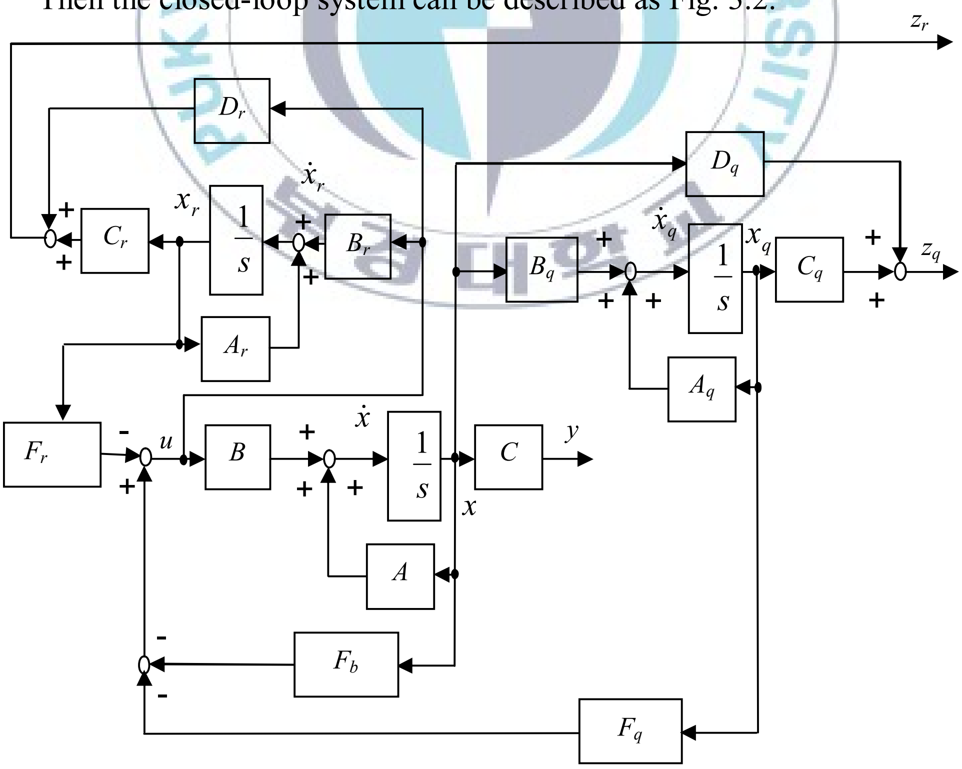

Fig. 3.2: Diagram of closed-loop system with frequency-shaped LQG method

In this approach, we must select weighting functions $\overline{Q}(j\omega)$ and $\overline{R}(j\omega)$, so that we can reduce the amplitude gain of the open-loop system especially in the resonance frequency range.

Chapter 4

Design of LQ Controller for Anti-sway System by MATLAB

4.1 Modeling and Simulating System

In this thesis, the weighting functions are frequency dependent as we studied in chapter 3, so we define $\overline{R}(j\omega)$ and $\overline{Q}(j\omega)$ following equations:

$$\overline{R}(j\omega) = \frac{K_1\omega_2^4[(j\omega)^2 + 2ze_1\omega_1 j\omega + \omega_1^2]^2}{\omega_1^4[(j\omega)^2 + 2ze_2\omega_2 j\omega + \omega_2^2]^2} \tag{4.1}$$

$$\overline{Q}(j\omega) = \frac{K_{21}\omega_3^4}{[(j\omega)^2 + 2ze_3\omega_3 j\omega + \omega_3^2]^2} \tag{4.2}$$

We need to prepare to combine weighting functions selection with iteration, because the translation of specification into $\overline{Q}$, $\overline{R}$ selection is imprecise. Here, we can change shape of bode diagram of $\overline{Q}$ and $\overline{R}$ by changing of ω_1, ω_2, ω_3, ze_1, ze_2, ze_3, K_1 and K_{21} to compare bode diagram of closed-loop with bode diagram of open-loop. Noting that, the first breakpoint on the bode diagram of $\overline{R}(j\omega)$ has same frequency with the breakpoint on the bode diagram of $\overline{Q}(j\omega)$.

Then M-files of plant of container crane (crane.m), weighting functions (fwfwt.m), and the Frequency-shaped Linear Quadratic controller (fslqg.m) were obtained as table I, II, and III in Appendix.

In figure 3.2, we obtain the diagram closed-loop system show as following figure.

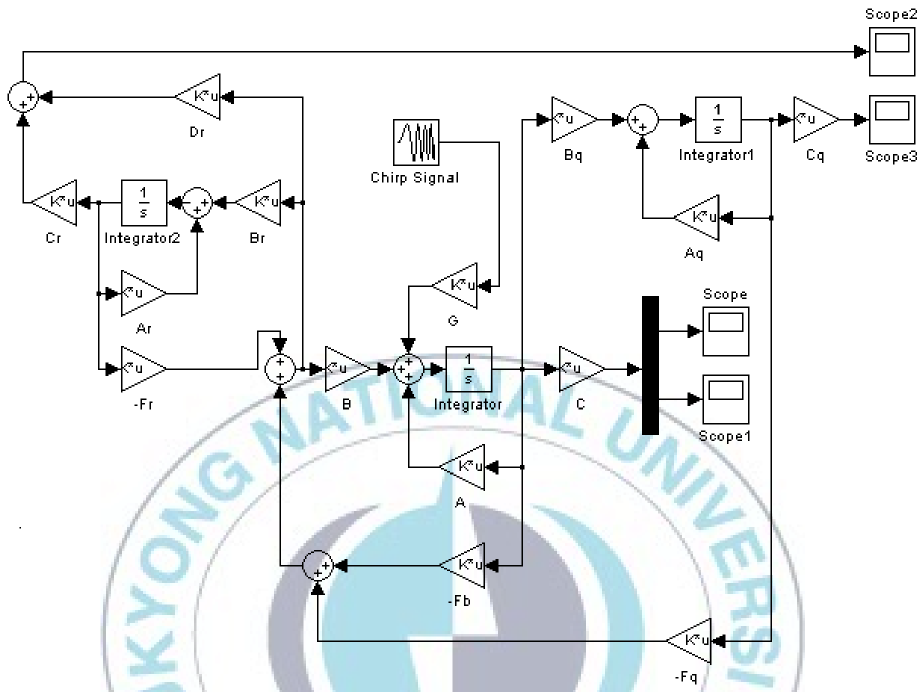

Fig. 4.1 Diagram closed-loop system with frequency shaped LQ method by MATLAB program.

4.2 Select Weighting Functions

First draw bode diagrams of $\overline{Q}$, $\overline{R}$, open-loop and closed-loop system, then we compare bode diagram of open-loop system with bode diagram of closed-loop system. If in resonance frequency range, the disturbance is reduced, then we obtain weighting functions as desired.

Draw bode diagrams of $\overline{Q}$, $\overline{R}$, open-loop and closed-loop system at $\omega_1 = \omega_3 = 20\pi$, $ze_1 = ze_2 = ze_3 = 0.6$, $\omega_2 = 60\pi$, $K_{21} = 1$, we obtain following figure:

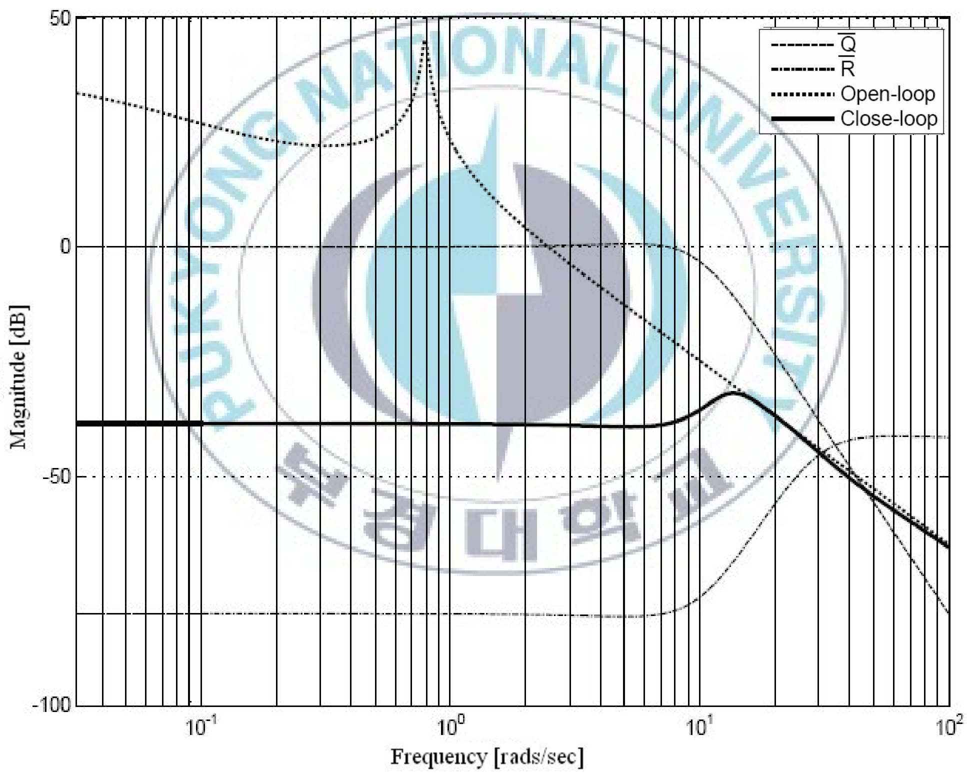

Fig. 4.2 Bode diagrams of $\overline{Q}$, $\overline{R}$, open-loop and closed-loop system at $\omega_1 = \omega_3 = 20\pi$, $ze_1 = ze_2 = ze_3 = 0.6$, $\omega_2 = 60\ \pi$, $K_{21} = 1$

In above figure, to increase magnitude of Closed-loop's bode diagram, we must increase K_{21}. To make the bode diagram of $\overline{Q}$ lies in the left of open-loop's bode diagram, we decrease parameter ω_3. Next figure show the bode diagrams of $\overline{Q}$, $\overline{R}$, open-loop and closed-loop system at $\omega_1 = \omega_3 = 3.4\pi$, $\omega_2 = 30\pi$, $K_{21} = 1.1$

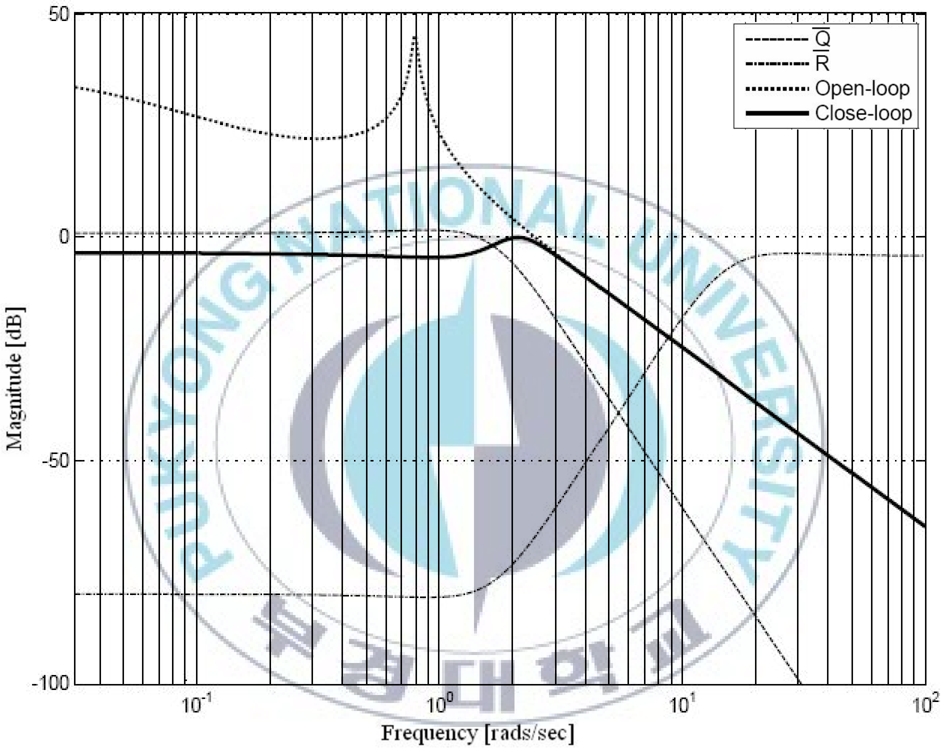

Fig. 4.3 Bode diagrams of $\overline{Q}$, $\overline{R}$, open-loop and closed-loop system at $\omega_1 = \omega_3 = 3.4\pi$, $\omega_2 = 30\pi$, $K_{21} = 1.1$

To increase magnitude of Closed-loop's bode diagram more, we try with K_{21}=1.4 and decrease ω_1 and ω_3 to 1.32. Next figure show the bode diagrams of $\overline{Q}$, $\overline{R}$, open-loop and closed-loop system when increase K_{21} to 1.4 and $\omega_1 = \omega_3 = 2.64\pi$.

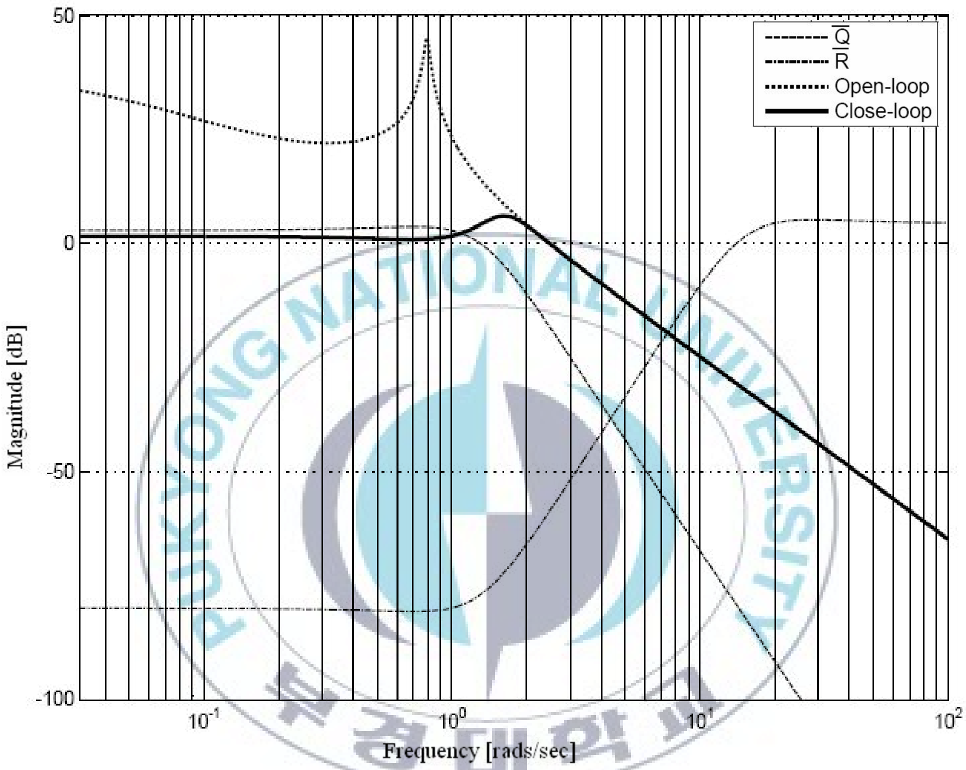

Fig. 4.4 Bode diagrams of $\overline{Q}$, $\overline{R}$, open-loop and closed-loop system when increase K_{21} to 1.4 and $\omega_1 = \omega_3 = 2.64\pi$.

We obtained the bode diagram of closed-loop as desired, such that the good control performance maybe obtained.

4.3 Simulation Result

For the weighting functions were selected, the controller is calculated as following based on the Frequency-shaped Linear Quadratic theory:

The $\overline{Q}$-loop:

$$A_q = \begin{bmatrix} -19.9 & -236.6 & -1369.2 & -4731.7 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad B_q = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_q = \begin{bmatrix} 0 & 0 & 0 & 6624.3 \end{bmatrix},\ D_q = 0.$$

The $\overline{R}$-loop:

$$A_r = \begin{bmatrix} -226.19 & -3.06\times 10^4 & -2.01\times 10^6 & -7.89\times 10^7 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_r = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C_r = \begin{bmatrix} -343.99 & -5.06\times 10^4 & -3.35\times 10^6 & -1.32\times 10^8 \end{bmatrix},$$

$$D_r = 1.6675.$$

The feedback matrices:

$$F_q = \begin{bmatrix} -2.32 & -40.99 & -384.88 & -1118.2 \end{bmatrix},$$

$$F_r = \begin{bmatrix} 1.9672 & 287.37 & 1.9\times 10^4 & 7.46\times 10^5 \end{bmatrix},$$

$$F_b = \begin{bmatrix} -0.5 & -0.067 & -0.015 & 3.74\times10^{-4} \end{bmatrix}$$

Consider the disturbance input as chirp signal.

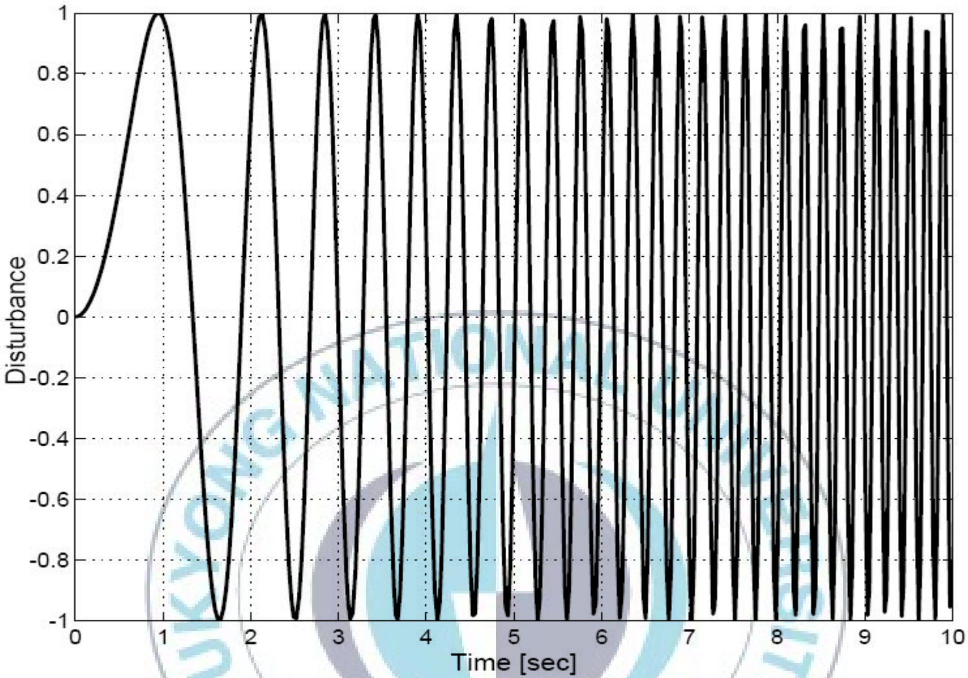

Fig. 4.5 Disturbance input

For this disturbance, the controlled outputs are calculated. Fig. 4.6 and Fig. 4.7 show the spreader displacement, and Fig. 4.8 shows the displacement of moving-mass in the actuator.

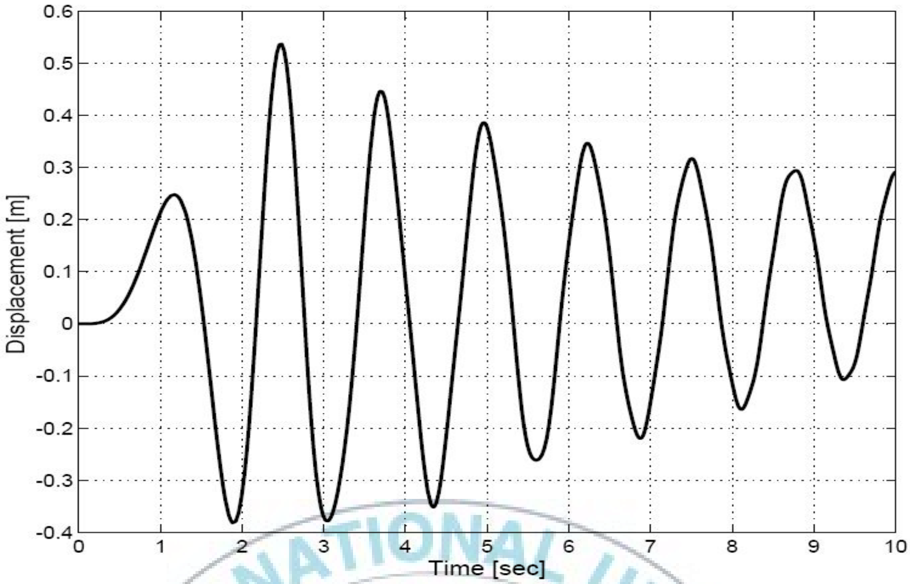

Fig. 4.6 Displacement of spreader with open-loop system

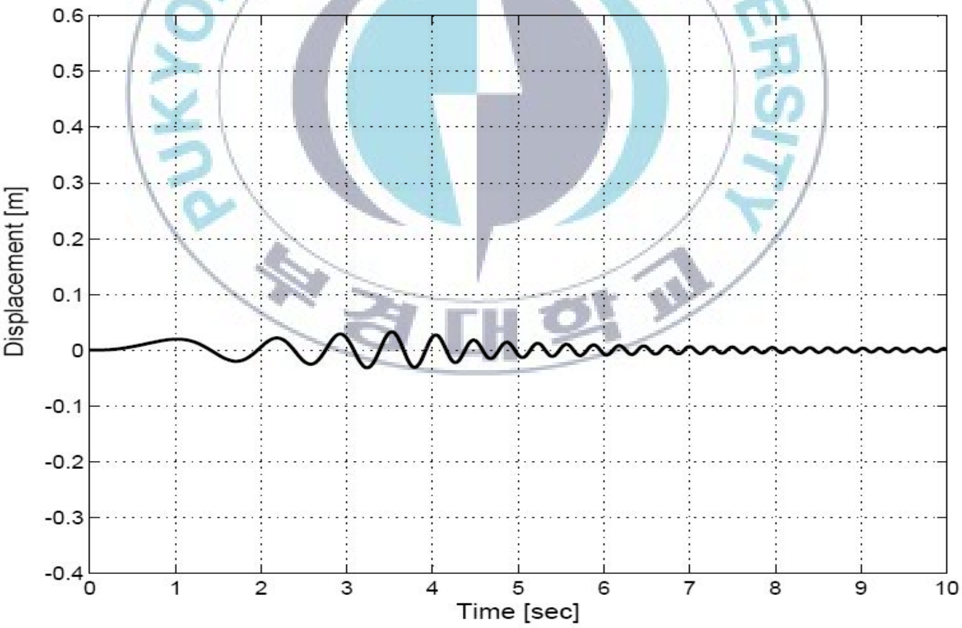

Fig. 4.7 Displacement of spreader with closed-loop system

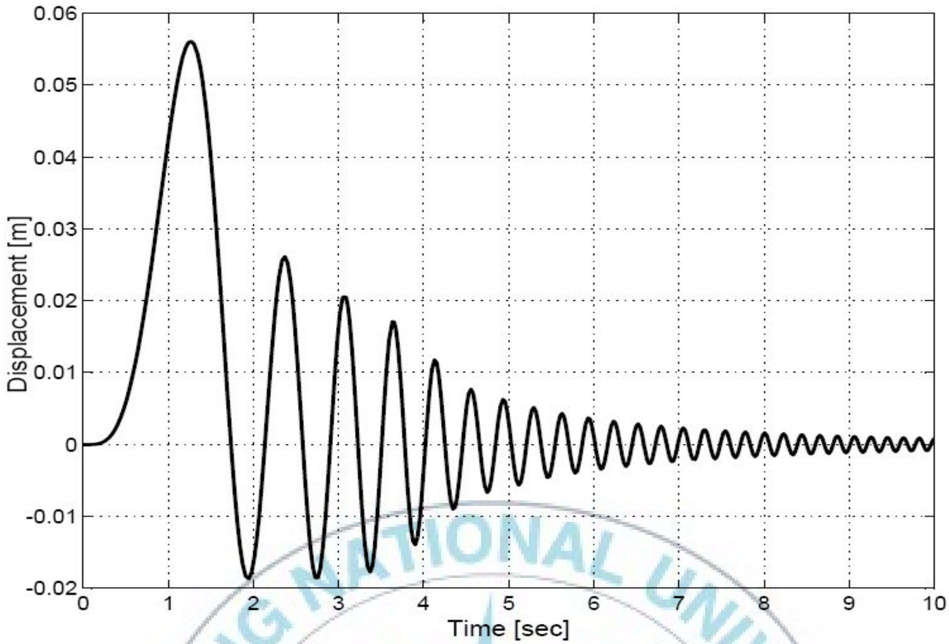

Fig. 4.8 Movement of mass-damper

The figures 4.7 show the displacement of spreader with Frequency-shaped Linear Quadratic controller that we designed above. Although, the swing motion still exist but it is reduced a lot. In figure 4.6, the biggest amplitude of vibration of spreader is 0.54 m, but when system has Frequency-shaped LQ controller, the biggest amplitude of vibration of spreader is only 0.045 m, and the spreader comes balance state very fast. So we obtain the good control performance.

Chapter 5

Conclusions

In this thesis, we introduced background of container crane system and the reason of swing motion in container crane and some methods reduce swing motion of container crane, and we obtained state space equation of anti-sway system using mass damper.

In order to design anti-sway system for container crane, an approach was recommended in this study. The new controller was designed by Frequency-shaped Linear Quadratic method.

In Frequency-shaped Linear Quadratic method, as same as other LQ methods, the most important work is selecting weighting functions. To select weighting functions, we draw bode diagrams of weighting functions, open-loop and closed-loop system. If in resonance frequency range, the magnitude of closed-loop system is reduced, then we obtain weighting functions as desired.

The steps to design Frequency-shaped Linear Quadratic controller are described as following:

1. Chapter 2 introduces the way to modeling plant of anti-sway system by state equation.

2. In chapter 3, we conformed bode diagram of open-loop system and bode diagram of closed-loop system by changing parameters of weighting functions, and selected the weighting function to obtain the

bode diagram of closed-loop as desired, so that the magnitude in the resonances frequency range could be reduced.

3. After that, the frequency dependent weighting functions were described as state equations in chapter 3.

4. In chapter 4, the control law is obtained by solving the Riccati equation.

The controller is obtained base on the solution of Riccati equation. Although, the swing motion still exist but it is reduced a lot, and the spreader comes balance state very fast. So we obtain the good control performance.

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APPENDIX

The M-file of plant of container crane (crane.m) was shows as following table.

```
g=9.8; l=0.4;
M=0.565; am=0.095;
cd=1.5856; kd=0.00095; km=150; c=0.005324;
a11=0;
a12=1;
a13=0;
a14=0;
a21=-g/l;
a22=-c/((M+am)*(l^2));
a23=-am*g/(M+am)/l;
a24=0;
a31=0;
a32=0;
a33=0;
a34=1;
a41=-g/l;
a42=0;
a43=-kd/am;
a44=-cd/am;
A=[a11 a12 a13 a14; a21 a22 a23 a24; a31 a32 a33 a34; a41 a42 a43 a44]
b1=0;
b2=-km/(M+am);
b3=0;
b4=km/am;
B=[b1;b2;b3;b4]
C=[1 0 0 0;0 0 1 0]
D=[0;0]
G=[0;1/((M+am)*l);0;0]
```

Table I M-file of plant of container crane (crane.m)

The M-file of weighting functions (fwfwt.m) was show as following table.

```
% WT=Rbar(w)
f1=10;
f2=30;
ze1=0.6;
ze2=0.6;
K1=0.0001;
%WS=Qbar(w)
f3=10;
ze3=0.6;
K21=1;
% WT
w1=2*pi*f1;
w2=2*pi*f2;
num1=[1 2*ze1*w1 w1^2];
num1=K1*w2^4*conv(num1,num1);
den1=[1 2*ze2*w2 w2^2];
den1=w1^4*conv(den1,den1);
w=logspace(-1,3,1000);
mag1=bode(tf(num1,den1),w);
mag1=20*log10(mag1(1,:));
% WS
w3=2*pi*f3;
num21=[0 0 0 0 K21*w3^4];
den21=[1 2*ze3*w3 w3^2];
den21=conv(den21,den21);
w=logspace(-1,3,1000);
mag2=bode(tf(num21,den21),w);
mag2=20*log10(mag2(1,:));
% Bode diagrams of Qbar and Rbar
f=w/pi/2;
semilogx(f,mag2,f,mag1)
Xlabel('Frequency    Hz')
Ylabel('Gain    dB')
axis([10^(-1),10^2,-100,10]);
```

Table II M-file of weighting functions $\overline{Q}$ and $\overline{R}$ (fwfwt.m)

The M-file (fslqg.m) of frequency-shaped LQG controller was shown as following table.

```
crane
fwfwt
% Q(jw)
[Aq,Bq,Cq,Dq]=tf2ss(num21,den21);
% R(jw)
[Ar,Br,Cr,Dr]=tf2ss(num1,den1);
Cd1=[1 0 0 0];
Dd1=0;
[n,m]=size(B);
[p,n]=size(C);
[n_q,m_q]=size(Bq);
[p_q,n_q]=size(Cq);
[n_r,m_r]=size(Br);
[p_r,n_r]=size(Cr);
Aag=[A zeros(n,n_q) zeros(n,n_r);
     Bq*Cd1 Aq zeros(n_q,n_r);
     zeros(n_r,n) zeros(n_r,n_q) Ar];
Bag=[B; zeros(n_q,m);Br];
Bw=[B; zeros(n_q,m);zeros(n_r,m)];
Cag=[C zeros(p,n_q) zeros(p,n_r)];
Qag=[[Cd1'*Dq';Cq']*[Dq*Cd1 Cq] zeros(n+n_q,n_r);
     zeros(n_r,n+n_q) Cr'*Cr];
Rag=Dr'*Dr;
Fag=lqr2(Aag,Bag,Qag,Rag);
Fb=Fag(1,1:4);
Fq=Fag(1,5:8);
Fr=Fag(1,9:12);
mag3=bode(ss(Aag-Bag*Fag,Bw,Cag,D),w);
mag3=20*log10(mag3(1,:));
mag4=bode(ss(A,B,C,D),w);
mag4=20*log10(mag4(1,:));
```

Table III M-file of designing LQG controller (fslqg.m)