Thesis for the Degree of Master of Education

Fuzzy Number Intuitionistic Judgment Matrix

by

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Graduate School of Education

Pukyong National University

August 2009

Fuzzy Number Intuitionistic Judgment Matrix
퍼지수 직관적 판단행렬

Advisor : Prof. Jin Han Park

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A thesis submitted in partial fulfillment of the requirement
for the degree of

Master of Education
Graduate School of Education
Pukyong National University

August 2009

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August 26, 2009

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요약

본 논문에서는 직관적 퍼지집합(intuitionistic fuzzy set)과 구간치 직관적 퍼지집합(interval-valued intuitionistic fuzzy set)을 소개하고 이들의 확장된 개념인 퍼지수 직관적 퍼지집합(fuzzy number intuitionistic fuzzy set)을 정의 하였다. 또한 그들의 연산 법칙과 스코어함수, 정확도 함수를 제시하고, 선호도 정보를 모으기 위한 집합연산자(FIFA, FIFWA, FIFOWA, FIFHA)를 소개하였다.

이를 기초로, 퍼지수 직관적 판단행렬(fuzzy number intuitionistic judgment matrix)과 그것의 스코어행렬(score matrix), 정확도행렬(accuracy matrix)과 같은 새로운 개념을 정의하고 그들의 기본적 성질을 밝힌다. 또한, 퍼지수 직관적 판단행렬을 이용하여 의사결정 문제 해결에의 응용과정과 그 예를 제공하였다.

Chapter 1

Introduction

In 1986, Atanassov [1, 2] introduced the concept of intuitionistic fuzzy set (IFS), which is characterized by a membership degree, nonmembership degree, and a hesitancy degree. Thus, it is more flexible and practical to deal with fuzziness and uncertainty [13] than the fuzzy set developed by Zadeh [32]. Since its appearance, the IFS theory has received more and more attention and has been used in a wide range of applications [6, 8], such as, logic programming [4], decision making [9, 12, 7, 27], medical diagnosis [10, 19, 26], pattern recognition [15, 17], and so on. Interval-valued intuitionistic fuzzy sets, introduced by Atanassov and Gargov [3], each of which is characterized by a membership function and a non-membership function whose values are intervals rather than exact numbers, are a very useful means to describe the decision information in the process of decision making. Some researcher have applied the interval-valued intuitionistic fuzzy set theory to the field of decision making. Xu and Chen [29] defined some operational laws of interval-valued intuitionistic fuzzy values and, based on these operational laws, developed some arithmetic aggregation operators, such as the interval-valued intuitionistic fuzzy weighted averaging (IIFWA) operator, the interval-valued intuitionistic fuzzy ordered weighted averaging (IIFOWA) operator and the interval-valued intuitionistic fuzzy hybrid aggregation (IIFHA) operator for aggregating interval-valued intuitionistic fuzzy information, and gave an application of the IIFHA operator to multiple attribute group decision making with interval-valued intuitionistic fuzzy information.

Recently, Liu and Yuan [16] also extended the IFS and introduced the fuzzy number intuitionistic fuzzy set (FNIFS) whose fundamental characteristic is that the values of its membership function and non-membership function are trigonometric fuzzy numbers rather than exact numbers. There is a little investigation on aggregation operators for aggregating fuzzy number intuitionistic fuzzy information. Wang [23] developed some geometric aggregation operator, such as the fuzzy number intuitionistic fuzzy weighted geometric (FIFWG) operator, the fuzzy number intuitionistic fuzzy ordered weighted geometric (FIFOWG) and the fuzzy number intuitionistic fuzzy hybrid geometric (FIFHG) operator, and applied them to multiple attribute decision making with fuzzy number intuitionistic fuzzy information. In [22], he also developed some arithmetic aggregation operators, such as the fuzzy number intuitionistic fuzzy weighted averaging (FIFWA) operator, the fuzzy number intuitionistic fuzzy ordered weighted averaging (FIFOWA) operator and the fuzzy number intuitionistic fuzzy hybrid aggregation (FIFHA) operator, and gave an application of the FIFHA operator to multiple attribute decision making with fuzzy number intuitionistic fuzzy information.

In this thesis, we have developed some operators for aggregating fuzzy number intuitionistic preference information and have defined some new concepts, such as, fuzzy number intuitionisitic judgment matrix, score matrices, and accuracy matrices of fuzzy number intuitionisitic judgment matrix, and so on, and then studied some of their desirable properties. Furthermore, we discuss the relations among fuzzy number intuitionistic judgment matrix, intuitionistic judgment matrix, and provide an approach to solving the group decision-making problems where the decision makers provide their preference information over alternatives by using fuzzy number intuitionistic judgment matrices.

Chapter 2

Preliminaries

The concept of intuitionistic fuzzy set was introduced by Atanassov [1, 2, 6] to deal with vagueness, which can be defined as follows.

Definition 2.0.1 Let X be the universe of discourse, then an intuitionistic fuzzy set (IFS) A in X is given by:

$$A = \{(x, \langle \mu_A(x), \nu_A(x) \rangle) : x \in X\}, \tag{2.1}$$

where $\mu_A : X \to [0, 1]$, $\nu_A : X \to [0, 1]$ with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$ for every $x \in X$. $\mu_A(x)$ and $\nu_A(x)$ denote the membership and non-membership degree of the element x to A.

Sometimes it is not approximate to assume that the membership degrees for certain elements of A are exactly defined, but a value range can be given. In such cases, Atanassov and Gargov [4] defined the notion of IVIFS as below.

Definition 2.0.2 An IVIFS $\tilde{A}$ in X is an object having the following form:

$$\tilde{A} = \{(x, \langle \tilde{\mu}_{\tilde{A}}(x), \tilde{\nu}_{\tilde{A}}(x) \rangle) : x \in X\}, \tag{2.2}$$

where $\tilde{\mu}_{\tilde{A}}(x) = (\tilde{\mu}^1_{\tilde{A}}(x), \tilde{\mu}^2_{\tilde{A}}(x)) \subset [0, 1]$ and $\tilde{\nu}_{\tilde{A}}(x) = (\tilde{\nu}^1_{\tilde{A}}(x), \tilde{\nu}^2_{\tilde{A}}(x)) \subset [0, 1]$ are intervals with $0 \le \tilde{\mu}^2_{\tilde{A}}(x) + \tilde{\nu}^2_{\tilde{A}}(x) \le 1$ for any $x \in X$.

However, interval lacks gravity center, cannot emphasis center point which is most impossible to be given value, and so it is more suitable that the degree of membership and the degree of non-membership are expressed by trigonometric fuzzy number [14]. In such cases, Liu and Yuan [16] defined the notion of FNIFS as follows.

Definition 2.0.3 The trigonometric fuzzy number [14] α on $I = [0, 1]$ denoted by $\alpha = (l, p, q)$ is fuzzy set with its membership function defined by

$$\mu_\alpha(x) = \begin{cases} \frac{x-l}{p-l}, & l \leq x \leq p, \\ \frac{x-q}{p-q}, & p \leq x \leq q, \\ 0, & \text{otherwise}, \end{cases} \tag{2.3}$$

where $x \in I$, $0 \leq l \leq p \leq q \leq 1$, l is lower limit of α, q is upper limit of α, and p is gravity center of α. The set of all trigonometric fuzzy numbers on I will be denoted $F(I)$.

On the basis of document [14], the mean of α is defined as follows:

$$E(\alpha)^{(\theta)} = \frac{(1-\theta)l + p + \theta q}{2}, \quad \theta \in [0, 1], \tag{2.4}$$

where θ is an index that reflects the decision-maker's risk-bearing attitude. If $\theta > 0.5$, then the decision-maker is a risk lover. If $\theta < 0.5$, then the decision maker is a risk averter. In general, let $\theta = 0.5$, then the attitude of the decision-maker is neutral to the risk, and

$$E(\alpha)^{(\theta)} = \frac{l + 2p + q}{4}. \tag{2.5}$$

Definition 2.0.4 A fuzzy number intuitionistic fuzzy set (FNIFS) [16] A in X is an object having the form:

$$\bar{A} = \{(x, \langle \bar{\mu}_{\bar{A}}(x), \bar{\nu}_{\bar{A}}(x) \rangle) : x \in X\}, \tag{2.6}$$

where $\bar{\mu}_{\bar{A}}(x) = (\bar{\mu}^1_{\bar{A}}(x), \bar{\mu}^2_{\bar{A}}(x), \bar{\mu}^3_{\bar{A}}(x))$ and $\bar{\nu}_A(x) = (\bar{\nu}^1_A(x), \bar{\nu}^2_A(x), \bar{\nu}^3_A(x))$ are two trigonometric fuzzy numbers satisfying the condition $0 \leq \bar{\mu}^3_A(x) + \bar{\nu}^3_A(x) \leq 1$

for every $x \in X$. Especially, if each of the trigonometric fuzzy numbers $\bar{\mu}_A(x)$ and $\bar{\nu}_A(x)$ contains exactly one element, then the given FNIFS is transformed to intuitionistic fuzzy set [3, 5].

For convenience, we call $\bar{\alpha} = \langle (a,b,c),(l,p,q) \rangle$ a FNIFN [23], where $(a,b,c) \in F(I),\ (l,p,q) \in F(I),\ 0 \le c+q \le 1$ and let Ω be the set of all FNIFNs.

Definition 2.0.5 Let $\bar{\alpha} = \langle (a,b,c),(l,p,q) \rangle$, $\bar{\alpha}_1 = \langle (a_1,b_1,c_1),\ (l_1,p_1,q_1) \rangle$ and $\bar{\alpha}_2 = \langle (a_2,b_2,c_2),(l_2,p_2,q_2) \rangle$ are three FNIFNs, Wang [22] defined two operational laws of FNIFNs defined as:

1. $\bar{\alpha}_1 \oplus \bar{\alpha}_2 = \langle (a_1+a_2-a_1a_2, b_1+b_2-b_1b_2, c_1+c_2-c_1c_2),(l_1l_2, p_1p_2, q_1q_2) \rangle$;
2. $\lambda\bar{\alpha} = \langle (1-(1-a)^\lambda, 1-(1-b)^\lambda, 1-(1-c)^\lambda, (l^\lambda, p^\lambda, q^\lambda)) \rangle$, $\lambda > 0$,

which can ensure that the operational results are also FNIFNs. Furthermore, we obtain the following relation:

1. $\bar{\alpha}_1 \oplus \bar{\alpha}_2 = \bar{\alpha}_2 \oplus \bar{\alpha}_1$;
2. $\lambda(\bar{\alpha}_1 \oplus \bar{\alpha}_2) = \lambda\bar{\alpha}_1 \oplus \lambda\bar{\alpha}_2$, $\lambda > 0$;
3. $(\lambda_1+\lambda_2)\bar{\alpha} = \lambda_1\bar{\alpha} \oplus \lambda_2\bar{\alpha}$, $\lambda_1, \lambda_2 > 0$.

Xu [30] and Wei and Wang [21] defined a score function and an accuracy function to measure an interval-valued intuitionistic fuzzy number, respectively. Based on these, Wang [22] defined a score function $\bar{s}$ to measure a FNIFN $\bar{\alpha}$ as follows:

Definition 2.0.6 Let $\bar{\alpha} = \langle (a,b,c),(l,p,q) \rangle$ be a FNIFN, then the score function $\bar{s}(\bar{\alpha})$ is defined:

$$\bar{s}(\bar{\alpha}) = \frac{a+2b+c}{4} - \frac{l+2p+q}{4}, \tag{2.7}$$

where $\bar{s}(\bar{\alpha}) \in [-1,1]$. The larger the value of $\bar{s}(\bar{\alpha})$, the higher the FNIFN $\bar{\alpha}$. Especially, if $\bar{s}(\bar{\alpha}) = 1$, then $\bar{\alpha} = \langle (1,1,1),(0,0,0) \rangle$, which is the largest FNIFN; if $\bar{s}(\bar{\alpha}) = -1$, then $\bar{\alpha} = \langle (0,0,0),(1,1,1) \rangle$, which is the smallest FNIFN.

Park et al. [18] defined an accuracy function to evaluate the accuracy degree of a FNIFN $\bar{\alpha}$ as follows:

Definition 2.0.7 Let $\bar{\alpha} = \langle (a, b, c), (l, p, q) \rangle$ be a FNIFN, then the accuracy function $\bar{h}(\bar{\alpha})$ is defined:

$$\bar{h}(\bar{\alpha}) = \frac{a + 2b + c}{4} + \frac{l + 2p + q}{4}, \tag{2.8}$$

where $\bar{h}(\bar{\alpha}) \in [0, 1]$. The larger the value of $\bar{h}(\bar{\alpha})$, the higher the degree of accuracy of $\bar{\alpha}$.

Moreover, Park et al. [18] also defined a method to compare two FNIFNs, which is based on the score function and the accuracy function:

Definition 2.0.8 Let

$$\bar{s}(\bar{\alpha}_1) = \frac{a_1 + 2b_1 + c_1}{4} - \frac{l_1 + 2p_1 + q_1}{4}, \ \bar{s}(\bar{\alpha}_2) = \frac{a_1 + 2b_2 + c_2}{4} - \frac{l_2 + 2p_2 + q_2}{4}$$

$$\bar{h}(\bar{\alpha}_1) = \frac{a_1 + 2b_1 + c_1}{4} + \frac{l_1 + 2p_1 + q_1}{4}, \ \bar{h}(\bar{\alpha}_2) = \frac{a_1 + 2b_2 + c_2}{4} + \frac{l_2 + 2p_2 + q_2}{4}$$

be the score and accuracy degrees of $\bar{\alpha}_1$, $\bar{\alpha}_2$, respectively, then

- if $\bar{s}(\bar{\alpha}_1) < \bar{s}(\bar{\alpha}_2)$, then $\bar{\alpha}_1$ is smaller than $\bar{\alpha}_2$, denoted by $\bar{\alpha}_1 < \bar{\alpha}_2$;
- if $\bar{s}(\bar{\alpha}_1) = \bar{s}(\bar{\alpha}_2)$, then
  1) if $\bar{h}(\bar{\alpha}_1) = \bar{h}(\bar{\alpha}_2)$, then $\bar{\alpha}_1$ and $\bar{\alpha}_2$ represent the same information, i.e, $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $l_1 = l_2$, $p_1 = p_2$ and $q_1 = q_2$, denoted by $\bar{\alpha}_1 = \bar{\alpha}_2$;
  2) if $\bar{h}(\bar{\alpha}_1) < \bar{h}(\bar{\alpha}_2)$, then $\bar{\alpha}_1$ is smaller than $\bar{\alpha}_2$, denoted by $\bar{\alpha}_1 < \bar{\alpha}_2$.

Chapter 3

Fuzzy number intuitionistic fuzzy aggregation operators

Up to now, many operators have been proposed for aggregating information. The weighted averaging (WA) [11] operator and the ordered weighted averaging (OWA) [31] operator are the most common operators for aggregating arguments. Let $a_j (j = 1, 2, \cdots, n)$ be a collection of real numbers, $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $a_j (j = 1, 2, \cdots, n)$, with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then a WA operator is defined as [11]:

$$\mathrm{WA}_w(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} w_j a_j.$$

An OWA operator [31] of dimension n is a mapping OWA: $R^n \to R$, that has an associated vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$, such that $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$. Furthermore,

$$\mathrm{OWA}_w(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} \omega_j b_j,$$

where b_j is the jth largest of $a_j (j = 1, 2, \cdots, n)$.

To aggregate the fuzzy number intuitionistic fuzzy information, in what follows, the concepts of the weighted averaging (FIFWA) operator, ordered weighted averaging (FIFOWA) operator and hybrid aggregation (FIFHA) operator of the FNIFNs are defined:

Definition 3.0.9 Let $\bar{\beta}_j = \langle (a_j, b_j, c_j), (l_j, p_j, q_j) \rangle (j = 1, 2, \cdots, n)$ be a collection of FNIFNs. A fuzzy number intuitionistic fuzzy weighted averaing (FIFWA) operator of dimension n is a mapping FIFWA: $\Omega^n \to \Omega$, which has the weight vector $w = (w_1, w_2, \cdots, w_n)^T$, $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$. Furthermore,

$$\mathrm{FIFWA}_w(\bar{\beta}_1, \bar{\beta}_2, \cdots, \bar{\beta}_n) = w_1 \bar{\beta}_{\sigma(1)} \oplus w_2 \bar{\beta}_{\sigma(2)} \oplus \cdots \oplus w_n \bar{\beta}_{\sigma(n)}. \tag{3.1}$$

Especially, if $w = (\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})^T$, then the FIFWA operator is reduced to a FIFA operator of dimension n:

$$\mathrm{FIFA}_w(\bar{\beta}_1, \bar{\beta}_2, \cdots, \bar{\beta}_n) = \frac{1}{n}(\bar{\beta}_1 \oplus \bar{\beta}_2 \oplus \cdots \oplus \bar{\beta}_n).$$

Theorem 3.0.10 *Let* $\bar{\beta}_j = \langle (a_j, b_j, c_j), (l_j, p_j, q_j) \rangle (j = 1, 2, \cdots, n)$ *be a collection of* FNIFNs, *then their aggregated value by using the* FIFWA *operator is also a* FNIFN *and*

$$\begin{aligned} \mathrm{FIFWA}_w(\bar{\beta}_1, \bar{\beta}_2, \cdots, \bar{\beta}_n) = \langle (1 - \prod_{j=1}^n (1 - a_j)^{w_j}, 1 - \prod_{j=1}^n (1 - b_j)^{w_j}, \\ 1 - \prod_{j=1}^n (1 - c_j)^{w_j}), (\prod_{j=1}^n l_j^{w_j}, \prod_{j=1}^n p_j^{w_j}, \prod_{j=1}^n q_j^{w_j}) \rangle, \end{aligned} \tag{3.2}$$

where $w = (w_1, w_2, \cdots, w_n)^T$ *is the weighting vector of* $\bar{\beta}_j (j = 1, 2, \cdots, n)$ *, with* $w \in [0, 1]$ *and* $\sum_{j=1}^n w_j = 1$.

Definition 3.0.11 Let $\bar{\beta}_j = \langle (a_j, b_j, c_j), (l_j, p_j, q_j) \rangle (j = 1, 2, \cdots, n)$ be a collection of FNIFNs. A fuzzy number intuitionistic fuzzy ordered weighted averaing (FIFOWA) operator of dimension n is a mapping FIFOWA: $\Omega^n \to \Omega$, which has an associated vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$. Furthermore,

$$\mathrm{FIFOWA}_\omega(\bar{\beta}_1, \bar{\beta}_2, \cdots, \bar{\beta}_n) = \omega_1 \bar{\beta}_{\sigma(1)} \oplus \omega_2 \bar{\beta}_{\sigma(2)} \oplus \cdots \oplus \omega_n \bar{\beta}_{\sigma(n)}, \tag{3.3}$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$ such that $\bar{\beta}_{\sigma(j-1)} \geq \bar{\beta}_{\sigma(j)}$ for all j.

Theorem 3.0.12 *Let $\bar{\beta}_j = \langle (a_j, b_j, c_j), (l_j, p_j, q_j) \rangle (j = 1, 2, \cdots, n)$ be a collection of* FNIFNs*, then their aggregated value by using the* FIFOWA *operator is also a* FNIFN *and*

$$\mathrm{FIFOWA}_{\omega}(\bar{\beta}_1, \bar{\beta}_2, \cdots, \bar{\beta}_n) = \langle (1 - \prod_{j=1}^{n} (1 - a_{\sigma(j)})^{\omega_j}, 1 - \prod_{j=1}^{n} (1 - b_{\sigma(j)})^{\omega_j},$$
$$1 - \prod_{j=1}^{n} (1 - c_{\sigma(j)})^{\omega_j}), (\prod_{j=1}^{n} l_{\sigma(j)}^{\omega_j}, \prod_{j=1}^{n} p_{\sigma(j)}^{\omega_j}, \prod_{j=1}^{n} q_{\sigma(j)}^{\omega_j}) \rangle, \quad (3.4)$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weighting vector of the FIFOWA *operator, with $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$,*

which can be determined by using the normal distribution based method [25]. The FIFOWA weights can be determined similar to the OWA weights. for example, we can use the normal distribution based method to the FIFOWA weights [25].

Consider that FIFWA operator weights only the FNIFNs, while the FIFOWA operator weights only the ordered positions of the FNIFNs instead of weighting the FNIFNs themselves. To overcome this limitation, motivated by the idea of combining the WA and the OWA operator [24, 20], in what follows, we develop a fuzzy number intuitionistic fuzzy hybrid aggregation (FIFHA) operator.

Definition 3.0.13 Let $\bar{\beta}_j = \langle (a_j, b_j, c_j), (l_j, p_j, q_j) \rangle (j = 1, 2, \cdots, n)$ be a collection of FNIFNs. A fuzzy number intuitionistic fuzzy hybrid aggregation (FIFHA) operator of dimension n is a mapping FIFHA: $\Omega^n \to \Omega$, which has an associated vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$. Furthermore,

$$\mathrm{FIFHA}_{w,\omega}(\bar{\beta}_1, \bar{\beta}_2, \cdots, \bar{\beta}_n) = \omega_1 \dot{\bar{\beta}}_{\sigma(1)} \oplus \omega_2 \dot{\bar{\beta}}_{\sigma(2)} \oplus \cdots \oplus \omega_n \dot{\bar{\beta}}_{\sigma(n)}, \quad (3.5)$$

where $\dot{\bar{\beta}}_{\sigma(j)}$ is the jth largest of the weighted FNIFNs $\dot{\bar{\beta}}_j (\dot{\bar{\beta}}_j = n w_j \bar{\beta}_j,\ j = 1, 2, \cdots, n,\ w = (w_1, w_2, \cdots, w_n)^T$ is the weight vector of $\bar{\beta}_j (j = 1, 2, \cdots, n)$ with $w \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1)$, and n is the balancing coefficient, which plays a role of balance. in this case, if the vector $w = (w_1, w_2, \cdots, w_n)^T$ approaches $(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})^T$, then the vector $(n w_1 \bar{\beta}_1, n w_2 \bar{\beta}_2, \cdots, n w_n \bar{\beta}_n)$ approaches $(\bar{\beta}_1,\ \bar{\beta}_2, \cdots, \bar{\beta}_n)$.

Theorem 3.0.14 *Let $\bar{\beta}_j = \langle (a_j, b_j, c_j), (l_j, p_j, q_j) \rangle (j = 1, 2, \cdots, n)$ be a collection of* FNIFNs*, then their aggregated value by using the* FIFHA *operator is also a* FNIFN *and satisfies:*

$$\begin{aligned} \mathrm{FIFHA}_{w,\omega}(\bar{\beta}_1, \bar{\beta}_2, \cdots, \bar{\beta}_n) = \langle (1 - \prod_{j=1}^{n} (1 - \dot{a}_{\sigma(j)})^{\omega_j}, 1 - \prod_{j=1}^{n} (1 - \dot{b}_{\sigma(j)})^{\omega_j}, \\ 1 - \prod_{j=1}^{n} (1 - \dot{c}_{\sigma(j)})^{\omega_j}), (\prod_{j=1}^{n} \dot{l}_{\sigma(j)}^{\omega_j}, \prod_{j=1}^{n} \dot{p}_{\sigma(j)}^{\omega_j}, \prod_{j=1}^{n} \dot{q}_{\sigma(j)}^{\omega_j}) \rangle, \end{aligned} \tag{3.6}$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weighting vector of the FIFHA *operator, with $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$, $\dot{\bar{\beta}}_{\sigma(j)}$ is the jth largest of the weighted* FNIFNs *$\dot{\bar{\beta}}_j (\dot{\bar{\beta}}_j = n w_j \bar{\beta}_j$, $j = 1, 2, \cdots, n$, $w = (w_1, w_2, \cdots, w_n)^T$ is the weight vector of $\bar{\beta}_j (j = 1, 2, \cdots, n)$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$), and n is the balancing coefficient.*

Chapter 4

Fuzzy number intuitionistic judgment matrix and decision-making approach

4.1 Fuzzy number intuitionistic judgment matrix

In many practical decision-making problems, decision makers are usually needed to provide their preferences over alternatives to a certain degree, however, it is possible that they are not so sure about it. In such cases, it is suitable and convenient to express the decision makers preferences in FNIFNs. In this chapter, the concept of the fuzzy number intuitionistic judgment matrix (whose elements are FNIFNs) will be defined, and some of its desirable properties studied in detail. For a decision-making problem, let $X = \{x_1, x_2, \cdots, x_n\}$ be the set of alternatives. The decision maker compares each pair of alternatives, and constructs a judgment matrix. In the following, initially the concept of intuitionistic judgment matrix is defined:

Definition 4.1.1 Let $C = (c_{ij})_{n\times n}$ be a judgment matrix, where $c_{ij} = (\mu_{ij}, \nu_{ij})$,

$i, j = 1, 2, \cdots, n$, μ_{ij} denotes the preference degree of the alternative x_i over x_j given by the decision maker, whereas, v_{ij} denotes the preference degree of the alternative x_j over x_i given by the decision maker, $1 - \mu_{ij} - \nu_{ij}$ denotes the uncertain degree and hesitation degree. If

$$\mu_{ij} \in [0,1],\ \nu_{ij} \in [0,1],\ \mu_{ji} = \nu_{ij},\ \nu_{ji} = \mu_{ij},$$
$$\mu_{ii} = \nu_{ii} = 0.5,\ \mu_{ij} + \nu_{ij} \leq 1,\ i, j = 1, 2, \cdots, n,$$

then C is called the intuitionistic judgment matrix.

Especially, if $1 - \mu_{ij} - \nu_{ij} = 0$, for all i, j, then the intuitionistic judgment matrix C can be decomposed into the following two complement judgment matrices formally : $C_1 = (\mu_{ij})_{n\times n}$ and $C_2 = (\nu_{ij})_{n\times n}$, where $\mu_{ij}, \nu_{ij} \in [0,1]$, $\mu_{ij} + \mu ji = 1$, $\nu_{ij} + \nu_{ji} = 1$, $\mu_{ii} = 0.5$, $\nu_{ii} = 0.5$, and $i, j = 1, 2, \cdots n$.

The increasing complexity of the real world and the fuzziness of human thinking make it difficult to express the decision makers preferences over alternatives in exact numerical values, however, it is very convenient and suitable to express the preferences with FNIFNs.

Definition 4.1.2 Let $\bar{C} = (\bar{c}_{ij})_{n\times n}$ be a judgment matrix, where $\bar{c}_{ij} = (\bar{\mu}_{ij}, \bar{\nu}_{ij})$ is an FNIFN, $i, j = 1, 2, \cdots, n$, $\bar{\mu}_{ij}$ denotes the preference range of the alternative x_i over x_j given by the decision maker, whereas, $\bar{\nu}_{ij}$ denotes the preference range of the alternative x_j over x_i given by the decision maker, and satisfies

$$\bar{\mu}_{ij} = (\bar{a}_{ij}, \bar{b}_{ij}, \bar{c}_{ij}) \in F(I),\ \bar{\nu}_{ij} = (\bar{l}_{ij}, \bar{p}_{ij}, \bar{q}_{ij}) \in F(I),$$
$$\bar{\mu}_{ji} = \bar{\nu}_{ij},\ \bar{\nu}_{ji} = \bar{\mu}_{ij},\ \bar{\mu}_{ii} = \bar{\nu}_{ii} = (0.5, 0.5, 0.5),$$
$$\bar{c}_{ij} + \bar{q}_{ij} \leq 1,\ i, j = 1, 2, \cdots, n,$$

where $\bar{a}_{ij}$ and $\bar{c}_{ij}$ denote the lower and upper limits of $\bar{\mu}_{ij}$ respectively, $\bar{l}_{ij}$ and $\bar{q}_{ij}$ denote the lower and upper limits of $\bar{\nu}_{ij}$ respectively, $\bar{b}_{ij}$ and $\bar{p}_{ij}$ denote the gravity center of $\bar{\mu}_{ij}$ and $\bar{\nu}_{ij}$ respectively, then $\bar{C}$ is called the fuzzy number intuitionistic judgment matrix.

Especially, if $\bar{a}_{ij} = \bar{b}_{ij} = \bar{c}_{ij}$, $\bar{l}_{ij} = \bar{p}_{ij} = \bar{q}_{ij}$ for all i, j, k, then the fuzzy number intuitionistic judgment matrix $\bar{C}$ is reduced to an intuitionistic judgment matrix. Therefore, the intuitionistic judgment matrix is the special case of fuzzy number intuitionistic judgment matrix.

Definition 4.1.3 Let $\bar{C} = (\bar{c}_{ij})_{n\times n}$ be a fuzzy number intuitionistic judgment matrix, if

$$\bar{c}_{ij} = \bar{c}_{ik} \cdot \bar{c}_{kj} \ for\ all\ i, j, k,$$

then $\bar{C}$ is called the consistent fuzzy number intuitionistic judgment matrix.

Definition 4.1.4 Let $\bar{C} = (\bar{c}_{ij})_{n\times n}$ be a fuzzy number intuitionistic judgment matrix, where $\bar{c}_{ij} = (\bar{\mu}_{ij}, \bar{\nu}_{ij})$, $\bar{\mu}_{ij} = (\bar{a}_{ij}, \bar{b}_{ij}, \bar{c}_{ij}) \in F(I)$, $\bar{\nu}_{ij} = (\bar{l}_{ij}, \bar{p}_{ij}, \bar{q}_{ij}) \in F(I)$ $i, j = 1, 2, \cdots, n$, then $\bar{S} = (\bar{s}_{ij})_{n\times n}$ is called the score matrix of $\bar{C}$. where $\bar{s}_{ij}$ is derived by the score function, that is,

$$\bar{s}_{ij} = \frac{\bar{a}_{ij} + 2\bar{b}_{ij} + \bar{c}_{ij}}{4} - \frac{\bar{l}_{ij} + 2\bar{p}_{ij} + \bar{q}_{ij}}{4},\ i, j = 1, \cdots, n.$$

Definition 4.1.5 Let $\bar{C} = (\bar{c}_{ij})_{n\times n}$ be a fuzzy number intuitionistic judgment matrix, where $\bar{c}_{ij} = (\bar{\mu}_{ij}, \bar{v}_{ij})$, $\bar{\mu}_{ij} \in F(I)$, $\bar{\nu}_{ij} \in F(I)$, $i, j = 1, 2, \cdots, n$, then $\bar{H} = (\bar{h}_{ij})_{n\times n}$ is called the accuracy matrix of $\bar{C}$. where $\bar{h}_{ij}$ is derived by the accuracy function, that is,

$$\bar{h}_{ij} = \frac{\bar{a}_{ij} + 2\bar{b}_{ij} + \bar{c}_{ij}}{4} + \frac{\bar{l}_{ij} + 2\bar{p}_{ij} + \bar{q}_{ij}}{4},\ i, j = 1, \cdots, n.$$

Theorem 4.1.6 *Let $\bar{C}$ be a fuzzy number intuitionistic judgment matrix, $\bar{S}$ be the score matrix of $\bar{C}$ and $\bar{S}^T$ the transpose of $\bar{S}$, then*

$$\bar{S}^T = -\bar{S}$$

that is, $\bar{S}$ is an antisymmetric matrix.

Proof By Definitions (4.2.1) and (4.2.4), it follows that,

$$\bar{s}_{ij} = \frac{\bar{a}_{ij} + 2\bar{b}_{ij} + \bar{c}_{ij}}{4} - \frac{\bar{l}_{ij} + 2\bar{p}_{ij} + \bar{q}_{ij}}{4},\ \bar{s}_{ji} = \frac{\bar{a}_{ji} + 2\bar{b}_{ji} + \bar{c}_{ji}}{4} - \frac{\bar{l}_{ji} + 2\bar{p}_{ji} + \bar{q}_{ji}}{4},$$

$i, j = 1, \cdots, n$, and then, $\bar{s}_{ij} + \bar{s}_{ji} = 0$. thus $\bar{S}^T = -\bar{S}$ □

Theorem 4.1.7 *Let $\bar{C}$ be a fuzzy number intuitionistic judgment matrix, $\bar{H}$ be the accuracy matrix of $\bar{C}$ and $\bar{H}^T$ the transpose of $\bar{H}$, then*

$$\bar{H}^T = \bar{H}$$

that is, $\bar{H}$ is an symmetric matrix.

Proof By Definitions (4.2.1) and (4.2.5), it follows that,

$$\bar{h}_{ij} = \frac{\bar{a}_{ij} + 2\bar{b}_{ij} + \bar{c}_{ij}}{4} + \frac{\bar{l}_{ij} + 2\bar{p}_{ij} + \bar{q}_{ij}}{4},\ \bar{s}_{ji} = \frac{\bar{a}_{ji} + 2\bar{b}_{ji} + \bar{c}_{ji}}{4} + \frac{\bar{l}_{ji} + 2\bar{p}_{ji} + \bar{q}_{ji}}{4},$$

$i, j = 1, \cdots, n$, and then, $\bar{h}_{ij} = \bar{h}_{ji}$. thus $\bar{H}^T = \bar{H}$ □

fuzzy number intuitionistic judgment matrices have the following properties:

Property 4.1.8 Let $\bar{C} = (\bar{c}_{ij})_{n\times n}$ be a fuzzy number intuitionistic judgment matrix, then the submatrix derived from deleting one line and the corresponding column from $\bar{C}$ is also a fuzzy number intuitionistic judgment matrix.

Property 4.1.9 Let $\bar{C} = (\bar{c}_{ij})_{n\times n}$ be a fuzzy number intuitionistic judgment matrix, where $\bar{c}_{ij} = (\bar{\mu}_{ij}, \bar{\nu}_{ij})$, $i, j = 1, 2, \cdots, n$, then

(1) If $\bar{c}_{ik} + \bar{c}_{kj} \geq \bar{c}_{ij}$, for all i, j, k, then it can be said that $\bar{C}$ satisfies the triangular condition.

(2) If $\bar{c}_{ik} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$ for all i, j, k, it follows that $\bar{c}_{kj} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle \Rightarrow \bar{c}_{ij} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$ then it can be

said that $\bar{C}$ satisfies the weak transitivity property.

(3) If $\bar{c}_{ij} \geq min\{\bar{c}_{ik}, \bar{c}_{kj}\}$, for all i, j, k, then it can be said that $\bar{C}$ satisfies the max-min transitivity property.

(4) If $\bar{c}_{ij} \geq max\{\bar{c}_{ik}, \bar{c}_{kj}\}$, for all i, j, k, then it can be said that $\bar{C}$ satisfies the max-max transitivity property.

(5) $\bar{c}_{ik} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$ for all i, j, k, it follows that $\bar{c}_{kj} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle \Rightarrow \bar{c}_{ij} \geq min\{\bar{c}_{ik}, \bar{c}_{kj}\}$, then it can be said that $\bar{C}$ satisfies the strict max-min transitivity property.

(6) $\bar{c}_{ik} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$ for all i, j, k, it follows that $\bar{c}_{kj} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle \Rightarrow \bar{c}_{ij} \geq max\{\bar{c}_{ik}, \bar{c}_{kj}\}$, then it can be said that $\bar{C}$ satisfies the strict max-max transitivity property.

(7) If min $\{\bar{c}_{ik}, \bar{c}_{kj}\} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle \Rightarrow \bar{c}_{ij} \geq$ max $\{\bar{c}_{ik}, \bar{c}_{kj}\}$, for all i, j, k then it can be said that $\bar{C}$ satisfies the strong stochastic transitivity property.

(8) If min $\{\bar{c}_{ik}, \bar{c}_{kj}\} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle \Rightarrow \bar{c}_{ij} \geq min\{\bar{c}_{ik}, \bar{c}_{kj}\}$, for all i, j, kthen it can be said that $\bar{C}$ satisfies the moderate stochastic transitivity property.

(9) If min$\{\bar{c}_{ik}, \bar{c}_{kj}\} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle, \Rightarrow \bar{c}_{ij} \geq \langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$ for all i, j, k, then it can be said that $\bar{C}$ satisfies the weak stochastic transitivity property.

4.2 Decision-making approach

On the basis of the earlier theoretic analysis, in what follows, an approach to solving the group decision-making problems is developed, where the decision makers provide their preferences over alternatives in the form of an fuzzy number intuitionistic judgment matrix, which involves the following steps:

Step 1. For a group decision-making problem, let $Y = \{y_1, y_2, \cdots, y_n\}$ be the set of alternatives, $D = \{d_1, d_2, \cdots, d_m\}$ be the set of decision makers, $\omega = (\omega_1, \omega_2, \cdots, \omega_m)^T$ be the weight vector of decision makers $d_k(k = 1, 2, \cdots, m)$, $\omega_k \geq 0$, $\sum_{j=1}^{n} \omega_j = 1$. The decision maker d_k compares each pair of alternatives $y_i(i = 1, 2, \cdots, n)$, and constructs an fuzzy number intuitionistic judgment matrix $\bar{C}_{(k)} = (\bar{c}_{ij}^{(k)})_{n\times n}$, where

$$\bar{c}_{ij}^{(k)} = \langle \bar{\mu}_{ij}^{(k)}, \bar{\nu}_{ij}^{(k)} \rangle,\ \bar{\mu}_{ij}^{(k)} \in F(I),\ \bar{\nu}_{ij}^{(k)} \in F(I),$$
$$\bar{\mu}_{ji}^{(k)} = \bar{\nu}_{ij}^{(k)},\ \bar{\nu}_{ji}^{(k)} = \bar{\mu}_{ij}^{(k)},\ \bar{\mu}_{ii}^{(k)} = \bar{\nu}_{ii}^{(k)} = (0.5, 0.5, 0.5),$$
$$\bar{c}_{ij} + \bar{q}_{ij} \leq 1,\ i, j = 1, 2, \cdots, n,\ k = 1, 2, \cdots, m$$

Step 2. Utilize the FIFA operator:

$$\bar{c}_i^{(k)} = \mathrm{FIFA}(\bar{c}_{i1}^{(k)}, \bar{c}_{i2}^{(k)}, \cdots, \bar{c}_{in}^{(k)}),\ i = 1, 2, \cdots, n,\ k = 1, 2, \cdots, m \tag{4.1}$$

to aggregate all the elements in each line of $\bar{C}_{(k)}$ so as to get the overall fuzzy number intuitionistic information.

Step 3. Utilize the FIFHA operator:

$$\bar{c}_i = \mathrm{FIFHA}_{\omega,w}(\bar{c}_i^{(1)}, \bar{c}_i^{(2)}, \cdots, \bar{c}_i^{(m)}) \tag{4.2}$$

to aggregate $\bar{c}_i^{(k)}(k = 1, 2, \cdots m)$, and then get the overall FNIFNs $\bar{c}_i(i = 1, 2, \cdots, n)$ of the alternatives $y_i(i = 1, 2, \cdots, n)$ provided by all the decision makers, where ω is the weight vector of the decision makers $d_k(k = 1, 2, \cdots, m)$, ω is the weight vector of the FIFHA operator.

Step 4. Calculate the scores $\bar{s}(\bar{c}_i)(i = 1, 2, \cdots, n)$ and the accuracy values $\bar{h}(\bar{c}_i)(i = 1, 2, \cdots, n)$ of $\bar{c}_i(i = 1, 2, \cdots, n)$ respectively.

Step 5. Utilize the scores $\bar{s}(\bar{c}_i)(i = 1, 2, \cdots, n)$ and the accuracy values $\bar{h}(\bar{c}_i)(i = 1, 2, \cdots, n)$ to rank the alternatives $y_i(i = 1, 2, \cdots, n)$, and then to select the optimal one.

4.3 Practical example

In a supply chain management, the enterprise usually needs to establish a partnership to (1) lower the total cost of supply chain; (2) lower inventory of enterprises; (3) enhance information sharing of enterprises; (4) improve the interaction of enterprises; and (5) obtain more competitive advantages for enterprises. There are many factors that can influence the cooperation among enterprises, in the process of selecting a partner for an enterprise, the following four factors are usually mainly considered: (1) y_1: respond time (delivery time) and supply capacity; (2) y_2: quality and technology level; (3) y_3: price and cost; and (4) y_4: service level. Suppose three decision makers d_k $(k = 1, 2, 3)$ (whose weight vector is $\omega = (0.35, 0.35, 0.30)^T$ are asked to provide their preferences over the factors y_i $(i = 1, 2, 3, 4)$. The decision makers d_k $(k = 1, 2, 3)$ compare each pair of the factors y_i $(i = 1, 2, 3, 4)$, and construct, the following three fuzzy number intuitionistic judgment matrices, respectively:

$$\bar{C}_1 = \begin{bmatrix} < (0.5,0.5,0.5), & < (0.3,0.4,0.5), & < (0.4,0.5,0.6), & < (0.1,0.2,0.3), \\ (0.5,0.5,0.5) > & (0.3,0.3,0.4) > & (0.2,0.3,0.4) > & (0.5,0.6,0.7) > \\ < (0.3,0.3,0.4), & < (0.5,0.5,0.5), & < (0.5,0.6,0.7), & < (0.6,0.6,0.6), \\ (0.3,0.4,0.5) > & (0.5,0.5,0.5) > & (0.2,0.2,0.3) > & (0.1,0.2,0.3) > \\ < (0.2,0.3,0.4), & < (0.2,0.2,0.3), & < (0.5,0.5,0.5), & < (0.4,0.6,0.7), \\ (0.4,0.5,0.6) > & (0.5,0.6,0.7) > & (0.5,0.5,0.5) > & (0.1,0.2,0.2) > \\ < (0.5,0.6,0.7), & < (0.1,0.2,0.3), & < (0.1,0.2,0.2), & < (0.5,0.5,0.5), \\ (0.1,0.2,0.3) > & (0.6,0.6,0.6) > & (0.4,0.6,0.7) > & (0.5,0.5,0.5) > \end{bmatrix}$$

$$\bar{C}_2 = \begin{bmatrix} <(0.5,0.5,0.5), & <(0.3,0.4,0.6), & <(0.4,0.5,0.6), & <(0.7,0.7,0.7), \\ (0.5,0.5,0.5)> & (0.2,0.3,0.4)> & (0.3,0.3,0.3)> & (0.1,0.2,0.3)> \\ <(0.2,0.3,0.4), & <(0.5,0.5,0.5), & <(0.7,0.7,0.8), & <(0.6,0.6,0.7), \\ (0.3,0.4,0.6)> & (0.5,0.5,0.5)> & (0.1,0.2,0.2)> & (0.1,0.2,0.3)> \\ <(0.3,0.3,0.3), & <(0.1,0.2,0.2), & <(0.5,0.5,0.5), & <(0.2,0.3,0.4), \\ (0.4,0.5,0.6)> & (0.7,0.7,0.8)> & (0.5,0.5,0.5)> & (0.1,0.2,0.3)> \\ <(0.1,0.2,0.3), & <(0.1,0.2,0.3), & <(0.1,0.2,0.3), & <(0.5,0.5,0.5), \\ (0.7,0.7,0.7)> & (0.6,0.6,0.7)> & (0.2,0.3,0.4)> & (0.5,0.5,0.5)> \end{bmatrix}$$

$$\bar{C}_3 = \begin{bmatrix} <(0.5,0.5,0.5), & <(0.6,0.7,0.7), & <(0.4,0.5,0.6), & <(0.3,0.5,0.6), \\ (0.5,0.5,0.5)> & (0.1,0.2,0.3)> & (0.2,0.3,0.4)> & (0.2,0.3,0.4)> \\ <(0.1,0.2,0.2), & <(0.5,0.5,0.5), & <(0.4,0.6,0.6), & <(0.5,0.6,0.7), \\ (0.6,0.7,0.7)> & (0.5,0.5,0.5)> & (0.1,0.1,0.2)> & (0.1,0.2,0.3)> \\ <(0.2,0.3,0.4), & <(0.1,0.1,0.2), & <(0.5,0.5,0.5), & <(0.2,0.3,0.4), \\ (0.4,0.5,0.6)> & (0.4,0.6,0.6)> & (0.5,0.5,0.5)> & (0.4,0.5,0.6)> \\ <(0.2,0.3,0.4), & <(0.1,0.2,0.3), & <(0.4,0.5,0.6), & <(0.5,0.5,0.5), \\ (0.3,0.5,0.6)> & (0.5,0.6,0.7)> & (0.2,0.3,0.4)> & (0.5,0.5,0.5)> \end{bmatrix}$$

On the basis of the data above, in what follows, the approach developed in this article is utilized, to find the most influential factor from $y_i(i = 1, 2, 3, 4)$. The authors first utilize the FIFA operator(4.1) to aggregate all the elements of each line in the fuzzy number intuitionistic judgment matrix $\bar{C}_k$ to obtain the overall fuzzy number intuitionistic information corresponding to each alternative given by the decision maker d_k:

$$\bar{C}_1(1) = \Big[<0.3407, 0.4114, 0.4856>, <0.3500, 0.4054, 0.4865> \Big],$$

$$\bar{C}_2(1) = \Big[<0.4856, 0.5135, 0.5644>, <0.2340, 0.2991, 0.3873> \Big],$$

$$\bar{C}_3(1) = \Big[<0.3381, 0.4215, 0.4990>, <0.3162, 0.4162, 0.4527> \Big],$$

$$\bar{C}_4(1) = \Big[<0.3292, 0.4019, 0.4616>, <0.3310, 0.4356, 0.5010> \Big],$$

$$\bar{C}_1(2) = \Big[< 0.4990, 0.5374, 0.6064 >, < 0.2340, 0.3080, 0.3663 > \Big],$$

$$\bar{C}_2(2) = \Big[< 0.5319, 0.5473, 0.6337 >, < 0.1968, 0.2991, 0.3663 > \Big],$$

$$\bar{C}_3(2) = \Big[< 0.2915, 0.3346, 0.3598 >, < 0.3440, 0.4325, 0.5180 > \Big],$$

$$\bar{C}_4(2) = \Big[< 0.2230, 0.2887, 0.3565 >, < 0.4527, 0.5010, 0.5595 > \Big],$$

$$\bar{C}_1(3) = \Big[< 0.4616, 0.5599, 0.6064 >, < 0.2115, 0.3080, 0.3936 > \Big],$$

$$\bar{C}_2(3) = \Big[< 0.3938, 0.4970, 0.5319 >, < 0.2340, 0.2893, 0.3807 > \Big],$$

$$\bar{C}_3(3) = \Big[< 0.2674, 0.3147, 0.3840 >, < 0.4229, 0.5233, 0.5733 > \Big],$$

$$\bar{C}_4(3) = \Big[< 0.3183, 0.3883, 0.4616 >, < 0.3500, 0.4606, 0.5384 > \Big].$$

Then, they utilize the FIFHA operator(4.2) (the weight vector derived by the normal distribution based method), which is $w = (0.243, 0.514, 0.243)^T$, to aggregate $\bar{c}(k)(k = 1, 2, 3)$, so as to get the overall FNIFNs $\bar{c}_i(i = 1, 2, 3, 4)$ of the alternatives $y_i(i = 1, 2, 3, 4)$ given by all the decision makers $d_k(k = 1, 2, 3)$:

$$\bar{C}_1 = \Big[< 0.5502, 0.4870, 0.4212 >, < 0.2487, 0.3251, 0.3946 > \Big],$$

$$\bar{C}_2 = \Big[< 0.5257, 0.4792, 0.4248 >, < 0.2195, 0.2921, 0.3644 > \Big],$$

$$\bar{C}_3 = \Big[< 0.6895, 0.6262, 0.5648 >, < 0.3390, 0.4367, 0.4881 > \Big],$$

$$\bar{C}_4 = \Big[< 0.7299, 0.6604, 0.5919 >, < 0.3909, 0.4702, 0.5352 > \Big].$$

Finally, the scores of $c_i(i = 1, 2, 3, 4)$ are calculated:

$$\bar{s}(\bar{C}_1) = 0.1541,$$

$$\bar{s}(\bar{C}_2) = 0.2270,$$

$$\bar{s}(\bar{C}_3) = -0.1115,$$

$$\bar{s}(\bar{C}_4) = -0.1361.$$

then the ranking of the factors $y_i(i = 1, 2, 3, 4)$ is as follows: $y_2 > y_1 > y_3 > y_4$. thus, the most influential factor is y_2. In the above example, we have utilized the FIFHA operator to aggregate the preference information provided by all the decision makers, and used the normal distribution based method to derive the weight vector of the FIFHA operator, which can relieve the influence of unfair arguments on the final results, by assigning low weights to the unduly high or unduly low ones, and hence make the decision results more precise and reasonable.

Chapter 5

Conclusions

In this thesis, we have investigated the aggregation of fuzzy number intuitionistic information, and developed some aggregation operators, such as, the ordered weighted aggregation operator and the hybrid aggregation operator of FNIFNs. we have defined a new judgment matrix called the fuzzy number intuitionistic judgment matrix and studied some of its desirable properties, and then defined the concepts of the consistent fuzzy number intuitionistic judgment matrix, and the score matrix, and accuracy matrix of the fuzzy number intuitionistic judgment matrix, and so on. we have shown that the score matrix and accuracy matrix are the antisymmetric matrix and symmetric matrix respectively, and discussed the relationships among fuzzy number intuitionistic judgment matrix, intuitionistic judgment matrix, and complement judgment matrix. Furthermore, on the basis of the arithmetic aggregation operator and hybrid aggregation operator, we have proposed an approach for solving the group decision-making problems where the decision makers provide their preferences over alternatives in the form of fuzzy number intuitionistic judgment matrices. In the future, the developed aggregation operators of FNIFNs can be applied to the fields of pattern recognition, artificial intelligence, data mining, fuzzy logic, and so on.

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