Thesis for the Degree of Doctor of Philosophy

Servo Controller Design Using Polynomial Differential Operator Method and Its Applications

by

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Servo Controller Design Using Polynomial Differential Operator Method and Its Applications

다항식 미분연산자법을 사용한 서보제어기 설계와 그 응용

Supervised by

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by

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A dissertation

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Dae Hwan Kim

Contents

Servo Controller Design Using Polynomial Differential Operator Method and Its Applications

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Abstract

A conventional PID controller has been used in most industrial fields so far. However, although the conventional PID controller can track a step type of reference without steady state error, it can have steady state error on disturbance, ramp, hyperbola, and higher order of references. Therefore, it is impossible to use the conventional PID controller in this cases. Moreover, a servo controller design problem for a given multi-input and multi-output system with disturbance and references which are polynomials attracting concern in control engineering field is one of most interested problems.

To solve this problem, a new servo control design method is needed deeply.

This dissertation proposes a new servo controller design method using polynomial differential operator method based on the internal model principle. To do this task, the followings are done. Firstly, modelings for an induction motor and an 4 wheel steering vehicle are proposed and are linearized. Secondly, a linearized system with disturbance is described. The disturbance and reference are expressed as the form of differential polynomial equations. Thirdly, it is shown that a closed loop system of the given system has zero steady errors by the final theorem when the controller includes the least common multiple of the denominators of reference and disturbance in its denominator using internal model principle. Fourthly, by operating the polynomial differential operator to the given system under the given conditions of reference and disturbance and an output error, an extended system is obtained. Fifthly, it is proven that the extended system is controllable. Sixthly, a full order observer of the extended system is designed to estimate its unknown states. To implement the proposed servo controller design method, a control system is developed to control speed of 1.5 Kw AC induction motor. Hardware for the proposed system is introduced. Hardware is designed to control 1.5 Kw AC induction motor. The TMS320F28335 DSP is selected as the digital controller for the system. Necessary peripheral and interface circuits are built for signal measurement, the three-phase inverter control and the system protection. Seventhly, the simulation and experimental results for an 1.5 Kw AC induction motor as a single-input and single-output system(SISO) and an 4wheel steering vehicle as a multi-input and multi-output system(MIMO) under a step type of disturbance and 3types of references such as step, ramp and parabola are shown to verify the effectiveness and the applicability of the proposed servo controller design method compared to PI controller. Finally, conclusions are presented and the future works are described.

Keywords: AC Induction Motor, Four Wheel Streering Vehicle, Internal Model Principle, Servo System, Speed Control, State Feedback, MIMO, SISO, Polynomial differential operator

Chapter 1: Introduction

In this chapter, first, the background and motivation for this dissertation are presented. Second, the objective and research method for this dissertation are described. At the end of this chapter, an outline and summary of contributions of the dissertation are provided.

1.1 Background and Motivation

Engineers have long known the benefits of integral feedback control for tracking constant reference signals in the presence of unknown constant disturbances. Indeed, proportional-integral(PI) and proportional-integral-derivative(PID) controllers are used as the popular controllers in industrial process control. Several literatures on the design and tuning of PID controllers for single-input (SISO) systems [1]-[5] has been reported.

For multi-input multi-output (MIMO) systems, the design of multivariable controllers with integral action was proposed in [7]-[10],[23]. The exciting techniques usually assume that either full-state information is available or require an observer to estimate the state. Hence, for large industrial systems, many MIMO controller design approaches are impractical. It is therefore understandable that both academic and industrial researchers have sought for and continue to search for MIMO controller design procedures that result in low-order controllers.

The industrial researchers has been inspired by their success in classical SISO problem using PID controllers that are the most popular MIMO low-order controllers. Various tuning methods have been developed for MIMO PID control [1],[10].

However, most of these design procedures require that the MIMO plant has special properties. For instance, some systems are assumed to be decoupled or be able to be decoupled, while others systems can be approximated by a low-order system with delay. Fortunately, by recognizing that there exist systems where the inter-loop interactions are not necessarily small and where the system cannot be approximated by a low-order system with delay, various researchers have managed to devise MIMO PID design procedures without making restrictive assumptions on the structure of the system. For example, in [11], a technique using pole placement was proposed, that is, it allowed the designer to partially place some of the poles of the closed-loop system. If the remaining poles are unstable or unsatisfactory, then the design parameters are modified and the pole-placement problem must be solved again. Another approach was taken in [12], where a MIMO optimal PID controller was obtained by numerically solving several simultaneous nonlinear matrix equations. In [13], a third approach for the design of an optimal PID controller was proposed, which involved solving a high-order static state-feedback problem and then reducing the size of the controller by retaining the "dominant" dynamics of the closed-loop system. Finally, in [14] and [15], a MIMO PID controller was obtained by solving a similar static output-feedback problem using linear matrix inequality (LMI) methods.

Several researchers have researched to solve the servomechanism problem. E. J. Davison and H. W. Smith[21] proposed the servo controller design problem for the special case that disturbance term is a constant un-measurable case. C. D. Johnson[22, 23] proposed the problem for the special case of full rank for output matrix S. P. Bhattacharyya and J. B. Pearson[26] proposed a generalization of the problem by introducing an “error system” and by using a geometric approach. They only obtained sufficient conditions for the stability of the closed-loop system, however, and did not investigate the minimality of the controller. E. J. Davison[18] proposed the output control of linear time-invariant multivariable systems with unmeasurable arbitrary disturbances as the robust control of a servomechanism problem for linear time-invariant multivariable systems theoretically. That is, he have considered the output control problem of multivariable systems as being a multivariable generalization of the classical single-input, single-output servomechanism problem. The method taken to develop the result relies extensively on properties of the asymptotic solution of a stable linear constant system subject to a specified class of forcing function inputs. It was initially shown that the closed loop system with the proposed controller is controllable if and only if the original system is controllable. To overcome the stated drawbacks, one has to challenge a new approach development. To solve robust servomechanism problem, S. B. Kim et al.[16] proposed bilinear transformation method to servo system design using internal model principle and pole assignment method in a specific region and position control for a cart system. That is, he introduced a servo control method with disturbance rejection and reference signal tracking by adopting the uses of the internal model principle and bilinear transformation method. The algorithm concept was shown to the servo design problem such that the types of reference input and disturbance are not matching polynomially and by using differential operator. The servo design was done by two steps: firstly constitute an extended system by incorporating the internal model principle into the servo controller synthesis approach and obtain the state feedback law for the extended system by solving the pole assignment problem in a specified region. In the method, there was not shown for the explicit condition and its proof for the controller existence of the extended system to achieve the robust servo control object.

Several researchers proposed estimators applied to some application fields. Sensorless vector control of an induction motor drive essentially refers to vector control without any speed sensor. Speed sensor is not suitable for the environmental condition, which suffers due to large shocks such as drive systems in an electric car or a conveyor system. Sensorless vector control is suitable from the point of reliability of the equipment, cost effectiveness and less maintenance. The speed is estimated from the measured terminal voltages and currents. Several methods for speed-sensorless control of induction machines have been proposed. Model-Reference Adaptive Systems (MRASs) [49, 50] are methods that have good performance over large speed range. Their disadvantage is the large influence of parameter deviation at low-speed and standstill operation. Also, the use of PI controllers with complicated gains creates difficulties in their implementation using a digital signal processor (DSP). Artificial intelligence methods [39-48] that use artificial intelligence techniques such as fuzzy logic and neural networks are very promising candidates to be robust to parameter deviation and measurement noise, but they need long development times and an expertise in several artificial intelligence procedures. With this method, it is possible to make the online estimation of states and perform the simultaneous identification of parameters in a relatively short time interval [53-55] by also taking system process and measurement noises directly into account. This is the reason why the servo system has widely applied to the sensorless control of IMs, in spite of its computational complexity. In conventional internal model principle estimation for IMs, the flux and speed are estimated [56-62].

However, the accuracy of the conventional internal model principle estimations still depends much on the motor parameters that can be changed by operating condition. Furthermore, the speed is estimated as a constant parameter, which gives rise to a significant estimation error in the speed during the transient state, particularly under instantaneous load variations. An accurate knowledge of stator resistance is also important for correct control of servo system induction motor in its low speed region since stator resistance inevitably varies with operating conditions. Stable and accurate operation at near-zero speed requires an appropriate online identification algorithm for the stator resistance. M. W. Naouar[17] propsed FPGA-based speed control of a synchronous machine using P-PI controller with an speed estimator using an absolute encoder. C. Kwon et al.[19] proposed rotor flux and speed observers to estimate the rotor speed for sensorless control of induction motor drives. C. Gabriela [20]proposed rotor speed estimation method using estimated rotor flux obtained from flux observer in dynamic control of the induction motor. H. K. Khalil et al[14-15]. proposed sensorless field –oriented speed control of induction motors without rotor position sensors using flux and speed observers with PI controllers via linarization to regulate the q-axis current to the its reference. H. K. Khalil and E. G. Strangas[14] augmented the traditional approach with flux and speed observers and derived a sixth-order nonlinear in field oriented coordinates. They formulated the flux and speed regulation problems based on the conventional PI controller. The nonlinear model was linearized to the third order linear model at the desired equilibrium point, and the well known PI controller was adopted for sensorless speed control. They showed that according to the product value of flux frequency and q-axis currents PI-approach based controller has some serious limitation for the robust property in the case of MIMO system. Therefore, they wished a development for a different type of more complex controller. As we know well, the PI controller is not adequate for more high order types of speed reference signals with disturbances.

From the above mentions, a conventional PID controller has been used in most industrial fields so far. However, although the conventional PID controller can track a step type of reference without steady state error, it can have steady state error on disturbance, ramp, hyperbola, and higher order of references. Therefore, it is impossible to use the conventional PID controller in this cases. Moreover, a servo controller design problem for a given system multi-input and multi-output system with disturbance and references which are polynomials attracting concern in control engineering field is one of most interested problems. To solve this problem a new servo controller design method is needed deeply.

1.2 Objective and research method of this dissertation

The purpose of this paper is to extend the results of [16] to include the more general formulation and show the controller existence conditions. The proposed algorithm differs from other servo design methods, for examples, frequency domain design methods [25]-[29], graphical design methods [31],[32], where there is usually only an implicit assumption made that a solution exists, perturbation analysis based state space method[33]-[39]. In those methods, two stage process are used : 1) using theory, determine the existence of a solution to the servo control problem, and the necessary controller structure to solve for it, 2) using parameter optimization methods, determine the controller parameters of the controller so as to minimize a performance index for the system, subject to certain constraint requirements.

To solve this problem, a new servo control design method is needed deeply. A new robust servo controller is obtained by modifying the well known concept of the internal model principle shown in S. B. Kim et al. [16]. This dissertation proposes a new servo controller design method using polynomial differential operator method based on the internal model principle. To do this task, the followings are done. Firstly, modelings for an AC induction motor as a single-input and single-output(SISO) system and an 4 wheel steering vehicle as a multi-input and multi-ouput(MIMO) system are proposed and are linearized. Their time-invariant linearized systems with disturbance are described, and the disturbance and the reference are expressed as the form of differential polynomial equations. Secondly, it is shown that a closed loop system of the given system has zero steady errors by the final theorem when the controller includes the least common multiple of the denominators of reference and disturbance in its denominator. Thirdly, it is shown that a closed loop system of the given system has zero steady errors by the final theorem when the controller includes the least common multiple in its denominator. Fourthly, by operating the polynomial differential operator to the given system under the given conditions of reference and disturbance and an output error, an extended system is obtained. Fifthly, it is proven that the extended system is controllable. Sixthly, to implement the proposed servo controller design method, a control system is developed to control speed of 1.5 Kw AC induction motor. Hardware for the proposed system is introduced. Hardware is designed to control 1.5 Kw AC induction motor. The TMS320F28335 DSP is selected as the digital controller for the system. Necessary peripheral and interface circuits are built for signal measurement, the three-phase inverter control and the system protection. A testing motor system is developed. The testing motor system consists of an 1.5 Kw AC induction motor, a torque sensor and a powder brake. They are connected in series. A voltage source inverter based on DSP 320F28335 is designed. The inverter uses smart power module FSBB30CH60 (IGBT module) as switching devices. A set of motor equations are given in various coordinates based on vector method. Seventhly, a full order observer of the extended system is designed to estimate its unknown states. Finally, the simulation and experimental results for an 1.5 Kw AC induction motor as a single-input and single-output(SISO) system and an 4wheel steering vehicle as a multi-input and multi-output(MIMO) system with a step type of disturbance and 3 references such as step, ramp, and parabola are shown to verify the effectiveness and the applicability of the proposed servo controller design method compared to PI controller.

1.3 Outline of the Dissertation and Summary of Contributions

In this section, contents of the dissertation and their contributions are summarized as follows:

Chapter 1: Introduction

In this chapter, first, the background and motivation for this dissertation are presented. Second, the objective and research method for this dissertation are described. At the end of this chapter, an outline and summary of contributions of the dissertation are provided.

Chapter 2: Problem Statement and System Modeling for Application

In this chapter, firstly, problem statements are introduced. Secondly, nonlinear mathematical modelings of an AC induction motor and 4 wheel steering vehicle, and their linearized modelings are introduced.

Chapter 3: Servo Controller Design Using Polynomial Differential Operator Method

In this chapter, a servo controller design method using polynomial differential operator is proposed. To do this task, the followings are done. Firstly, a given linear time invariant system with disturbance is described. Secondly, disturbance and reference are expressed as the form of differential polynomial equations. Thirdly, it is shown that a closed loop system of the given system has zero steady errors by the final theorem when the controller includes the least common multiple of the denominators of reference and disturbance in its denominator. Fourthly, by operating the polynomial differential operator to the given system under the given conditions of reference and disturbance and an output error, an extended system is obtained. Fifthly, it is proven that the extended system is controllable. Sixthly, a feedback control law is designed by a well known regulator design method.

Chapter 4: Application for SISO System of AC Induction Motor

This chapter first describes the prototype of the experimental AC induction motor drive system. The motor testing system including AC induction motor, torque sensor and powder brake that are connected in series is set up. Voltage source inverter based on DSP is developed. The analog amplifiers for measuring two current signals and DC voltage from power source signal are described. Secondly, a full order observer of the extended system is designed to estimate its unknown states. Thirdly, to verify the effectiveness and the applicability of the proposed servo controller design method in chapter 3, simulation and experimental results of the proposed robust servo controller for a step type of disturbance and 3 references such as step, ramp and parabola are shown for AC induction motor system as a single-input and single-output(SISO) system compared with those of the conventional PI controller.

Chapter 5: Application to MIMO System of 4 Wheel Steering Vehicle

In the chapter, to verify the effectiveness and the applicability of the proposed servo controller design method, two simulation and experimental results of the proposed robust servo controller for a step type of disturbance and 3 references such as step, ramp and parabola are shown a 4 wheel steering vehicle as a multi-input and multi-output (MIMO) system with two inputs and two outputs compared with those of the conventional PI controller.

Chapter 6: Conclusions and Future Works

In this chapter, some conclusions of this dissertation and some ideas for future work are presented.

Chapter 2: Problem Statement and System Modeling for Application

In this chapter, firstly, problem statement is introduced. Secondly, the mathematical modelings of an AC induction motor and a 4 wheel steering vehicle are introduced.

2.1 Problem Statement

PID controller problem has inefficient operation. PID controller can track a step type of reference without steady state error, but can have steady state error on disturbance, ramp, hyperbola, and higher order of references. In this dissertation to solve this problem, the following subjects are considered and solved. PI-approach based controller has some serious limitation for the robust property in the case of MIMO system. MIMO PID control design procedure requires that the MIMO plant has special properties decoupling problem and transformation into low-order system. Therefore, we need a development for a different type of more complex controller. As we know well, the PI controller is not adequate for more high order types of speed reference signals with disturbances.

This dissertation is to extend the results of [17] to include more general information and shows the controller existence conditions. In [17], there was not shown for the explicit condition and its proof for the controller existence of an extended system to achieve the robust servo control objectives.

The problem statements to design a robust servo controller of a given system are as follows :

- To introduce polynomial differential operator with respect to references and disturbances
- To introduce nonlinear modelings for an AC induction motor and a 4 wheel steering vehicle, and their linearized modelings
- Based on he internal model principle, to obtain an extended system to construct a servo system by operating polynomial differential operator to a given system with disturbance and an output error in case that the types of reference inputs and disturbance are polynomials
- To design a state feedback control law for the extended system with disturbance to track the given reference input
- To design a full order observer to estimate the unknown states of an extended system
- To develop a control system of 1.5 Kw AC induction motor and 4wheel steering vehicle for implementing the proposed controller and the servo system.
- To perform the simulation and experiment for showing that the system with a step typed disturbance can track 3 types of the references such as step, ramp and parabola using the proposed robust servo controller and the conventional PI controller

2.2 Introduction to Motor Drive System

The purpose of this dissertation is to develop a motor drive system for AC induction motor. The drive system is required being fast response and high efficiency.

AC induction motor is selected for the drive system due to its simple construction, reliability and robustness. Fig. 2.1 shows a typical motor drive system using AC induction motor.

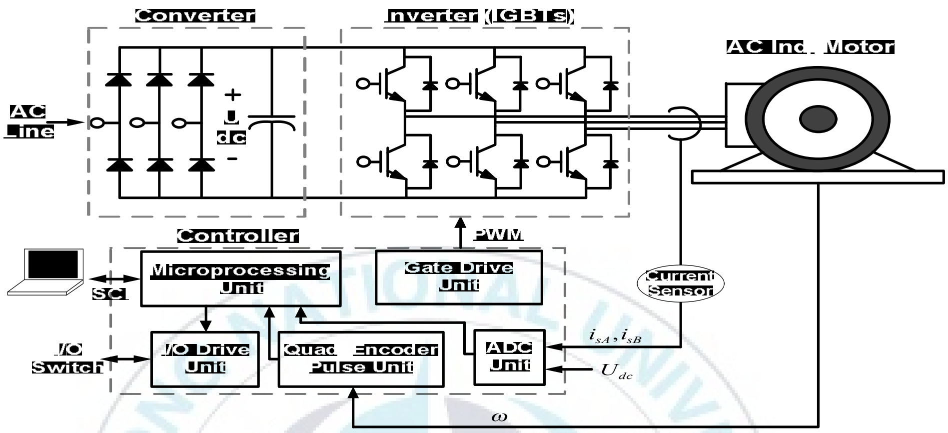

Fig. 2.1 Typical motor drive system using AC induction motor

Controllers are required to match the characteristics of the motor with that of load. A number of control strategies have been developed for various motor drives. The electric controller in general controls the current and voltage or flux linkage and torque within the PEC. The electric sensors (estimators) refer to voltage, current, flux as measured or calculated state variables. The electric sensors get their input from both the power source and the PEC output. The output of this controller is commands for improving the performance of power converter such as improving the power factor, reducing harmonics etc.. Motion sensors refer to mean position, speed and torque as measured or calculated state variables. The motion controller gets input from motion sensors and delivers output in the form of commands relating to motions such as speed, position and torque. The electric and motion controllers are combined as single controller and are realized with analog or digital circuits. In this dissertation, a high performance Digital Signal Processors (DSP) TMS320F28335 is used as controller for the drive system

2.3 Modeling of AC Induction Motor as SISO(Single Input and Single Output)

The induction motor can be represented in the state frame of reference by [15]

$$\dot{\lambda}_r = \left(-\alpha_r I + p\omega I_\lambda\right)\lambda_r + \alpha_r L_m i_s \tag{2.1}$$

$$\dot{i}_s = -\beta\left(-\alpha_r I + p\omega I_\lambda\right)\lambda_r - \left(\beta\alpha_s + \beta\alpha_r L_m\right) i_s + \gamma\upsilon_s \tag{2.2}$$

$$\dot{\omega} = -\mu\lambda_r^T I_\lambda i_s - b\omega - \frac{T_L}{J} \tag{2.3}$$

where $\lambda_r = \begin{bmatrix}\lambda_{rd} & \lambda_{rq}\end{bmatrix}^T \in R^2$ is the rotor flux vector, $i_s = \begin{bmatrix}i_{sd} & i_{sq}\end{bmatrix}^T \in R^2$ is the stator current vector, $\upsilon_s = \begin{bmatrix}\upsilon_{sd} & \upsilon_{sq}\end{bmatrix}^T \in R^2$ is the stator voltage vector, ω is the rotor speed, L_s, L_r, L_m denote rotor, stator and mutual inductances, R_r, R_s are rotor and stator resistances, p is the number of pole pairs, J is rotor's moment of inertia, b_1 is a friction coefficient, T_L is load torque $I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $I_\lambda = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, and $\sigma = 1 - \frac{L_m^2}{L_s L_r}$, $\alpha_r = \frac{R_r}{L_r}$, $\alpha_s = \frac{R_s}{L_s}$, $b = \frac{b_1}{J}$, $\mu = \frac{3pL_m}{2JL_r}$, $\beta = \frac{1-\sigma}{\sigma L_m} = \frac{L_m}{\sigma L_s L_r}$, $\gamma = \frac{1}{\sigma L_s}$, and $\eta = \frac{1}{\sigma}$.

Under the conditions of estimates $\hat{R}_s, \hat{R}_r, \hat{m}, \hat{b}_1$ of uncertain parameters R_s, R_r, J, b_1, Eqs. (2.1)-(2.3) can be represented based on the direct-axis components $\lambda_{rd}, i_{sd}, \upsilon_{sd}$,

e_d and on the quadrature-axis components λ_q (=0:flux of rotor in q - $axis$), $i_{sq}, \upsilon_{sq}, e_q$, e_d as follows:

$$\dot{\lambda}_{rd} = -\hat{\alpha}_r \lambda_{rd} + \hat{\alpha}_r L_m i_{sd} \tag{2.4}$$

$$\dot{i}_{sd} = \beta\alpha_r\lambda_{rd} - \left(\beta\alpha_s + \beta\alpha_r L_m\right) i_{sd} + \gamma\upsilon_{sd} + p\omega_{ref} i_{sq} + \hat{\alpha}_r L_m \, {i_{sq}^2}/{\lambda_{rd}} - \alpha_r\beta e_d - \beta p\omega e_q \tag{2.5}$$

$$\begin{aligned}\dot{i}_{sq} = &-\beta p\omega\lambda_{rd} - p\omega_{ref} i_{sd} - \left(\beta\alpha_s + \beta\alpha_r L_m\right) i_{sq} + \gamma\upsilon_{sq} - \hat{\alpha}_r L_m \, {i_{sd} i_{sq}}/{\lambda_{rd}} \\ &+ \beta p\omega e_d - \alpha_r \beta e_q\end{aligned} \tag{2.6}$$

$$\dot{\omega} = \mu[i_{sq}(\lambda_{rd} - e_d) + i_{sd} e_d] - b\omega - \frac{T_L}{J} \tag{2.7}$$

where $\hat{\alpha}_s = \frac{\hat{R}_s}{L_s}$, $\hat{\alpha}_r = \frac{\hat{R}_r}{L_r}$, $\hat{b} = \frac{\hat{b}_1}{\hat{J}}$, $\hat{\mu} = \frac{3pL_m}{2\hat{J}L_r}$

Error dynamics and the rotor flux estimation error vector a represented as follows :

$$\dot{e}_d = \hat{\lambda}_{rd} - \lambda_{rd} = -\alpha_r e_d + (p\omega_{ref} - p\omega + {\hat{\alpha}_r L_m i_{sq}}/{\lambda_{rd}}) e_q + (\hat{\alpha}_r - \alpha_r)(L_m i_{sd} - \lambda_{rd}) \tag{2.8}$$

$$\begin{aligned}\dot{e}_q = \hat{\lambda}_{rq} - \lambda_{rq} = &-(p\omega_{ref} - p\omega + {\hat{\alpha}_r L_m i_{sq}}/{\lambda_{rd}}) e_d - \alpha_r e_q + (\hat{\alpha}_r - \alpha_r) L_m i_{sq} \\ &+ p(\omega_{ref} - \omega)\lambda_{rd}\end{aligned} \tag{2.9}$$

$$e_\lambda = \hat{\lambda}_r - \lambda_r \tag{2.10}$$

where $e_\lambda = \begin{bmatrix} e_d & e_q \end{bmatrix}^T$ is the rotor flux estimation error vector, $\hat{\lambda}_r = [\hat{\lambda}_d \quad \hat{\lambda}_q]^T$ is the estimated vector of λ_r which can be estimated by an observer and also regulated by the well known PI controller[15], and ω_{ref} is the reference rotor speed.

An observer to estimate the rotor flux vector is proposed as follows :

$$\dot{\hat{\lambda}}_r = \left(-\hat{\alpha}_r I + p\omega_{ref} I_\lambda\right)\hat{\lambda}_r + \hat{\alpha}_r L_m i_s \tag{2.11}$$

where $e_\lambda = \begin{bmatrix} e_d & e_q \end{bmatrix}^T$ is the rotor flux estimation error vector, $\hat{\alpha}_s = \frac{\hat{R}_s}{L_s}$, $\hat{\alpha}_r = \frac{\hat{R}_r}{L_r}$, $\hat{b} = \frac{\hat{b}_1}{\hat{J}}$, $\hat{\mu} = \frac{3pL_m}{2\hat{J}L_r}$, $\hat{J}$ is the estimate of J, $\hat{\lambda}_r = [\hat{\lambda}_d \quad \hat{\lambda}_q]^T$ is the estimated vector of λ_r which can be estimated by an observer and also regulated by the well known PI controller[15], and ω_{ref} is the reference rotor speed.

In this paper, the sensorless speed controller design on the state space is the main goal. So how to simplify the model of Eqs. (2.4)-(2.10) is important to accomplish the target goal.

It is assumed that the speed is estimated by an observer shown in [15,16] and the flux regulator acts fast enough to regulate the reference rotor flux of λ_{rd} to the reference rotor flux in $d\text{-}axis$ of λ_{ref} ($\lambda_{rd} = \lambda_{ref}$), and the stator current in $d\text{-}axis$ of $i_{sd} = \lambda_{ref}/L_m$, then the speed controller can be designed as the third-order nonlinear model as follows :

$$\dot{e}_d = -\alpha_r e_d + (p\omega_{ref} - p\omega + \hat{\alpha}_r L_m i_{sq}/\lambda_{ref})e_q \tag{2.12}$$

$$\dot{e}_q = -(p\omega_{ref} - p\omega + \hat{\alpha}_r L_m i_{sq}/\lambda_{ref})e_d - \alpha_r e_q + (\hat{\alpha}_r - \alpha_r)L_m i_{sq} + p(\omega_{ref} - \omega)\lambda_{ref} \tag{2.13}$$

$$\dot{\omega} = \mu[i_{sq}(\lambda_{ref} - e_d) + {}^{\lambda_{ref}}\!/_{L_m} e_g] - b\omega - \frac{T_L}{J} \tag{2.14}$$

$$\Omega = \left(\frac{\lambda_{ref} - e_d}{\lambda_{ref}} \right)\omega + \frac{\alpha_r e_q}{p\lambda_{ref}} - a i_{sq} \tag{2.15}$$

$$a = \left[(\hat{\alpha}_s - \alpha_s) \right]\eta + (\hat{\alpha}_r - \alpha_r)\beta L_m] / (\beta p \lambda_{ref})$$

where Ω is viewed as the measured output under the equilibrium point with $\Omega = \omega_{ref}$ as the followings:

$$\begin{cases} \bar{e}_d = \bar{e}_q = 0 \\ \bar{\omega} = \omega_{ref} + \dfrac{(\hat{\alpha}_s - \alpha_s) L_m \bar{i}_{sq}}{p\lambda_{ref}} \\ \bar{i}_{sq} = \dfrac{b\omega_{ref} + T_L / m}{\mu\lambda_{ref} - \dfrac{b(\hat{\alpha}_s - \alpha_s) L_m}{p\lambda_{ref}}} \end{cases} \tag{2.16}$$

The nonlinear model of Eqs. (2.12)-(2.15) under the above equilibrium point with $\hat{\alpha}_s = \alpha_s$ can be linearized as the following linear model:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \tag{2.17}$$

$$y = C\mathbf{x}(t) + du(t) \tag{2.18}$$

$$A = A(t)\big|_{\bar{\mathbf{x}},\bar{i}_{sq}} = \begin{bmatrix} -\alpha_r & \dfrac{\alpha_r L_m}{\lambda_{ref}}\bar{i}_{sq} & 0 \\ -\dfrac{\alpha_r L_m}{\lambda_{ref}}\bar{i}_{sq} & -\alpha_r & -p\lambda_{ref} \\ -\mu\bar{i}_{sq} & \mu\dfrac{\lambda_{ref}}{L_m} & -b \end{bmatrix}, \quad B = B(t)\big|_{\bar{\mathbf{x}},\bar{i}_{sq}} = \begin{bmatrix} 0 \\ (\hat{\alpha}_r - \alpha_r)L_m \\ \mu\lambda_{ref} \end{bmatrix},$$

$$C = C(t)\big|_{\bar{\mathbf{x}},\bar{i}_{sq}} = \begin{bmatrix} \dfrac{-\bar{\omega}}{\lambda_{ref}} & \dfrac{\alpha_r}{p\lambda_{ref}} & 1 \end{bmatrix}, \quad d = d(t)\big|_{\bar{\mathbf{x}},\bar{i}_{sq}} = -\frac{(\hat{\alpha}_r - \alpha_r)L_m}{p\lambda_{ref}},$$

$$\mathbf{x}(t) = \begin{bmatrix} e_d & e_q & \omega \end{bmatrix}^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T, \ u = i_{sq}, \ y = \Omega, \ \bar{X} = \begin{bmatrix} \bar{e}_d & \bar{e}_q & \bar{w} \end{bmatrix}^T$$

where the speed can be estimated by an observer[15] or measured by sensor, and $\mathbf{x}, y, u$ are state variable vector, output variable and input variable.

At the equilibrium points with $\hat{\alpha}_s = \alpha_s$ and $\hat{\alpha}_r = \alpha_r$, the followings are obtained.

$$A = \begin{bmatrix} -\alpha_r & \dfrac{\alpha_r L_m}{\lambda_{ref}}\bar{i}_{sq} & 0 \\ -\dfrac{\alpha_r L_m}{\lambda_{ref}}\bar{i}_{sq} & -\alpha_r & -p\lambda_{ref} \\ -\mu\bar{i}_{sq} & \mu\dfrac{\lambda_{ref}}{L_m} & -b \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \mu\lambda_{ref} \end{bmatrix}, \ C = \begin{bmatrix} \dfrac{-\bar{\omega}}{\lambda_{ref}} & \dfrac{\alpha_r}{p\lambda_{ref}} & 1 \end{bmatrix}, \ d = 0,$$

$$\mathbf{x}(t) = \begin{bmatrix} e_d & e_q & \omega \end{bmatrix}^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T, \ u = i_{sq}, \ y = \Omega - \omega_{ref}, \ \tilde{\omega} = \bar{\omega} - \omega_{ref} = 0,$$

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{e}_d & \bar{e}_q & \bar{\omega} \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & \omega_{ref} \end{bmatrix}^T, \ \bar{u} = \bar{i}_{sq} = \frac{b\omega_{ref} + T_L / J}{\mu\lambda_{ref}}, \quad \bar{y} = \Omega\big|_{\bar{\mathbf{x}},\bar{i}_{sq}} = \omega_{ref} = \bar{\omega} = y_r$$

where $\bar{\mathbf{x}}, \bar{y}, \bar{u}$ are state variable vector, output variable and input variable at the equilibrium point respectively, and y_{ref} is the reference output.

Proof of Eqs. (2.17) and (2.18) are written in **Appendix A.**

2.4 Modeling of 4 Wheel Steering Vehicle as MIMO(Multi-Input and Multi-Output)

A vehicle is a very complex system, so many of parameters are usually required if a multi-body dynamics method is adopted to model it in detail. To minimize the complexity and difficulty of developing a vehicle dynamics control system, it is common to build a relatively simple vehicle model and thus validate the feasibility in the concept design stage.

In order to study the essential vehicle dynamics and simplify the analysis procedure, the influence of roll on lateral motion is assumed to be small and not taken into account. A 2DOF model containing sideslip and yaw rate, commonly used in vehicle dynamics control, is utilized to study the handling stability of 4 wheel steering vehicle. As shown in Fig. 2.1.

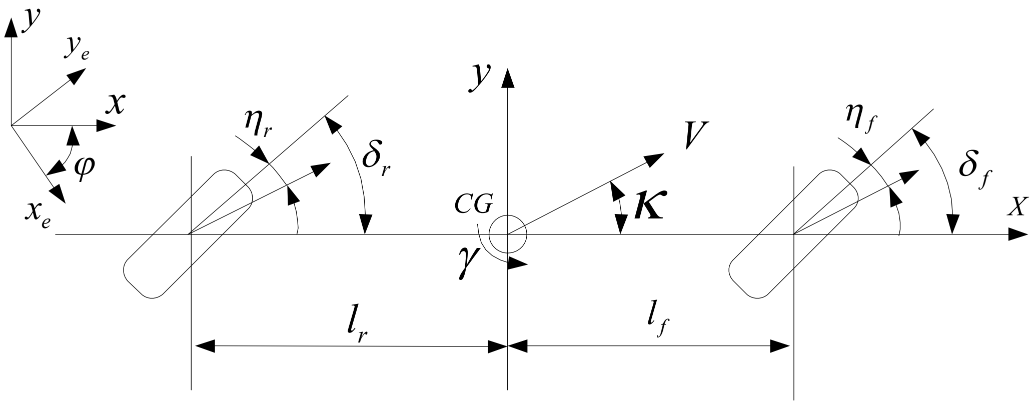

Fig. 2.1 2 DOF vehicle lateral dynamic model

The initial coordinate system (x_e, y_e, z_e) is fixed on the ground, where the z_e axis reperesents the direction normal to the (x_e, y_e) plane. This serves as a reference frame for vehicle motions. The body(chassis) coordinate system, denoted by (x,y,z) with its origin at the CG, is introduced to describe vehicle motion. Next, the chassis system, (x,y,z) is rotated a yaw angle φ with respect to the inertial system (x_e, y_e, z_e).

In Fig. 4, κ and γ denote the sideslip angle and yaw rate of vehicle at the CG; m is the vehicle mass ; J_z is the yaw moment of inertia about its mass centre $z-axis$; V denotes the linear velocity of vehicle; e_f and e_r are the distances from the CG (centre of gravity) to the front and rear axles; δ_f and δ_r denote the steering angles of front and rear tires; η_f and η_r are the slip angles of front and rear tires; C_f and C_r denote the lateral stiffnesses of the front and rear tires, respectively.

Only considering lateral and yaw motions, the vehicle dynamic equations can be derived by applying Newton's second law.

Lateral motion:

$$mV(\dot{\kappa}+\gamma)=F_f\cos\delta_f+F_r\cos\delta_r \tag{2.19}$$

where F_f, F_r are the lateral forces generated by the front and rear tires.

Yaw motion, or moments about the vertical $z-axis$ through the CG:

$$J_z\dot{\gamma}=l_fF_r\cos\delta_r \tag{2.20}$$

In general, lateral tire force is a non-linear function of the slip angle. In this study, the cornering stiffness for the front(rear) wheel is denoted by C_f (C_r) and its value depends on the tire-road interaction. As long as the tire slip angle is small, a linear relationship between tire force and slip angle can be justified. Then, the lateral forces generated by the front and rear tires vary linearly with their slip angles.

$$F_f=C_f\eta_f \text{ and } F_r=C_r\eta_r \tag{2.21}$$

If the sideslip angle κ is small and vehicle linear velocity V varies slowly, η_f and η_r will be given by:

$$\eta_f=\delta_f-\kappa-\frac{l_f}{V}\gamma \text{ and } \eta_r=\delta_r-\kappa-\frac{l_r}{V}\gamma \tag{2.22}$$

In addition, δ_f and δ_r are generally small,

$$\cos\delta_f\approx 1 \text{ and } \cos\delta_r\approx 1 \tag{2.23}$$

Hence, the vehicle motion equations are expressed by

$$\begin{cases} mV(\dot{\kappa}+\gamma) = -2(C_f + C_r)\kappa - \dfrac{2(l_f C_f - l_r C_r)}{V}\gamma + 2C_f\delta_f + 2C_r\delta_r \\ J_z\dot{\gamma} = -2(l_f C_f - l_r C_r)\kappa - \dfrac{2(l_f^2 C_f + l_r^2 C_r)}{V}\gamma + 2l_f C_f\delta_f - 2l_r C_r\delta_r \end{cases} \tag{2.24}$$

By defining the state vector $x = \begin{bmatrix} \kappa & \gamma \end{bmatrix}^T$, input vector $u = \begin{bmatrix} \delta_f & \delta_r \end{bmatrix}^T$ and output vector $y = \begin{bmatrix} \kappa & \gamma \end{bmatrix}^T$, the vehicle model can be written in the state-space form as follows:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{2.25}$$

where the coefficient matrices are:

$$A = \begin{bmatrix} -\dfrac{2\left(C_f + C_r\right)}{mV} & -\dfrac{2\left(l_f C_f - l_r C_r\right)}{mV^2} - 1 \\ -\dfrac{2\left(l_f C_f - l_r C_r\right)}{J_z} & -\dfrac{2\left(l_f^2 C_f - l_r^2 C_r\right)}{J_z V} \end{bmatrix},\ B = \begin{bmatrix} \dfrac{2C_f}{mV} & \dfrac{2C_r}{mV} \\ \dfrac{2C_f}{J_z} & -\dfrac{2C_r}{J_z} \end{bmatrix}$$

Chapter 3: Servo Controller Design Using Polynomial Differential Operator Method

In this chapter, a servo controller design method using polynomial differential operator is proposed. To do this task, the followings are done. Firstly, a given linear time invariant system with disturbance is described. Secondly, disturbance and reference are expressed as the form of differential polynomial equations. Thirdly, it is shown that a closed loop system of the given system has zero steady errors by the final theorem when the controller includes the least common multiple of the denominators of reference and disturbance in its denominator. Fourthly, by operating the polynomial differential operator to the given system under the given conditions of reference and disturbance and an output error, an extended system is obtained. Fifthly, it is proven that the extended system is controllable.

3.1 Preliminaries

$$\frac{dx}{dt} = Ax + Bu + \varepsilon \tag{3.1}$$

$$y = Cx \tag{3.2}$$

where $A \in R^{n\times m}$ is the system matrix, $B \in R^{n\times m}$ is the input matrix, $C \in R^{p\times n}$ is the out matrix, $x = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^T \in R^n$ is the system state vector, $u = \begin{bmatrix} u_1 & u_2 & \cdots & u_m \end{bmatrix} \in R^m$ is the control input vector, $y = \begin{bmatrix} y_1 & y_2 & \cdots & y_p \end{bmatrix}^T \in R^p$ is the system output vector, $\varepsilon = \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{bmatrix}^T \in R^n$ is the unmeasurable disturbance vector, and $m \geq p$.

The output error vector is defined by

$$e = y_r - y \tag{3.3}$$

where $y_r = \begin{bmatrix} y_{r1} & y_{r2} & \cdots & y_{rp} \end{bmatrix}^T \in R^p$ is the reference output vector, and $e = \begin{bmatrix} e_1 & e_2 & \cdots & e_p \end{bmatrix}^T \in R^p$ is the output error vector.

It is assumed that the following homogeneous differential equation forms for the i^{th} disturbance $\varepsilon_i(t)$ and the i^{th} reference output vector y_{ri} are satisfied, respectively:

$$L_r(D) y_{ri}(t) = 0 \quad for\ i = 1 \sim p \tag{3.4}$$

$$L_\varepsilon(D) \varepsilon_i(t) = 0 \quad for\ i = 1 \sim n \tag{3.5}$$

where $L_\varepsilon(D)$ and $L_r(D)$ are assumed as the following differential polynomial operators with constant coefficients .

$$L_r(D) = D^\sigma + \rho_{\sigma-1} D^{\sigma-1} + \cdots + \rho_0 \tag{3.6}$$

$$L_\varepsilon(D) = D^l + \mu_{l-1} D^{l-1} + \cdots + \mu_0 \tag{3.7}$$

where $D = d/dt$ is the differential operator, ρ_i, μ_i are constant coefficients, σ, l are orders of differential polynomials.

This includes the case of most common type of disturbance and reference signals occurring in practice such as polynomial, sinusoidal type signals, etc. $R(D)$ is the greatest common divisor of $L_r(D)$ and $L_\varepsilon(D)$ as follows:

$$L_r(D) = R(D)U(D) \tag{3.8}$$

$$L_\varepsilon(D) = R(D)V(D) \tag{3.9}$$

where $U(D), V(D)$ are factors of $L_r(D)$ and $L_\varepsilon(D)$, respectively.

If the greatest common divisor $R(D)$ of the differential polynomial operators $L_r(D)$ and $L_\varepsilon(D)$ is equal to a constant, $L_r(D)$ and $L_\varepsilon(D)$ are coprime. Furthermore, finding the least common multiple of the two differential polynomial operators involves finding their common multiple with the smallest order polynomial.

$L(D)$ is defined as the least common multiple of $L_r(D)$ and $L_\varepsilon(D)$ and can be obtained from Eqs. (3.6) ~ (3.9) using the differential polynomial operator with constant coefficients as follows:

$$\begin{aligned} L(D) &= \frac{L_\varepsilon(D)L_r(D)}{R(D)} = U(D)R(D)V(D) = V(D)L_r(D) \; or \; U(D)L_\varepsilon(D) \\ &= D^q + \alpha_{q-1}D^{q-1} + \cdots + \alpha_0 \end{aligned} \tag{3.10}$$

where $dim\{V(D)\} = q - \sigma$ and $dim\{U(D)\} = q - l$, $dim\{R(D)\} = l + \sigma - q$ and $dim\{L(D)\} = q \geq \dim\{L_r(D)\} \; or \; \dim\{L_\varepsilon(D)\}$.

For simplicity, let us consider a SISO system case with disturbance in Eq. (3.1). Then the block diagram of the closed loop control system can be shown in Fig. 3.1.

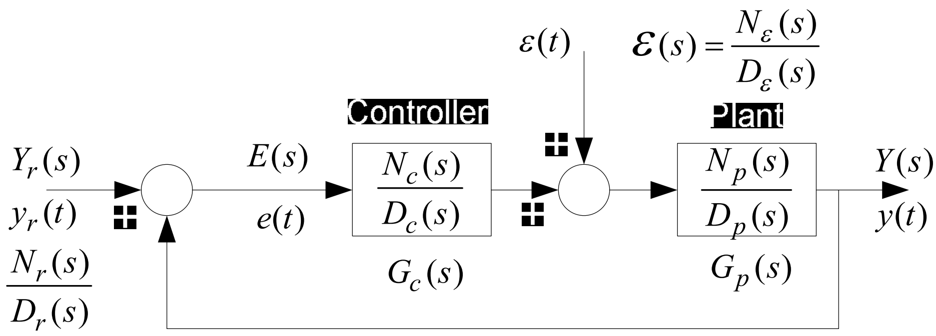

Fig. 3.1 Block diagram of a closed-loop control system

From Fig. 3.1, the output error after Laplace transform for the reference signal with disturbance $\varepsilon(s)$ is obtained as:

$$E(s)=\frac{1}{1+G_c(s)G_p(s)}Y_r(s)-\frac{G_p(s)}{1+G_c(s)G_p(s)}\varepsilon(s)=E_r(s)-E_\varepsilon(s) \tag{3.11}$$

where

$$\begin{cases} E_r(s)=\dfrac{D_p(s)D_c(s)}{D_p(s)D_c(s)+N_p(s)N_c(s)}\dfrac{N_r(s)}{D_r(s)} \\ E_\varepsilon(s)=\dfrac{D_c(s)N_p(s)}{D_p(s)D_c(s)+N_p(s)N_c(s)}\dfrac{N_\varepsilon(s)}{D_\varepsilon(s)} \end{cases} \tag{3.12}$$

The following polynomial equation obtained from Eq. (3.12) is the closed-loop characteristic polynomial equation of the closed-loop system depicted in Fig. 3.1 consisting of the polynomial poles and polynomial zeros of transfer functions of plant and controller and its roots are the closed-loop poles:

$$D_p(s)D_c(s)+N_p(s)N_c(s)=0 \tag{3.13}$$

[Theorem 1] <Internal model principle based on least common polynomial model>

Let us assume that the given system of Eq. (3.1) and the controller of $G_c(s)$ has no transmission zeros at the origin point and the closed loop poles of Fig. 3.1 are located in open left half plane under disturbance condition. Under assumptions of Eqs. (3.4) and (3.5) for the disturbance and reference signals, the output error $e(t)$ of Eq. (3.3) becomes zero: $\lim_{t\to\infty} e(t)=0$ if and only if the least common multiple polynomial for disturbance and reference signals is a factor of $D_c(s)$.

[Proof of Theorem 1]

By the final value theorem, the error function shown in Fig. 3.1 can be given as follows:

$$\lim_{t\to\infty} e(t)=\lim_{s\to 0} sE(s)=\lim_{s\to 0} sE_r(s)-\lim_{s\to 0} sE_\varepsilon(s) \tag{3.14}$$

For the sufficient condition, in order to obtain, firstly, $\lim_{t\to\infty} e(t) = 0$ the first term in the right side of Eq. (3.14) becomes $\lim_{s\to 0} sE_r(s) \to 0$.

The denominator $D_r(s)$ of $Y_r(s)$, Laplace transform of the reference y_r, must be included as a factor of the open-loop characteristic polynomial $D_p(s)D_c(s)$. That is, there is a polynomial, say $Q(s)$, such that $D_p(s)D_c(s) = Q(s)D_r(s)$. This means that the tracking controller must be designed in such way that the open-loop transfer function, $G_p(s)G_c(s)$, contains a model of the reference signal to be tracked.

For the reference signal of Eq. (3.4), its Laplace transform can be written by (SISO case)

$$Y_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{N_{r0}(s)}{L_r(s)} y_r(0) \text{ for } y_r^{(i)}(0) = 0 \ (i = 1, L, \sigma) \tag{3.15}$$

where $N_{r0}(s) = s^{\sigma-1} + \rho_{\sigma-1}s^{\sigma-2} + \cdots + \rho_2 s + \rho_1$,

$D_r(s) = L_r(s) = s^{\sigma} + \rho_{\sigma-1}s^{\sigma-1} + \cdots + \rho_1 s + \rho_0$, and $N_r(s) = N_{r0}(s)y_r(0)$.

When the open loop characteristic polynomial $D_p(s)D_c(s)$ includes the reference model of $L_r(s)$, $D_p(s)D_c(s)$ yields:

$$D_p(s)D_c(s) = L_r(s)D_{pc}(s) \text{ or } D_r(s)D_{pc}(s) \tag{3.16}$$

where $D_{pc}(s)$ is a common factor of the denominator $D_p(s)D_c(s)$ of the open-loop transfer function $G_p(s)G_c(s)$.

Then the 1^{st} term in the right side of Eq. (3.14) using Eqs. (3.15) and (3.16) can be rearranged as

$$\begin{aligned} \lim_{t\to\infty} e_r(t) &= \lim_{s\to 0} sE_r(s) \\ &= \lim_{s\to 0} \frac{D_r(s)D_{pc}(s)}{D_r(s)D_{pc}(s)+N_p(s)N_c(0)} \frac{N_{r0}(s)y_r(0)}{D_r(s)} \\ &= \lim_{s\to 0} s\frac{N_{r0}(s)D_{pc}(s)y_r(0)}{D_r(s)D_{pc}(s)+N_p(s)N_c(ds)} \\ &= \lim_{s\to 0} s\frac{[s^{\sigma-1}+\rho_{\sigma-1}s^{\sigma-2}+\cdots+\rho_2 s+\rho_1]D_{pc}(s)y_r(0)}{[s^{\sigma}+\rho_{l-1}s^{\sigma-1}+\cdots+\rho_1 s+\rho_0]D_{pc}(s)+N_p(s)N_c(s)} \\ &= \lim_{s\to 0} s\frac{\rho_1(0)D_{pc}(0)y_r(0)}{\rho(0)D_{pc}(s)+N_p(0)N_c(0)} = 0 \end{aligned} \tag{3.17}$$

where $N_p(0)$ and $N_c(0)$ are not zero because the transmission zeros are not at the origin that is $N_p(s) = det[RSM(s)] = det[(sI - A)G_p(s)] \neq 0$ at $s = 0$ and $N_c(s)$ is also done similarly.

$$RSM(s) = \begin{bmatrix} sI - A & B \\ -C & 0 \end{bmatrix} : Rosenbrock\ syetem\ matrix \tag{3.18}$$

Using the above shown proof for the 1st term, the 2nd term of Eq. (3.14) can be easily proven.

Secondly, the 2nd term in the right side of Eq. (3.14) must satisfy that the tracking controller $G_c(s)$ stabilizes the closed loop system and must obtain the same order model or above the order model of the disturbance signal. For the disturbance signal of Eq. (3.5), its Laplace transform can be expressed by (SISO case) the form:

$$\varepsilon(s) = \frac{N_\varepsilon(s)}{D_\varepsilon(s)} = \frac{N_{\varepsilon 0}(s)}{L_\varepsilon(s)}\varepsilon(0) \text{ for } \varepsilon^{(i)}(0) = 0 \ \ (i = 1, \cdots, l) \tag{3.19}$$

where $N_{\varepsilon 0}(s) = s^{l-1} + \mu_{l-1}s^{l-2} + \cdots + \mu_2 s + \mu_1$, $D_\varepsilon(s) = L_\varepsilon(s) = s^l + \mu_{l-1}s^{l-1} + \cdots + \mu_1 s + \mu_0$ and $N_\varepsilon(s) = N_{\varepsilon 0}(s)\varepsilon(0)$.

When the open loop characteristic polynomial $D_c(s)$ includes the disturbance model of $L_\varepsilon(s)$, $D_c(s)$ can be written by the form:

$$D_c(s) = L_\varepsilon(s) D_{\varepsilon c}(s) \tag{3.20}$$

where $D_{\varepsilon c}(s)$ is a common factor of $D_c(s)$.

Then the 2nd term in the right side of Eq. (3.14) using (3.19) and (3.20) can be rearranged as

$$
\begin{aligned}
\lim_{t\to\infty} e_{\varepsilon}(t) &= \lim_{s\to 0} sE_{\varepsilon}(s) \\
&= \lim_{s\to 0} s \frac{[s^{l-1} + \mu_{\sigma-1}s^{l-2} + \cdots + \mu_2 s + \mu_1] D_{\varepsilon c}(s) N_p(s)\varepsilon(0)}{[s^{l} + \mu_{l-1}s^{l-1} + \cdots + \mu_1 s + \mu_0] D_{\varepsilon c}(s) D_p(s) + N_p(s) N_c(s)} \\
&= \lim_{s\to 0} s \frac{D_{\varepsilon c}(0) N_p(0)\mu_1}{D_{\varepsilon c}(0) D_p(0)\mu_0 + N_p(0) N_c(0)} \varepsilon(0) = 0
\end{aligned}
\tag{3.21}
$$

Using the least common multiple of disturbance and reference signals $L(s)$, the denominator of the controller can be expressed as

$$
D_c(s) = L(s) D_{\varepsilon r}(s) = D_r(s) V(s) D_{\varepsilon r}(s) = D_{\varepsilon}(s) U(s) D_{\varepsilon r}(s) \tag{3.22}
$$

where $D_{\varepsilon r}(s)$ is a common factor of two signals.

The steady state error with respect to reference signal using Eqs. (3.14), (3.15) and (3.22) can be obtained as

$$\begin{aligned}
\lim_{t\to\infty} e_r(t) &= \lim_{s\to 0} sE_r(s) = \lim_{s\to 0} \frac{D_p(s)L(s)D_{\varepsilon r}(s)}{D_p(s)L(s)D_{\varepsilon r}(s)+N_p(s)N_c(s)} \frac{N_{r0}(s)y_r(0)}{D_r(s)} \\
&= \lim_{s\to 0} \frac{D_p(s)D_r(s)V(s)D_{\varepsilon r}(s)}{D_p(s)L(s)D_{\varepsilon r}(s)+N_p(s)N_c(s)} \frac{N_{r0}(s)y_r(0)}{D_r(s)} \\
&= \lim_{s\to 0} \frac{N_{r0}(s)D_p(s)L(s)D_{\varepsilon r}(s)y_r(0)}{D_p(s)L(s)D_{\varepsilon r}(s)+N_p(s)N_c(s)} \\
&= \lim_{s\to 0} s\frac{[s^{\sigma-1}+\rho_{\sigma-1}s^{\sigma-2}+\cdots+\rho_2 s+\rho_1]D_p(s)D_{\varepsilon r}(s)y_r(0)}{[s^q+\alpha_{q-1}s^{q-1}+\cdots+\alpha_1 s+\alpha_0]D_p(s)D_{\varepsilon r}(s)+N_p(s)N_c(s)} \\
&= \lim_{s\to 0} s\frac{\rho_1 D_p(0)D_{\varepsilon r}(0)y_r(0)}{\alpha_0 D_p(0)D_{\varepsilon r}(0)+N_p(0)N_c(0)} = 0
\end{aligned} \tag{3.23}$$

And the steady state error with respect to disturbance signal using Eqs. (3.14), (3.19) and (3.22) can be obtained as

$$\begin{aligned}
\lim_{t\to\infty} e_\varepsilon(t) &= \lim_{s\to 0} sE_\varepsilon(s) = \lim_{s\to 0} s\frac{L(s)D_{\varepsilon r}(s)N_p(s)}{D_p(s)L(s)D_{\varepsilon r}(s)+N_p(s)N_c(s)} \frac{N_{\varepsilon 0}(s)\varepsilon(0)}{D_\varepsilon(s)} \\
&= \lim_{s\to 0} s\frac{N_{\varepsilon 0}(s)U(s)D_{\varepsilon r}(s)N_p(s)\varepsilon(0)}{D_\varepsilon(s)D_p(s)U(s)D_{\varepsilon r}(s)+N_p(s)N_c(s)} \\
&= \lim_{s\to 0} s\frac{[s^{l-1}+\mu_{\sigma-1}s^{l-2}+\cdots+\mu_2 s+\mu_1]U(s)D_{\varepsilon r}(s)N_p(s)\varepsilon(0)}{[s^l+\alpha_{q-1}s^{q-1}+\cdots+\alpha_1 s+\alpha_0]D_p(s)D_{\varepsilon r}(s)+N_p(s)N_c(s)} \\
&= \lim_{s\to 0} s\frac{\mu_1 U(0)D_{\varepsilon r}(0)N_p(0)\varepsilon(0)}{\alpha_0 D_p(0)D_{\varepsilon r}(0)+N_p(0)N_c(0)} = 0
\end{aligned} \tag{3.24}$$

To satisfy $e(t) \to 0$ for $t \to \infty$, the two error conditions of $e_r(t) \to 0$ and $e_\varepsilon(t) \to 0$ must be satisfied simultaneously. Therefore, the controller $G_c(s)$ must includes the least common multiple $L(s)$ in its demoninator $D_c(s)$.

As we can see, from the results of Eqs. (3.23) and (3.24), if the controller $G_c(s)$ includes the order of the least common multiple polynomial model for reference and disturbance signals as a factor of $D_c(s)$, the tracking error $e(t)$ becomes zero for $t \to \infty$.

The proof of the necessary condition is trivial.

EOD

In Theorem 1, the polynomial differential operator $L(D)$ should become a factor of the open-loop characteristic polynomial $D_p(D)D_c(D)$ such that $D_p(D)D_c(D) = Q(D)L(D)$.

From the above stated result, the IMP can be incorporated into the state equation form based on the polynomial differential operator.

In order to satisfy the internal model principle (IMP) for the robust tracking control system, the reference polynomial differential operator of $L(D)$ must be operated on the plant of Eqs. (3.1)~(3.3).

To show the applicability of the polynomial differential operator, let us consider a modified tracking control system of $G_c(s)$ in Fig. 1 defined as :

$$G_c(s) = KG_{ck}(s) \tag{3.25}$$

where $G_{ck}(s)=\dfrac{N_{ck}(s)}{D_{ck}(s)}$ and K is an appropriate constant gain, which can be obtained by some design procedure in the following. In the case of PI controller, it can be treated as proportional or integral gain.

$E_c(s)$ is defined as the output of $G_{ck}(s)$ with respect to $E(s)$ as follows:

$$E_c(s)=G_{ck}E(s)=\frac{N_{ck}(s)}{D_{ck}(s)}E(s)=\frac{Z(s)}{D_{ck}(s)} \tag{3.26}$$

From Eq. (3.26) , $Z(s)$ can be obtained as the following:

$$Z(s)=D_{ck}(s)E_c(s)=N_{ck}(s)E(s) \tag{3.27}$$

When the closed loop system is stabilized by $G_{ck}(s)$ with gain K, Eq. (3.25) plays in role of the IMP based tracking controller. For example, if there is no disturbance, Eq. (3.27) can be considered in the time domain as the following operator property:

$$z(t)=P_c(D)e_c(t) \tag{3.28}$$

where $P_c(D)=D_{ck}(D)$ in which its operator dimension can be described according to the reference model.

Operating inverse of $P_c(D)$ in Eq. (3.28) for both sides yields:

$$e_c(t)=P_c^{-1}(D)z(t) \tag{3.29}$$

which is given as the input for the gain K.

Also, under disturbance, the above introduced concept for the tracking controller gives us the basic idea of the IMP in the time domain. Using the polynomial differential operator, the tracking controller can be obtained in the following section.

Operating $L(D)$ for ε_i and y_{ri} of Eqs. (3.4)-(3.5), the following are obtained

$$\begin{Bmatrix} L(D)y_{ri} = U(D)R(D)V(D)y_{ri} = V(D)L_r(D)y_{ri} = 0 \\ L(D)\zeta_i = U(D)R(D)V(D)\omega_i = U(D)L_\zeta(D)\zeta_i = 0 \end{Bmatrix} \tag{3.30}$$

where the dimension of q holds $q \geq l \ or \ q \geq \sigma$.

The adaption of IMP to the robust MIMO servo controller system design is attempted by 3 steps :

[Step 1] by operating the polynomial differenetial operator to the given system of Eq. (3.1) and also the output error vector of Eq. (3.3),

[Step 2] an extended system is obtained by using the operated system and output errors obtained through the step 1,

[Step 3] for the extended system, a regulator problem is solved based on the well known design method such as pole assignment or optimal control.

3.2 Operating Polynomial Differential Operator

Firstly, to eliminate the effect of disturbance in Eq. (3.1), operating the polynomial differential operator of $L(D)$ to both sides of Eq. (3.1) by using Eq. (3.30), Eq. (3.1) can be written as

$$\frac{d}{dt}\{L(D)x\} = AL(D)x + BL(D)u \tag{3.31}$$

The i^{th} output error of Eq. (3.3) can be written as

$$e_i(t) = y_i(t) - y_{ri}(t) \text{ for } i = 1 \sim p \tag{3.32}$$

Secondly operating $L(D)$ to Eq. (3.32) and using the property of (3.30), the followings can be obtained.

$$\begin{aligned} L(D)e_i(t) &= D^q e_i + \alpha_{q-1} D^{q-1} e_i + \cdots + \alpha_1 De_i + \alpha_0 e_i \\ &= L(D)y_i - L(D)y_{ri} = c_i^T L(D)x \quad for \quad i = 1 \sim p \end{aligned} \tag{3.33}$$

or

$$D^q e_i = -\alpha_{q-1} D^{q-1} e_i - \cdots - \alpha_1 De_i - \alpha_0 e_i + c_i^T L(D)x \; for \quad i = 1, \cdots, p \tag{3.34}$$

Eq. (3.34) can be described into the matrix form as follows:

$$\dot{z}_i = M_i L(D)x + Nz_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ c_i^T \end{bmatrix} L(D)x + Nz_i \tag{3.35}$$

where

$$N = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & -\alpha_3 & \cdots & -\alpha_{q-1} \end{bmatrix} \in R^q \quad ,$$

$$M_i = \begin{bmatrix} 0 \\ \cdots \\ c_i^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} c_i^T \in R^{q \times q},\ c_i^T = \begin{bmatrix} c_{i1} & c_{i2} & \cdots & c_{in} \end{bmatrix} \in R^{1 \times n},$$

$$c^T = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{p1} & c_{p2} & \cdots & c_{pn} \end{bmatrix} = \begin{bmatrix} c_1^T \\ c_2^T \\ \vdots \\ c_p^T \end{bmatrix} \in R^{n \times p},\ \text{and}\ z_i = \begin{bmatrix} e_i & e_i^{(1)} & \cdots & e_i^{(q-1)} \end{bmatrix}^T \in R^q$$

3.3 Extended System and Controller Design

By combining the operated system , Eqs. (3.31) and (3.35), an extended system can be obtained as follows:

$$\dot{x}_e = A_e x_e + B_e v \tag{3.36}$$

where

$$A_e = \begin{bmatrix} A & 0 & \cdots & 0 & 0 \\ \begin{bmatrix} 0 \\ c_1^T \end{bmatrix} & N & 0 & \vdots & 0 \\ \begin{bmatrix} 0 \\ c_2^T \end{bmatrix} & 0 & N & 0 & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \begin{bmatrix} 0 \\ c_p^T \end{bmatrix} & 0 & \cdots & 0 & N \end{bmatrix} \in R^{(n+pq)\times(n+pq)},\ B_e = \begin{bmatrix} B \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \in R^{(n+pq)\times m},$$

$$x_e = \begin{bmatrix} L(D)x \\ z_1 \\ z_2 \\ \vdots \\ z_p \end{bmatrix} \in R^{n+pq}$$

$x_e = \begin{bmatrix} L(D)x^T & z^T \end{bmatrix}^T$ is an extended system state variable vector, $v = L(D)u \in R^m$ is a new control law for the extended system , and $z = \begin{bmatrix} z_1^T & z_2^T & \cdots & z_p^T \end{bmatrix}^T \in R^{pq}$ is an error variable vector for the extended system.

A new control law for the extended system is defined by the following form:

$$v = L(D)u = -Fx_e \in R^m \tag{3.37}$$

$F = \begin{bmatrix} F_x & F_z \end{bmatrix} \in R^{m\times(n+pq)}$ is a feedback control gain matrix, and $F_x \in R^{m\times n}$ and $F_z \in R^{m\times pq}$ are feedback control gain matrices for $L(D)x$ and z, respectively.

A new error variable vector for the extended system can be defined as

$$\zeta = L^{-1}(D)z \tag{3.38}$$

where $\zeta = \begin{bmatrix} \zeta_1^T & \zeta_2^T & \cdots & \zeta_p^T \end{bmatrix}^T \in R^{pq}$, and $\zeta_i \in R^q$ for $i = 1, \cdots, p$

Using Eq. (3.37)~(3.38), the control law of Eq. (3.1) can be obtained as follows:

$$u = -Fx_\zeta = -\begin{bmatrix} F_x & F_z \end{bmatrix} \begin{bmatrix} x \\ \zeta \end{bmatrix} \tag{3.39}$$

where $x_\zeta \in R^{n+pq}$ is a new extended system variable vector.

[**Theorem 2**] <Controllability of the extended system >

Given the system of Eq. (3.1) with the assumptions of Eqs. (3.4)~(3.5) for the disturbance and reference inputs, the extended system Eq. (3.36) obtained by operating $L(D)$ is controllable if the following two conditions are held:

(1) The system (A, B) of Eq. (3.1) is controllable.

(2) The following matrix $\bar{V}_e$ of Eq. (3.40) related with the extended system of Eq. (3.36) has

$$rank(\bar{V}_e) = n + pq.$$

where

$$\bar{V}_e = \begin{bmatrix} B & \gamma_1 & \gamma_2 & \cdots & \gamma_{q-1} & \gamma_q \\ 0 & CB & CAB & \cdots & CA^{q-2}B & CA^{q-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & CB & CA \\ 0 & 0 & 0 & \cdots & 0 & C \end{bmatrix} \in R^{(n\times pq)\times(n\times mq)} \quad \text{and} \quad m \geq p \tag{3.40}$$

with

$\gamma_1 = AB + \alpha_{q-1}B \quad \in R^{n\times m}$

$\gamma_2 = A^2B + \alpha_{q-1}AB + \alpha_{q-2}B \quad \in R^{n\times m}$

$\gamma_2 = A^2B + \alpha_{q-1}AB + \alpha_{q-2}B \quad \in R^{n\times m}$

$\vdots$

$\gamma_{q-1} = A^{q-1}B + \alpha_{q-1}A^{q-2}B + \cdots + \alpha_1 B \quad \in R^{n\times m}$

$\gamma_q = A^q + \alpha_{q-1}A^{q-1} + \alpha_{q-2}A^{q-2} + \cdots + \alpha_1 A + \alpha_0 I \quad \in R^{n\times n}$

Full proof of theorem 2 and Eq. (3.40) are written in **Appendix B**.

From Eqs. (3.36) and (3.37), the closed loop system of the extended system is obtained as

$$\dot{x}_e = (A_e - B_e F)x_e \tag{3.41}$$

Theorem 2 shows that the servo controller problem for Eq. (3.1) with reference and disturbance of Eqs. (3.4) and (3.5) becomes a regulator design problem for the extended system Eq. (3.36) such that the closed loop system of Eq. (3.41) is asymptotically stabilized by designing the feedback control law of Eq. (3.37) with a feedback control matrix F so as to be $\mathrm{Re}\{\lambda_i(A_e - B_e F\} < 0$.

[Theorem 3] < Regulator design[37]>

Consider the system of Eq. (3.1) and assume that Theorem 2 holds; then there exists gain matrix $F = \begin{bmatrix} F_x & F_z \end{bmatrix}$ so that the closed loop control system obtained by applying the feedback control law of Eq. (3.37) to the extended system of Eq. (3.36) is asymptotically stable, i.e. There exists a gain matrix $F = \begin{bmatrix} F_x & F_z \end{bmatrix}$ so that the following matrix is asymptotically stable.

$$A_F = A_e - B_e F = A_e - B_e \begin{bmatrix} F_x & F_z \end{bmatrix} \tag{3.42}$$

[Corollary]

When the feedback control law of the extended system of Eq. (3.36) is designed based on Theorems 2 and 3, the output error vector of Eq. (3.32) becomes $e(t) \to 0 \;\; as \;\; t \to \infty$.

[Proof of corollary]

Since $x_e \to 0$ by regulator design result, that is $x_e \to 0$ means $\begin{bmatrix} Lx^T & z^T \end{bmatrix}^T \to 0$ as follows:

$$\begin{aligned} x_e &= \begin{bmatrix} L(D)x^T & z_1^T & z_1^T & \cdots & z_p^T \end{bmatrix}^T \\ &= \begin{bmatrix} L(D)x^T & \begin{bmatrix} e_1 & e_1^{(1)} & \cdots & e_1^{(q-1)} \end{bmatrix}^T & \cdots & \cdots & \begin{bmatrix} e_p & e_p^{(1)} & \cdots & e_p^{(q-1)} \end{bmatrix}^T \end{bmatrix}^T \end{aligned} \tag{3.43}$$

As the result, the error $e(t) = \begin{bmatrix} e_1(t) & e_2(t) & \cdots & e_p(t) \end{bmatrix}^T \to 0 \; as \; t \to \infty$.

EOD

[**Theorem 4**] < Stabilizing servo compensator>

Let a new control input v in Eq. (3.37) be given for the extended system of Eq. (3.36) such that the extended system satisfies Theorem 2 and also based on Theorem 3, the new control input is obtained by well known regulator design methods[23, 24, 17, 31-39]. Then, the robust servo compensator is given by the following form:

$$\frac{d\zeta}{dt} = N_z \zeta + I_\zeta e \tag{3.44}$$

$$N_z = \begin{bmatrix} N & 0 & \cdots & 0 \\ 0 & N & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & N \end{bmatrix} \in R^{pq \times pq},$$

$$I_\zeta = \begin{bmatrix} \lambda & 0 & \cdots & 0 \\ 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda \end{bmatrix} \in R^{pq \times n} \quad and \quad \lambda = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T \in R^q$$

where ζ is a new error variable vector defined by

$$\zeta = L^{-1}(D)z = \begin{bmatrix} \zeta_1^T & \zeta_2^T & \cdots & \zeta_p^T \end{bmatrix}^T \in R^{pq}, \quad \zeta_i \in R^q \quad \text{for } i = 1, \cdots, p \ or \ 1 \sim p$$

<Proof of theorem 4>

Using the new input vector v, the extended system of Eq. (3.36) can be written by

$$\frac{d}{dt}\{L(D)x\} = [A - BF_x]L(D)x - BF_z z + L(D)\varepsilon \tag{3.45}$$

$$\frac{d}{dt}z = N_z z + I_\zeta L(D)e \tag{3.46}$$

$$\begin{cases} L(D)y_{ri} = U(D)R(D)V(D)y_{ri} = V(D)L_r(D)y_{ri} = 0 \\ L(D)\varepsilon_i = U(D)R(D)V(D)\varepsilon_i = U(D)L_\varepsilon(D)\varepsilon_i = 0 \end{cases} \tag{3.47}$$

where $L(D)\varepsilon = 0$ from Eq. (3.47).

The 2nd term for the polynomial differential operator of $L(D)$ in Eq. (3.46) due to Eq. (3.47) has the following relation:

$$L(D)e = L(D)(y - y_r) = CL(D)x \tag{3.48}$$

By operating the inverse polynomial differential operator $L^{-1}(D)$ for Eqs. (3.45) and (3.46), the following equations can be obtained due to $L^{-1}(D)(L(D)\varepsilon) = \varepsilon$ by $L(D)\varepsilon = \mathbf{0}$:

$$\frac{d}{dt}x = [A - BF_x]x - BF_z\zeta + \varepsilon = Ax + Bu + \varepsilon \tag{3.49}$$

$$\frac{d}{dt}\zeta = N_z\zeta + I_\zeta e \tag{3.50}$$

$$u = -Fx_{\zeta} = -\begin{bmatrix} F_x & F_z \end{bmatrix} \begin{bmatrix} x \\ \zeta \end{bmatrix} \tag{3.51}$$

which holds the servo compensator of Eq. (3.44).

EOD

In Theorem 4, the servo compensator of Eq. (3.44) includes the model of reference and disturbance signals since the matrix N_z is composed of the least common multiple model of two signals. It proposes the internal model principle based on the polynomial differential operator.

The configuration of the proposed servo control system can be described as shown in Fig. 3.2.

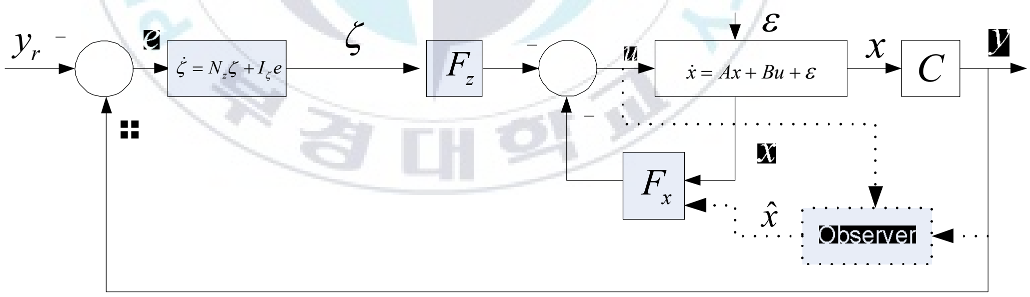

Fig. 3.2. Configuration of the proposed servo control system

The observer in Fig. 3.2 can be used when unknown states exist in the given system, and is expressed in next section.

Chapter 4: Application for SISO System of AC Induction Motor

This chapter first describes the prototype of the experimental AC induction motor drive system. The motor testing system including AC induction motor, torque sensor and powder brake that are connected in series is set up. Voltage source inverter based on DSP is developed. The analog amplifiers for measuring two current signals and DC voltage from power source signal are described. Secondly, a full order observer of the extended system is designed to estimate its unknown states. Thirdly, to verify the effectiveness and the applicability of the proposed servo controller design method in chapter 3, simulation and experimental results of the proposed servo controller for a step type of disturbance and 3 references such as step, ramp and parabola are shown for AC induction motor system as single-input and single-output(SISO) system compared with those of the conventional PI controller.

4.1 Motor Testing System

The motor testing system used in the experiment is shown in Fig. 4.1 It consists of an AC induction motor, an encoder, a torque sensor, a powder brake and two couplings. AC induction motor for this paper has 3 phase, four poles and $2Hp / 1.5Kw$. The powder brake works as a load and it can be controlled by the adjustment of the voltage from DC $0V$ to $24V$ represent constant load form 0 Nm to 33 Nm. A torque transducer rated at 10 $Kgmf$ and an encoder with 1024 counts/rev are used for the verification of the load torque and speed estimation.

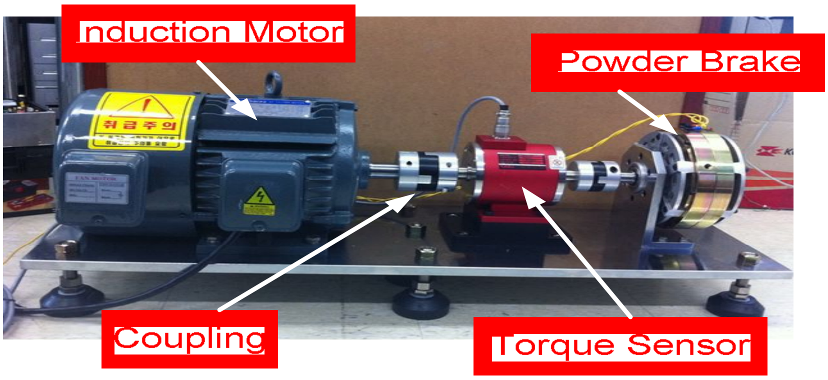

Fig. 4.1 Motor testing system

The induction motor under consideration has three phase, four poles and 2HP/1.5Kw with its specification as shown in Table 4.1.

Table 4.1 Specification of 1.5*Kw* induction motor

Description	Value	Unit
Rated Power	1.5	*Kw*
Rated Voltage	220	*V*
Rated Current	6.0	*A*
Rated Frequency	60	*Hz*
Rated speed	1750	*rpm*
Rated torque	10	*Nm*
Rated flux	0.49	*Wb*

The powder brake works as a load and it can be controlled by the adjustment of the voltage from DC 0 V to 24 V . Fig. 4.2 shows the torque curve of the powder brake with respect to the current.

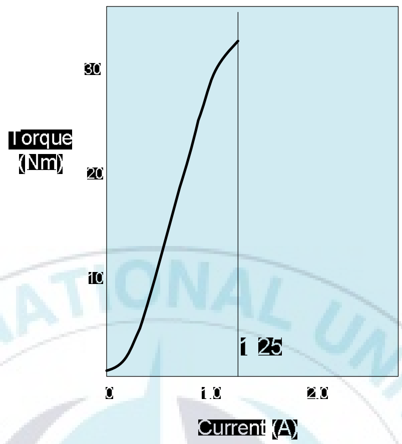

Fig. 4.2 Torque curve with respect to the current of the powder brake.

A torque transducer rated at 10 *Kgfm* and an encoder with 1024 counts/rev are used for the verification of the load torque and speed estimation. Fig. 4.3 shows the wire connection of the torque sensor. The excitation voltage is recommended at 10V.

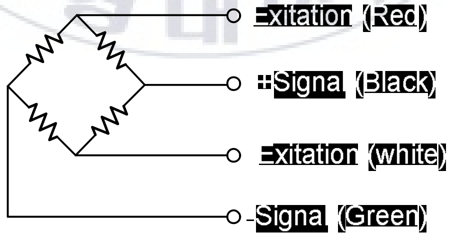

Fig. 4.3 Wire connection of the torque sensor.

4.2 Voltage Source Inverter

Fig. 4.4 shows voltage source inverter (VSI) based on DSP 320F28335 used in experiments. The inverter includes two parts: the controller board and the IGBT driver. The controller board is designed based on DSP 320F28335 of Texas Instrument. The IGBT driver is designed based on smart power module FSBB30CH60 of Fairchild.

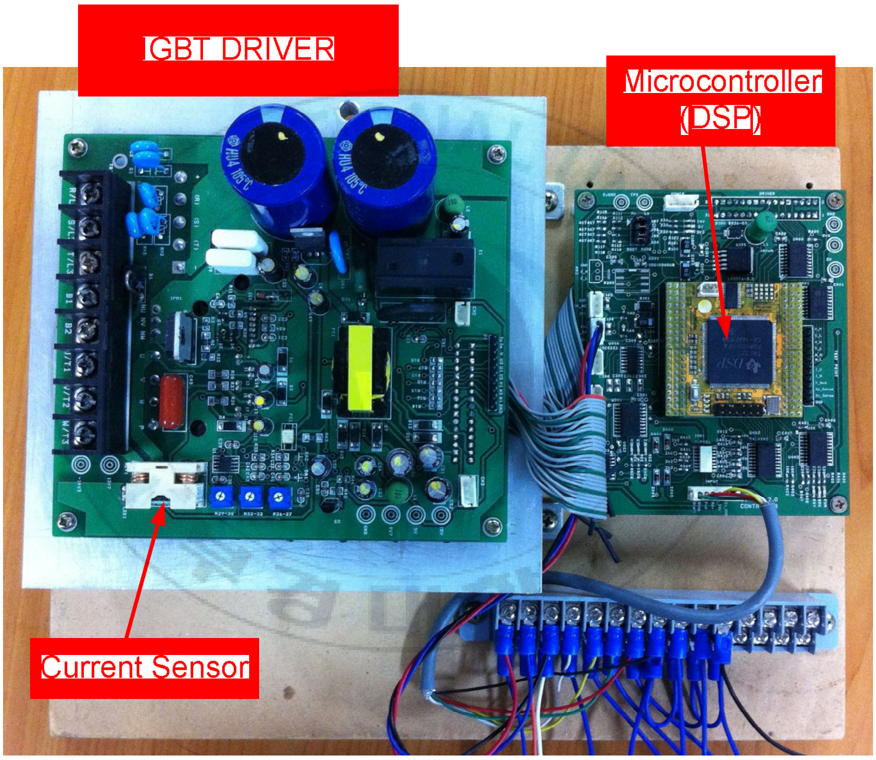

Fig. 4.4 Voltage source inverter

Fig. 4.5 shows two versions of controller board used in experiment. In the first version, a DSP module (a) is plugged in the socket of the controller board. In the second version, the DSP chip (b) is soldered directly on the controller board.

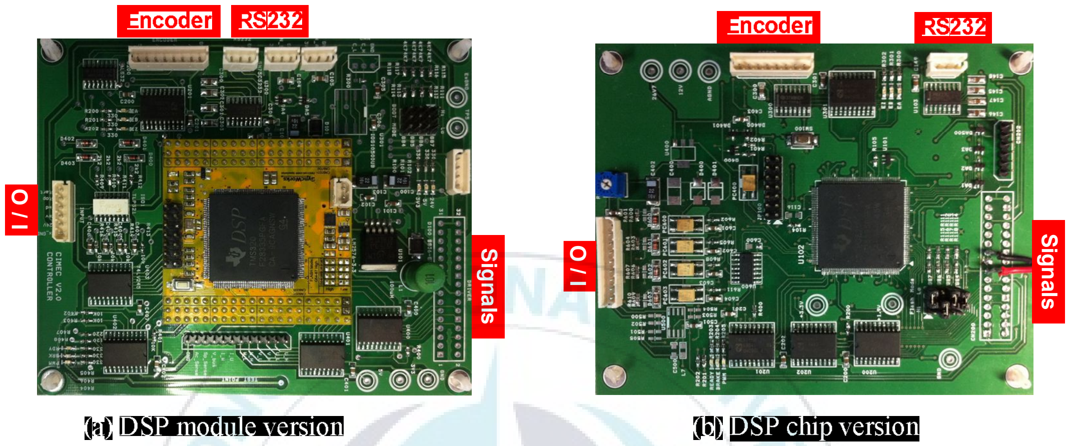

Fig. 4.5 DSP based controller board

4.2.1 Overview of the DSP 320F28335

The controller board is based on DSP 320F28335. The DSP has a 32-bit CPU and a single-precision 32-bit floating-point unit (FPU), which enables the floating-point computation to be performed in hardware. Moreover, the CPU of the F28335 has an 8-stage pipeline structure, which makes the CPU be able to execute eight instructions simultaneously in one system clock period. The 150*Mhz* system clock is provided by an on-chip oscillator and a phase-locked loop (PLL) circuit. The physical memory of the F28335 comprises of a 34Kx16 single-access random-access memory (SARAM), a 256Kx16 Flash, an 8Kx16 read-only memory (ROM), a 1Kx16 one-time programmable memory (OTP) and the registers. F28335 also has the feature of direct memory access (DMA). With the DMA bus, the data can be transferred from one part of the DSP to the other part without the interaction of the CPU, which increases the data transmission speed. As the F28335 is designed mainly for industrial applications, it has plenty of peripheral circuits. For example, the 16-chanels, 12-bits ADC module, the PWM module and the encoder module can be used for motor control purposes. Five kinds of communications can be achieved by the controller area network (CAN) module, the serial communication interface (SCI) module, the serial peripheral interface (SPI), the multichannel buffered serial port (McBSP) module and the inter-integrated circuit (I2C) module. 96 interrupts are supported by F28335. These interrupts are governed by the peripheral interrupt expansion (PIE), which enables or disables some interrupts, decides the interrupts' priorities and informs the CPU of the occurrence of a new interrupt.

4.2.2 Switching Devices

FSBB30CH60 (600V-30A) is an advanced smart power module that Fairchild has newly developed and designed to provide very compact and high performance AC motor drives. FSBB30CH60 is mainly targeting low-power inverter. It combines optimized circuit protection with drive matched to low-loss IGBTs. Its reliability is further enhanced by the integrated under-voltage lock-out and short-circuit protection. The high speed built-in HVIC provides optocoupler-less single-supply IGBT gate driving capability that further reduces the overall size of the inverter design.

4.2.3 Analog Input Voltage

The analog-to-digital converter (ADC) samples the analog signal and converts it into the bit form which can be processed by the DSP. The ADC is used to measure three phase currents and the DC voltage. The precision of these measurements is very important for the performance of the whole vector control system.

In practical, only two phases of current are required for computation of the control algorithm. Fig. 4.6 shows the circuit for one current sensor (**U**-phase). Another sensor is similar.

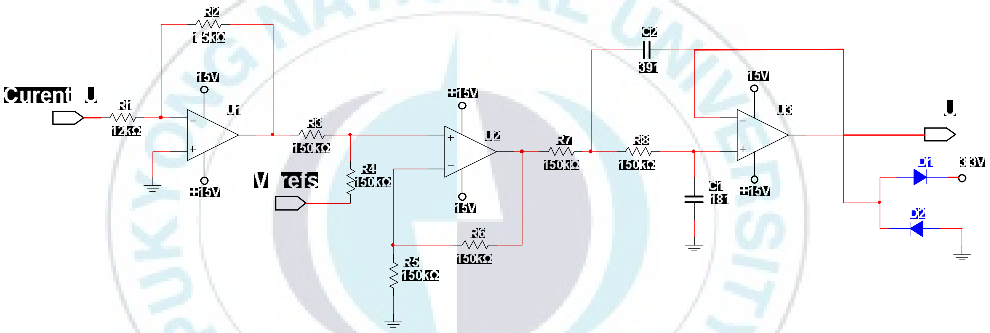

Fig. 4.6 Current sensor circuit using OP-AMP.

The current sensor has a supply voltage of ±15 V and a output voltage of ±4 V while the analog inputs for the DSP are in the range from $0V$ to $3V$. Due to this fact, the input signals need to be scaled down and biased with an OP-AMP circuit as shown in Fig. 4.6. In Fig. 4.6, following the signal from the input, the first OP-AMP is connected as an inverting amplifier. The two resistances of $1.5K\Omega$ and $12K\Omega$ scale the input voltage of ±4 V to ±0.5 V. The second OP-AMP is a summing amplifier, which shifts the input voltage by 1.5 V. The output of this OP-AMP is

$1V - 2V$. The third OP-AMP is voltage following amplifier with two capacitors $\mathbf{C_1}$, $\mathbf{C_2}$ for filtering.

The reference voltage, **V_refs** $= 1.5\ V$, in Fig. 5.6 can be obtained by using voltage follower in Fig. 4.7.

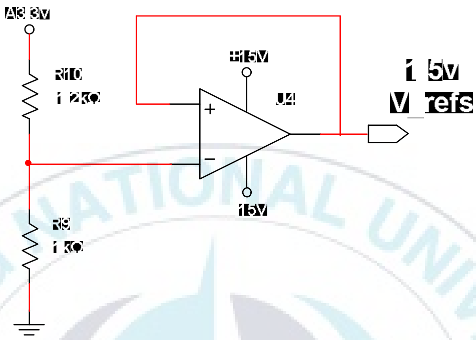

Fig. 4.7 Voltage follower

DC voltage signal can be measured by ADC module of DSP by using differential amplifier circuit shown in Fig. 4.8.

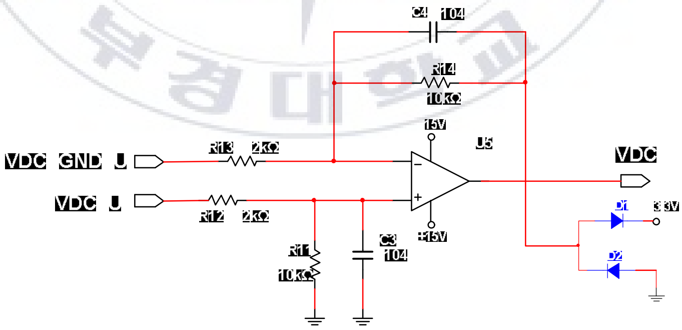

Fig. 4.8 Differential amplifier

4.3 Observer Design in the Case with Speed Sensorless

In Eq. (2.17), e_d, e_q are unknown variables. To get these variables, the following observer is adopted.

Full-order observer of Eq. (2.17) and estimation error are defined as

$$\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + Bu + K(y - C\hat{\mathbf{x}}) \tag{4.1}$$

$$\sigma = \mathbf{x} - \hat{\mathbf{x}} \tag{4.2}$$

where $\hat{\mathbf{x}}$ is the estimated state vector of $\mathbf{x}$ and K is the observer gain matrix.

From Eqs. (2.17), (4.1) and (4.2), the following is obtained.

$$\dot{\sigma} = (A - KC)\sigma \tag{4.3}$$

To be $\lim_{t \to \infty} \sigma = 0$, that is, $\mathrm{X} \to \hat{\mathrm{X}}$, K is designed to make $A - KC$ be a stable matrix by well known regulator design methods[37] .

4.4 SISO Case : Simulation and Experiment Results

Three reference inputs are chosen: reference of step type $y_r = 1000rpm$, reference of ramp type $y_r(t) = 100t$, and reference of parabola type $y_r(t) = 10t^2$. It is assumed that the disturbance is given as step type model of the following:

$$\dot{\varepsilon} \equiv \frac{d\varepsilon}{dt} = 0 \tag{4.4}$$

Table 4.2 shows parameter and initial values of 1.5kw AC induction motor.

Table 4.2 Parameter and initial values of 1.5Kw induction motor

Symbol	Description	Value	Unit
p	Number of pole pairs	2	EA
m	Moment of inertia	0.089	Kgm^2
R_s	Stator resistance	3.285	Ω
R_r	Rotor resistance	2.715	Ω
L_s	Stator inductance	0.387117	H
L_r	Rotor inductance	0.387117	H
L_m	Mutual inductance	0.374	H
b_1	Friction coefficient	0.01	Nms
$x(0)$	Initial value of state vector	$[0\ 0\ 0]^T$	-

$\hat{x}(0)$	Initial value of estimated state vector	$\left[0\ 0\ 0\right]^T$	-
$\zeta(0)$	Initial value of new error variable vector	$\begin{cases}[0] \text{ for step reference} \\ [0\ 0]^T \text{ for ramp referece} \\ [0\ 0\ 0]^T \text{ for parabola reference}\end{cases}$	-

Including the parameter and initial value from Table 4.2 to the AC induction motor mathematical model of Eq. (2.17) under disturbance with $\hat{\alpha}_r = \alpha_r$, the following can be obtained:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu + \varepsilon \tag{4.5}$$

$$y = C\mathbf{x}(t) \tag{4.6}$$

$$A = \begin{bmatrix} -701.33 & 25151.9 & 0 \\ -25151.9 & -701.33 & -0.98 \\ -2.2931 & 0.0639 & -0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 0.0239 \end{bmatrix}$$

$$C = \begin{bmatrix} -2040.8 & 715.6514 & 1 \end{bmatrix}$$

The full order observer is designed based on pole assignment method. The pole are chosen as $[-100, -101, -102]^T$, and the observer gain is obtained as $K = [-0.0246 \quad 1.2076 \quad 0.2307]^T$.

The parameter and controller gains for 3 types of reference signals are obtained by using the proposed servo controller design method as shown in Table 4.3 and is shown in Appendix D.

Table 4.3 Parameters and gains for the proposed method

Design Parameter	Reference types		
	Step	Ramp	Parabolic
Reference Model	$\dot{y}_r = 0$ $y_r(0) = 1000$	$\ddot{y}_r = 0$ $\dot{y}_r(0) = 100$ $y_r(0) = 0$	$\dddot{y}_r = 0$ $\ddot{y}_r(0) = 10$ $\dot{y}_r(0) = 0$ $y_r(0) = 0$
Matrix N	$N = [0]$	$N = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	$N = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
Matrix A_e	$\begin{bmatrix} -3.41 & -0.9045 & 0 & 0 \\ 46.5451 & 3.173 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -3.41 & -0.9045 & 0 & 0 & 0 & 0 \\ 46.5451 & 3.173 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -3.41 & -0.9045 & 0 & 0 & 0 & 0 & 0 & 0 \\ 46.5451 & 3.173 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Matrix B_e	$\begin{bmatrix} 1000 & 2069 \\ 18.046 & -37.3367 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1000 & 2029 \\ 18.049 & -37.3367 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1000 & 2029 \\ 18.049 & -37.3367 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
Assigned Poles	{-60, -61, -62, -63}	{-19,-20,-21,-22,-23}	{-18,-19,-20,-21,-22,-23}
Gain matrix F_x	$\begin{bmatrix} 2.75\times10^{7} \\ 1.49\times10^{9} \\ 471.1 \end{bmatrix}^T$	$\begin{bmatrix} 6.9724\times10^{6} \\ 4.5207\times10^{8} \\ 224.1418 \end{bmatrix}^T$	$\begin{bmatrix} 6.2852\times10^{7} \\ 4.5207\times10^{9} \\ 203.4504 \end{bmatrix}^T$
Gain matrix F_z	$\begin{bmatrix} 0.1672 \end{bmatrix}$	$\begin{bmatrix} 0.0209 \\ 0.0294 \end{bmatrix}^T$	$\begin{bmatrix} 0.0201 \\ 0.0374 \\ 0.0270 \end{bmatrix}^T$

Compensator of Eq. (3.44) or (3.50) is as follows: $\dot{\zeta} = e$ for reference of step type, $\dot{\zeta} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\zeta + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e$ for reference of ramp type and $\dot{\zeta} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\zeta + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e$ for reference of parabola type.

4.4.1 Step Reference

Figs. 4.9~4.12 show simulation and experimental results for reference of step type of $y_r(t) = 1000rpm$. Fig. 4.9 shows the control laws of the proposed method and PID method. It shows that both control laws as current in q - $axis$ converge to 0.8A after about 2 seconds. Fig. 4.10 shows the outputs for both methods. It shows that both outputs as motor speed in $q\text{-}axis$ converge to the step reference of 1000rpm. The outputs for PID shows overshoot and converges to the reference input after about 2 seconds, while the outputs for the proposed method converges to the reference input smoothly without overshoot after about 0.3 second. Fig. 4.11 shows the output errors for both methods. The output error for PID converges to zero after about 2 seconds, while the output error for the proposed method converges to zero smoothly after about 0.3 second. Fig. 4.12 shows the extimated state variables for the proposed method. $\hat{e}_d$, $\hat{e}_q$, $\hat{\omega}$ converge to about 0.014 Wb about 0.001 Wb and about 1000rpm after 2seconds, repectively. Compensator of Eq. (3.45) can be described as follows: $\dot{\zeta} = e$ for step reference.

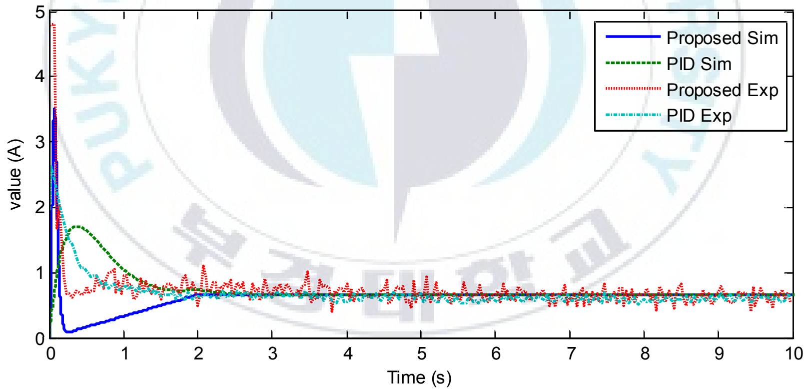

Fig. 4.9 Control law $u = i_{sq}$ for reference step input

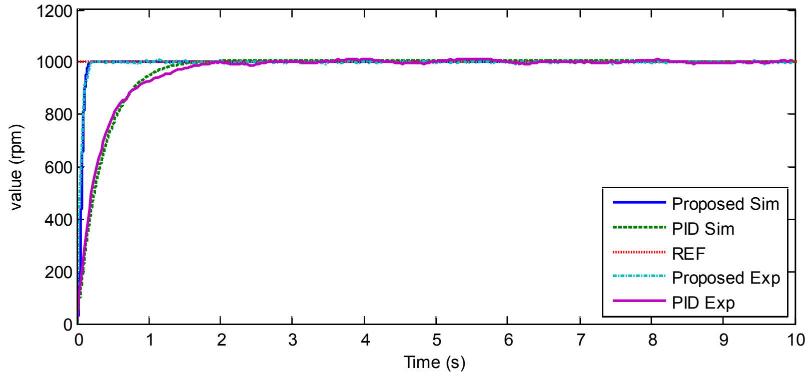

Fig. 4.10 Output y for step reference input

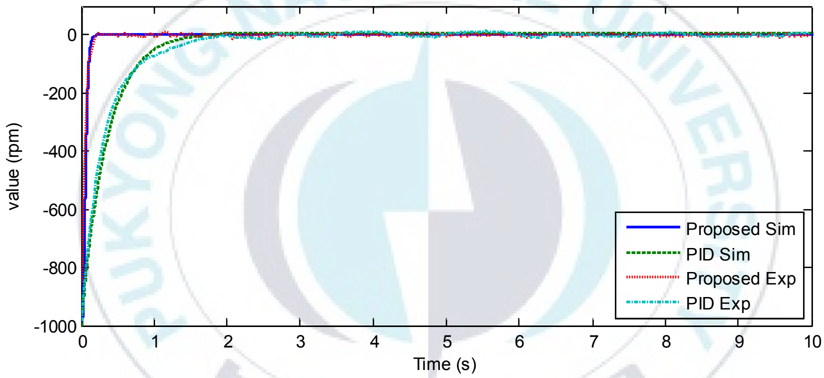

Fig. 4.11 Output error $\tilde{\omega}$ for step reference input

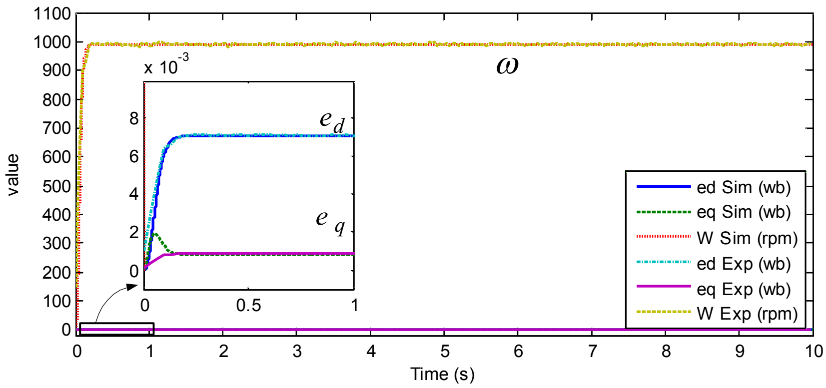

Fig 4.12 Estimated state variables for step reference input.

4.4.2 Ramp Reference

Figs. 4.13~4.16 show simulation and experimental results for the ramp reference of $y_r(t) = 100t$. Fig. 4.13 shows the control laws for both methods. It shows that both control laws as current in q-$axis$ are changed to ramp type after about 2 seconds. Fig. 4.14 shows the outputs for both methods. The outputs for PID do not converge to the reference input and has steady state errors, while the outputs for the proposed method converge to the reference input smoothly without overshoot after about 1.2 second. Fig. 4.15 shows the output errors for both methods. The output errors for PID converge about -40rpm after about 4 seconds, while the output errors for the proposed method converge to zero smoothly after about 1.2 second. Fig. 4.16 shows the estimated state variables for ramp reference using the proposed method.

Compensator of Eq. (3.45) can be described as follows: $\dot{\zeta} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e$ for ramp reference.

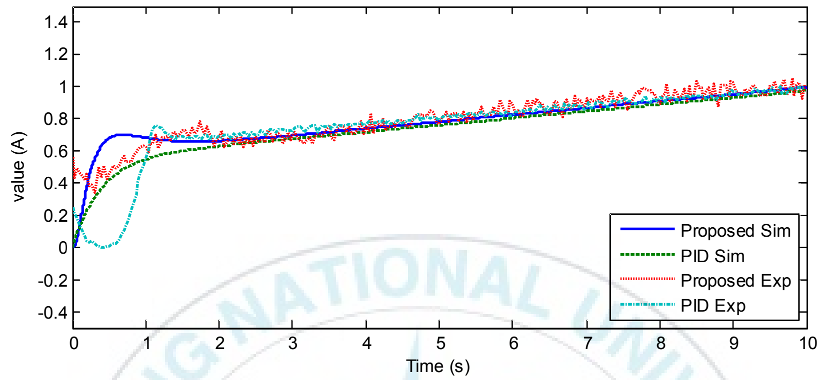

Fig. 4.13 Control law $u = i_{sq}$ for ramp reference input

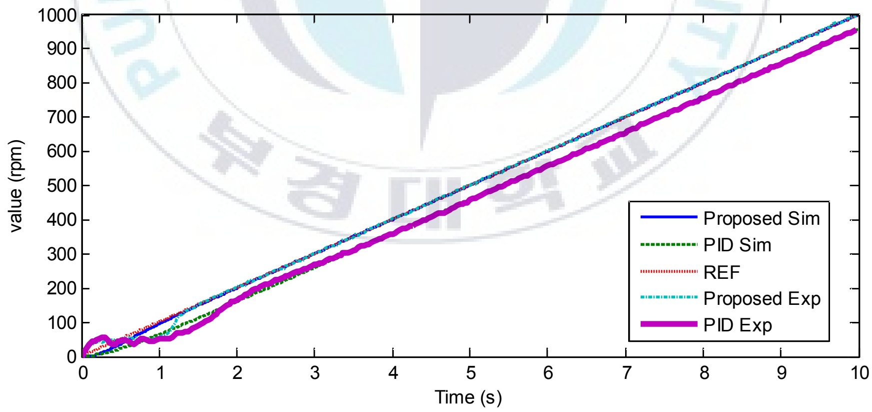

Fig. 4.14 Output y for ramp reference input

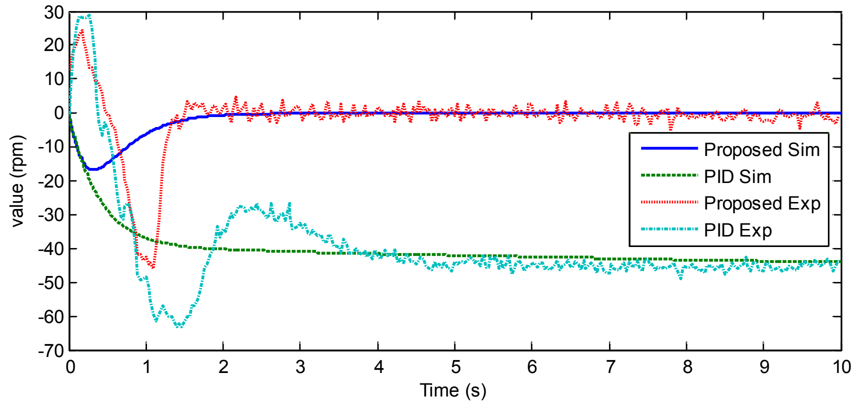

Fig. 4.15 Output error $\tilde{\omega}$ for ramp reference input

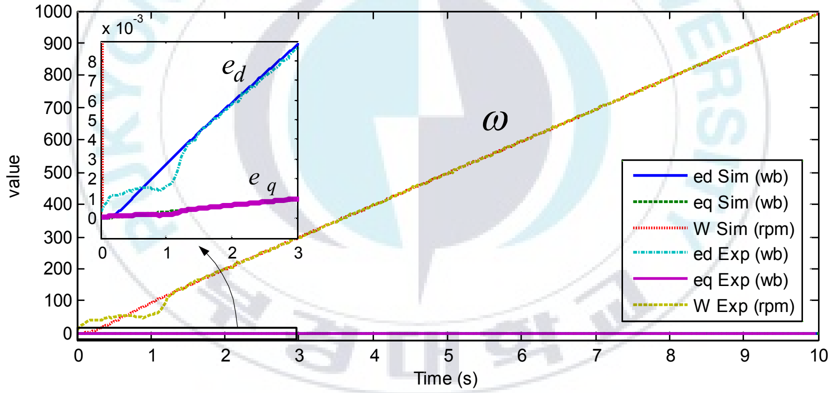

Fig. 4.16 Estimated state variables for ramp reference input

4.4.3 Parabola Reference

Figs. 4.17~4.20 show simulation and experimental results for parabola reference of $y_r(t) = 10t^2$. Fig. 4.17 shows control laws for the proposed method and PID method. It shows both control laws as current in q-$axis$ are changed to parabola type and increase to 0.5A after about 10 seconds. Fig. 4.18 shows the outputs for both methods. The outputs for PID have steady state errors without converging to the reference input and then have bigger and bigger steady state error as time goes, while the outputs for the proposed method converge to the reference input smoothly without overshoot after about 0.1 second. Fig. 4.19 shows the output errors for both methods. The output errors for PID are always increasing and never converging to zero, while the output errors for the proposed method converge to zero smoothly after about 0.1 second. Fig. 4.20 shows the estimated state variables for the proposed method. Compensator of Eq. (3.45) can be described as follows:

$$\dot{\zeta} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e \text{ for parabola reference.}$$

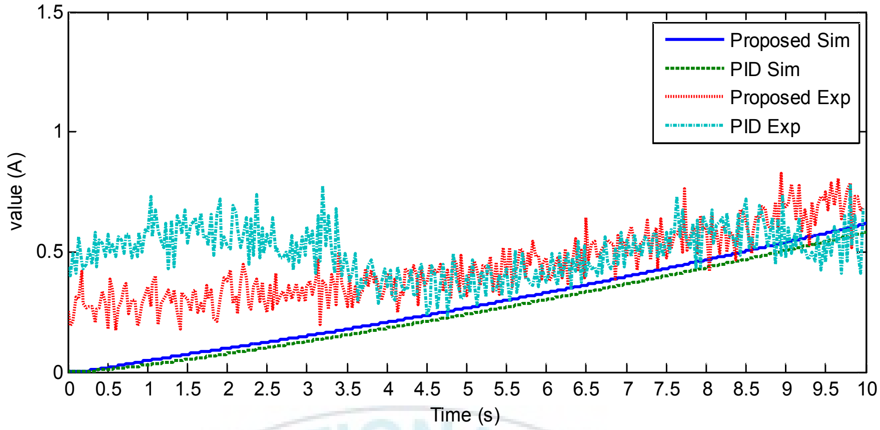

Fig. 4.17 Control law $u = i_{sq}$ for parabolic reference input

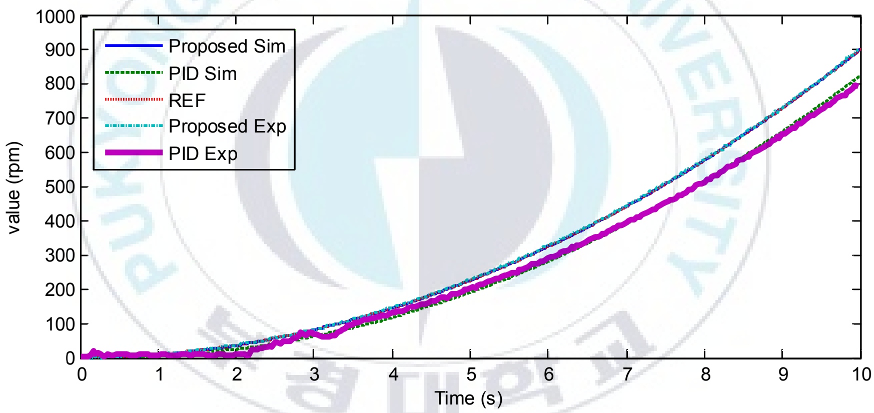

Fig. 4.18 Output y for parabolic reference input

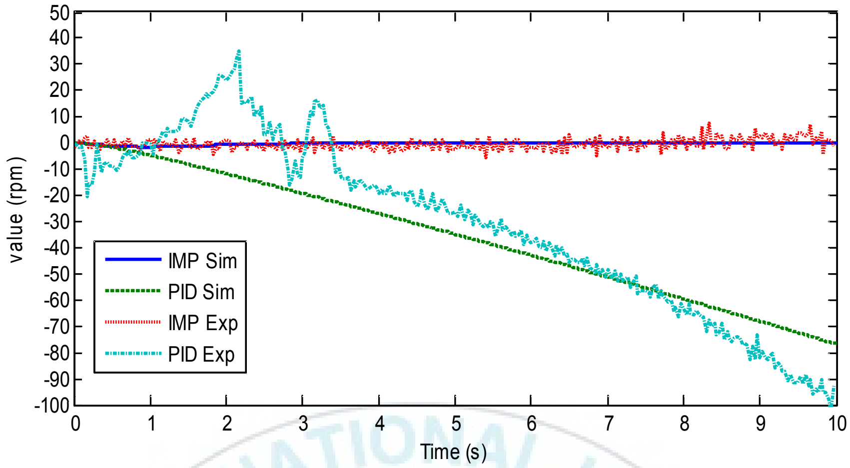

Fig. 4.19 Output error $\tilde{\omega}$ for parabolic reference input

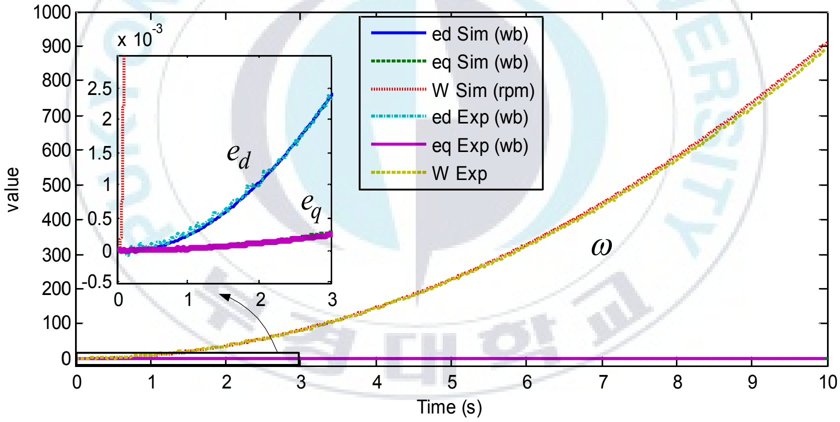

Fig. 4.20 Estimated state variables for parabolic reference input

4.5 Summaries

In this chapter, the followings were presented:

- A testing motor system was developed to implement the proposed controller. The testing motor system consisted of an AC induction motor 1.5 Kw, a torque sensor and a powder brake. They were connected in series. A voltage source inverter based on DSP 320F28335 was designed. The inverter used smart power module FSBB30CH60 (IGBT module) as switching devices.
- An full order observer was adopted to estimate unknown variables.
- To verify the effectiveness and the applicability of the proposed robust servo controller design method simulation and experimental results of the proposed robust servo controller for a step type of a disturbance and 3 references such as step, ramp and parabola were shown for AC induction motor system as a single-input and single-output(SISO) system compared with the conventional PI controller. The simulation and experimental results were as follows :

[Step input] The output for the conventional PID controller converged to the reference input after about 2 seconds, while the outputs for the proposed method converged to the reference input smoothly without overshoot after about 0.3 second. The output error for PID converged to zero after about 2 seconds, while the output error for the proposed method converged to zero smoothly after about 0.3 second. Estimated state values of w, e_d, e_q for step reference input were estimated as 1000rpm, $7\times10^{-3}W_b$ and $1.2\times10^{-3}W_b$ after 0.2second by the proposed full order observer, respectively.

[Ramp input] The outputs for PID did not converged to the reference input and has steady state errors of about 40 rad/s after about 4 seconds, while the outputs for the proposed method converged to the reference input smoothly without overshoot and without no steady state error after about 1.2 second. Estimated state values of w, e_d, e_q for ramp reference input were estimated well.

[Parabola input] The outputs for PID had steady state errors without converging to the reference input and then had bigger and bigger steady state error as time went, while the outputs for the proposed method converged to the reference input smoothly without overshoot after about 0.1 second. Estimated state values of w, e_d, e_q for parabola reference input were estimated well.

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Chapter 5: Application to MIMO System of 4 Wheel Steering Vehicle

5.1 MIMO Case Simulation Results(A Four-Wheel Steering-4WS)

In chapter 2, modeling of the 4 wheel steering vehicle are with the following step type of ε described as

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = 0 \tag{5.1}$$

Three references inputs are chosen as follows :

Reference of step type $y_{r1}(t) = 0.1$, $y_{r2}(t) = 0.01$, reference of ramp type $y_{r1}(t) = 0.1t$, $y_{r2}(t) = 0.01t$, and reference of parabola type $y_{r1}(t) = 0.05t^2$, $y_{r1}(t) = 0.005t^2$.

By defining the state vector $x = \begin{bmatrix} k & \gamma \end{bmatrix}^T$, input vector $u = \begin{bmatrix} \delta_f & \delta_r \end{bmatrix}^T$ and output vector $y = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T$, the 4 wheel steering vehicle model with disturbance ε from Eq. (2.25) in chapter 2 can be written in the state-space form as follows:

$$\dot{x} = Ax + Bu + \varepsilon \tag{5.2}$$

$$y = Cx \tag{5.3}$$

where the coefficient matrices are:

$$A=\begin{bmatrix} -\frac{2\left(C_f+C_r\right)}{mV} & -\frac{2\left(l_fC_f-l_rC_r\right)}{mV^2}-1 \\ -\frac{2\left(l_fC_f-l_rC_r\right)}{J_z} & -\frac{2\left(l_f^2C_f-l_r^2C_r\right)}{J_zV} \end{bmatrix},\ B=\begin{bmatrix} \frac{2C_f}{mV} & \frac{2C_r}{mV} \\ \frac{2C_f}{J_z} & -\frac{2C_r}{J_z} \end{bmatrix},$$

$$C=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Table 5.1 shows parameter and initial values of the 4 wheel steering vehicle.

Table 5.1 Parameter and initial values of the 4 wheel steering vehicle

Parameter	**Value**
Vehicle mass (m)	1740 Kg
Yaw inertia (J_z)	3214 Kg$\cdot m^2$
Front axle to CG (l_f)	1.058 m
Rear axle to CG (l_r)	1.756 m
Front tire cornering stiffness (C_f)	29000 N/rad
Rear tire cornering stiffness (C_r)	60000 N/rad
Vehicle linear velocity (V)	0.01 m/s
Reference yaw rate time lag (τ_r)	0.1 s
Reference slideslip angle time lag (τ_β)	0.1 s
Driver's lead time (T_p)	1 s
Driver's compensatory gain (C_0)	1.7 s^2/m
Driver's correction time (T_c)	0.2 s
Driver's cognitive time delay (t_d)	0.28 s

Driver's neuromuscular time delay (T_h)	0.1 s
Initial value of state vector $(x(0))$	$[0\ 0]^T$
Initial value of estimated state vector $(\hat{x}(0))$	$[0\ 0]^T$
Initial value of new error variables $(\zeta(0))$	$\begin{cases} [0\ 0]^T & \text{for step reference} \\ [0\ 0\ 0\ 0]^T & \text{for ramp reference} \\ [0\ 0\ 0\ 0\ 0\ 0]^T & \text{for parabola reference} \end{cases}$

To verify the effectiveness of proposed controller design method, our controller is compared with PI controller for MIMO system with two inputs and two outputs as given in [29]. System matrices of the given MIMO system using Table 5.1 are given as follows:

$$A = \begin{bmatrix} -3.41 & -0.9045 \\ 46.5451 & 3.173 \end{bmatrix},\ B = \begin{bmatrix} 1000 & 2069 \\ 18.046 & -37.3367 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{5.4}$$

5.1.1 PI-MIMO Controller[35]

The tunable gain matrices of K_P and K_I have eight tunable variables. Initial values for the controller are generated as shown in [35]. The gain value for PI-MIMO controller in this simulation were chosen as:

$$K_P = \begin{bmatrix} 0.5 & 2 \\ 2 & 0.5 \end{bmatrix} \; , \; K_I = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.5 \end{bmatrix} \tag{5.5}$$

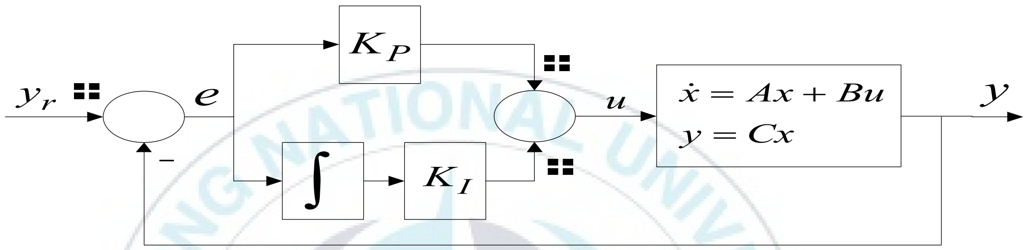

Fig. 5.1 PI-MIMO controller

5.1.2 Proposed Servo Control System

The proposed servo control system is shown in Fig. 3.2 The parameter and controller gains and servo compensators for 3 types of reference signals are obtained by using the proposed servo controller design method as shown in Table 5.2 and are shown in Appendix D.

Table 5.2 Parameters and gains for the proposed method

Design Parameters	**Reference types**		
	Step	**Ramp**	**Parabolic**
Reference Model	$y_{r1}(t)=0.1, y_{r2}(t)=0.01$	$\dot{y}_{r1}(t)=0.1, \dot{y}_{r2}(t)=0.01$ $y_{r1}(0)=0, y_{r2}(t)=0$	$\ddot{y}_{r1}(t)=0.1, \ddot{y}_{r2}(t)=0.01$ $\dot{y}_{r1}(0)=0, \dot{y}_{r2}(0)=0$ $y_{r1}(0)=0, y_{r2}(0)=0$
Matrix N	$N=[0]$	$N=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	$N=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
Assigned Poles	{-10,-11,-12,-13}	{-10,-11,-12,-13,-14,-15}	{-10,-11,-12,-13,-14,-15,-16,-17}
Gain matrix F_x	$\begin{bmatrix} 1.3004 & 0.6693 \\ -0.6181 & -0.3239 \end{bmatrix}$	$\begin{bmatrix} 1.3449 & 1.1334 \\ -0.6337 & -0.5475 \end{bmatrix}$	$\begin{bmatrix} 1.2630 & 1.5923 \\ -0.5862 & -0.7711 \end{bmatrix}$
Gain matrix F_z	$\begin{bmatrix} 0.0780 & 3.0478 \\ 0.0377 & -1.4731 \end{bmatrix}$	$\begin{bmatrix} 6.848 & -2.4031 \\ 1.1859 & -0.3509 \\ 54.1180 & -26.0385 \\ 13.0744 & -6.3002 \end{bmatrix}^T$	$\begin{bmatrix} -100.7003 & 66.9356 \\ -23.1968 & 15.7626 \\ -1.5543 & 1.2687 \\ 899.5088 & -437.1666 \\ 271.1928 & -131.6198 \\ 30.4197 & -14.7434 \end{bmatrix}^T$

5.1.3 Step Reference

Simulation results using step reference for the proposed method and the PI-MIMO controller are shown in Figs. 5.3~5.5. Fig. 5.3 shows the control inputs using for the proposed method and the PI-MIMO. Fig. 5.4 shows that the outputs of the controllers used for both methods track the reference values and stabilize the plant after finite time. As we can see, the outputs of the controller using the proposed method can track the step reference signals well after about 0.5 second, while the output of the controller using the PI-MIMO can track the step reference signals well after about 3 seconds. Fig. 5.5 shows that the output errors of the controllers using both methods converge to zero and stabilize the plant after finite times of about 0.5 second and 3 seconds, respectively. Therefore, the output errors of the closed loop system using the proposed method can become zero faster than the output errors of the controller using the PI-MIMO.

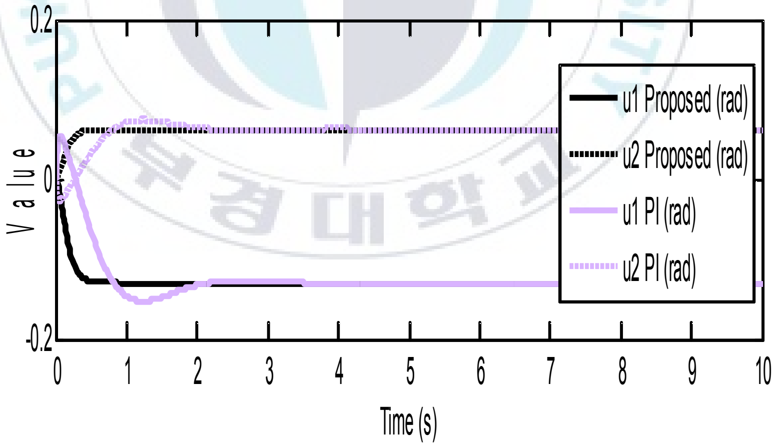

Fig. 5.3 Control inputs of PI and proposed methods for step reference

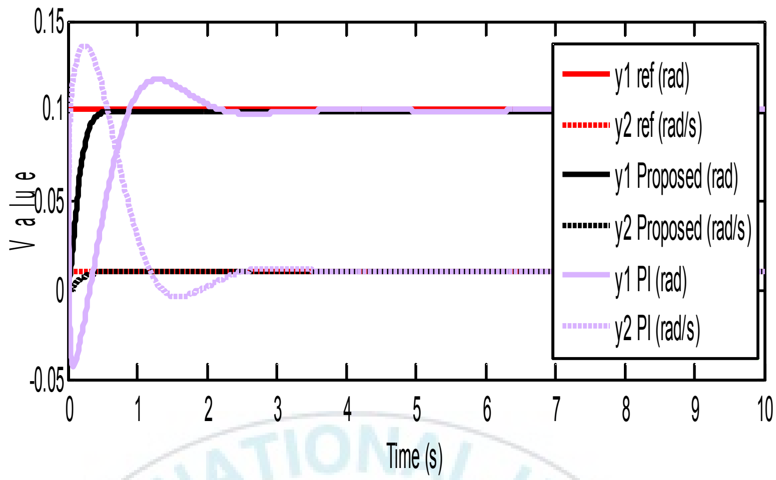

Fig. 5.4 Outputs of PI and proposed methods for step reference

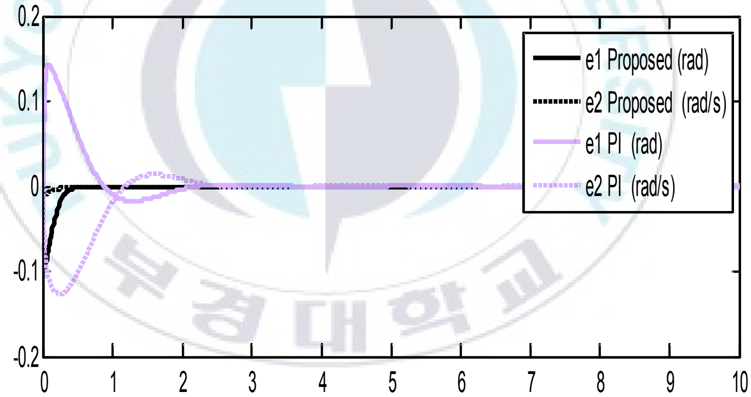

Fig. 5.5 Output errors of PI and proposed methods for step reference

5.1.4 Ramp Reference

Simulation results using ramp input for the proposed method and the PI-MIMO controller are shown in Figs. 5.6-5.8. Fig. 5.6 shows the control inputs using both methods. Fig. 5.7 shows that only the outputs of the controller using the proposed method can track the ramp reference signals and stabilize the plant after finite time. The outputs of the controller using the PI-MIMO cannot track the ramp reference signals. Furthermore, Fig. 5.8 shows that the output errors of the controller using the proposed method converge to zero and stabilize the plant after finite time. On the other hand, in the controller using the PI-MIMO, the steady state errors of $+0.05$ rad and -0.08 rad/s exist.

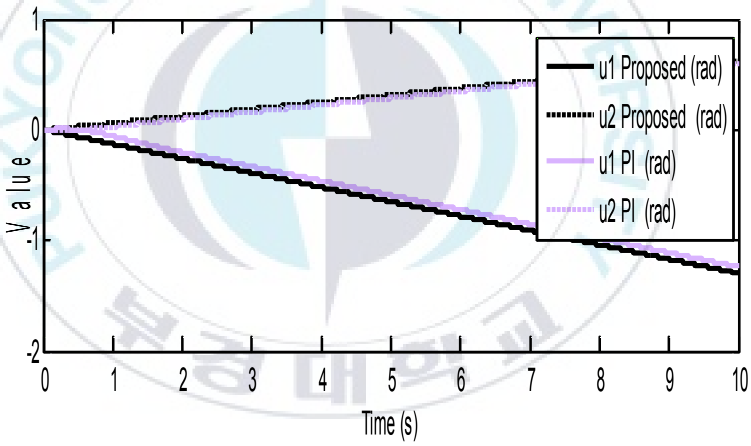

Fig. 5.6 Control inputs of PI and proposed methods for ramp reference

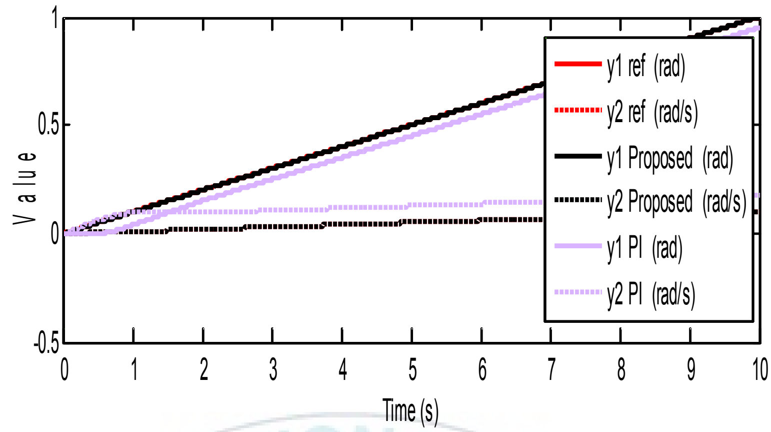

Fig. 5.7 Outputs of PI and proposed methods for ramp reference

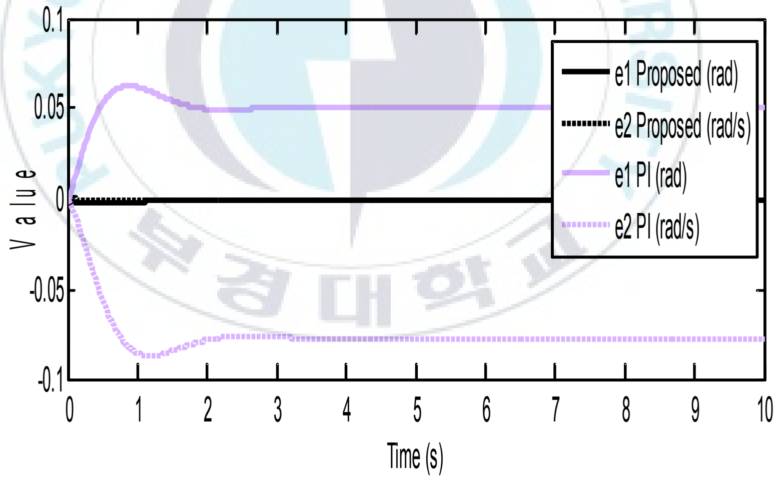

Fig. 5.8 Output errors of PI and proposed methods for step reference

5.1.5 Parabolic Reference

Simulation results using parabolic input for the proposed control method and the PI-MIMO controller are shown in Figs. 5.9~5.11. Fig. 5.9 shows the control inputs using both methods. Fig. 5.10 shows that the outputs of the controller using the proposed control method can track the parabola reference signals well and stabilize the plant after finite time. The outputs of the controller using the control method PI – MIMO cannot track the parabola reference signals. Fig 5.11 shows that the output errors of the controller using the proposed method converges to zero and stabilize the plant finite time. However, PI controller has increasing steady errors for the given reference parabolic signals.

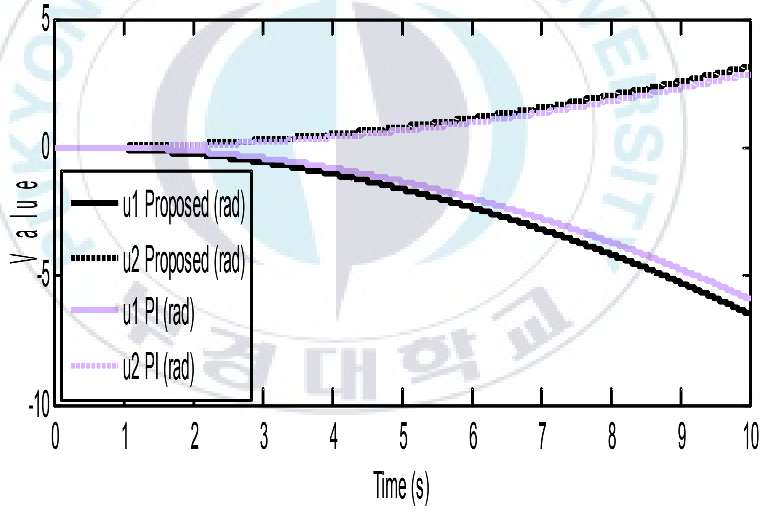

Fig. 5.9 Control inputs of PI and proposed methods for parabolic reference

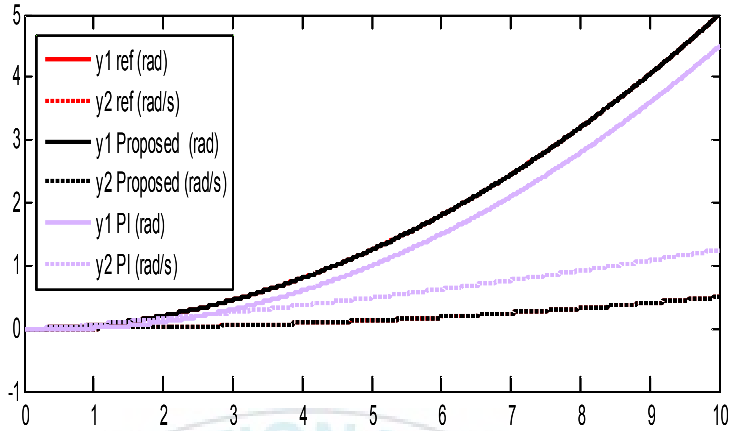

Fig. 5.10 Outputs of PI and proposed methods for parabolic reference

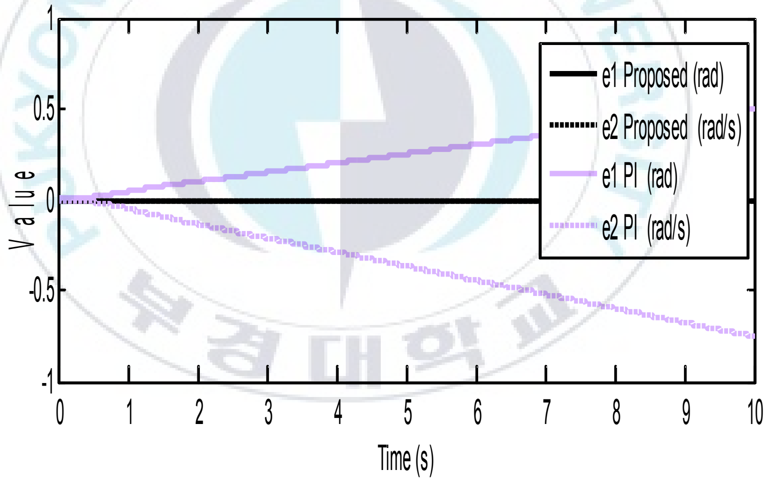

Fig. 5.11 Output errors of PI and proposed methods for parabolic reference

5.2 Summaries

In this chapter, the followings were presented :

To verify the effectiveness and the applicability of the proposed robust servo controller design method simulation and experimental results of the proposed robust servo controller for a step type of a disturbance and 3 references such as step, ramp and parabola were shown for a 4 wheel steering vehicle system as a multi-input and multi-output(MIMO) system compared with the conventional PI controller. The simulation and experimental results were as follows :

[Step input] The outputs of controller using the proposed method could track the reference signal faster than the output of the controller using the PI-MIMO. Fig. 5.5 shows that the output errors of the controllers using both methods converged to zero and stabilized the plant after finite time of about 0.5 second for the proposed method and 3 seconds for the PI-MIMO method, respectively. Therefore, the output errors of the closed loop system using the proposed method could become zero faster than the output errors of the controller using the PI-MIMO.

[Ramp input] The output errors using the proposed control method converged to zero after finite time, while the output errors using the PI-MIMO control method had the steady state errors of + 0.05 rad and − 0.08 rad/s, respectively. The outputs using the PI-MIMO control method could not track the ramp reference signals.

[Parabola input] The outputs of the proposed control method for the 4 wheel steering vehicle method could track the parabola reference signals after finite time. However, the outputs of the PI-MIMO control method could not track the parabola reference signals. The output errors using the proposed control method converged to zero after finite time, while the output errors using the PI-MIMO control method were increased more as $t \to \infty$.

Chapter 6: Conclusions and Future Works

6.1 Conclusions

This dissertation proposed a servo controller design using polynomial differential operator method based on internal model principle and its applications. The conclusions of this dissertation were summarized as follows.

- ❖ **In chapter 2**, firstly, problem statement were introduced. Secondly, the mathematical modelings of an AC induction motor and 4 wheel steering vehicle were introduced.
- ❖ **In chapter 3**, a servo controller design method using polynomial differential operator were proposed. To do this task, the followings were done. Firstly, a given linear time invariant system with disturbance was described. Secondly, disturbance and reference were expressed as the form of differential polynomials. Thirdly, it was shown that a closed loop system of the given system had zero steady errors by the final theorem when the controller includes the least common multiple of the denominators of reference and disturbance in its denominator using internal model principle. Fourthly, by operating the polynomial differential operator to the given system under the given conditions of reference and disturbance and an output error, an extended system was obtained. Fifthly, it was proven that the extended system was controllable. Finally, a feedback control law was designed by a well known regulator design method.
- ❖ **In chapter 4**, A prototype of the experimental AC induction motor drive system was presented. Firstly, the motor testing system including AC induction

motor, torque sensor and powder brake that were connected in series was set up. Secondly, voltage source inverter based on DSP was developed. The analog amplifiers for measuring two current signals and DC voltage from power source signal were described. Thirdly, a full order observer of the extended system was designed to estimate its unknown states. Finally, Simulation and experimental results for a step type of disturbance and 3 references such as step, ramp and parabola were shown for an AC induction motor with single input and single output(SISO) system compared with PI controller. The simulation and experiment results were as follows :

[Step input] The output for the conventional PID controller converged to the reference input after about 2 seconds, while the outputs for the proposed method converged to the reference input smoothly without overshoot after about 0.3 second. The output error for PID converged to zero after about 2 seconds, while the output error for the proposed method converged to zero smoothly after about 0.3 second. Estimated state values of w, e_d, e_q for step reference input were estimated as 1000rpm, $7\times10^{-3}W_b$ and $1.2\times10^{-3}W_b$ after 0.2second by the proposed full order observer, respectively.

[Ramp input] The outputs for PID did not converge to the reference input and has steady state errors of about 40 rad/s after about 4 seconds, while the outputs for the proposed method converged to the reference input smoothly without overshoot and without no steady state error after about 1.2 second. Estimated state values of w, e_d, e_q for ramp reference input were estimated well.

[Parabola input] The outputs for PID had steady state errors without converging to the reference input and then had bigger and bigger steady state error as time went, while the outputs for the proposed method converged to the reference input smoothly without overshoot after about 0.1 second. Estimated state values of w, e_d, e_q for parabola reference input were estimated well.

- **In chapter 5**, to verify the effectiveness and the applicability of the proposed servo controller design method, simulation and experimental results for a step type of disturbance and 3 references such as step, ramp and parabola were shown in a 4 wheel steering vehicle as a multi-input and multi-output (MIMO) system with two inputs and two outputs compared with PI controller. The simulation results were as follows :

[Step input] The outputs of controller using the proposed method could track the reference signal faster than the output of the controller using the PI-MIMO. Fig. 5.5 showed that the output errors of the controllers using both methods converged to zero and stabilized the plant after finite time of about 0.5 second for the proposed method and 3 seconds for the PI-MIMO method, respectively. Therefore, the output errors of the closed loop system using the proposed method could become zero faster than the output errors of the controller using the PI-MIMO.

[Ramp input] The output errors using the proposed control method converged to zero after finite time, while the output errors using the PI-MIMO control method had the steady state errors of +0.05 rad and –0.08 rad/s, respectively. The outputs using the PI-MIMO control method could not track the ramp reference signals.

[Parabola input] The outputs of the proposed control method for the 4 wheel steering vehicle method could track the parabola reference signals after finite time. However, the outputs of the PI-MIMO control method could not track the parabola reference signals. The output errors using the proposed control method converged to zero after finite time, while the output errors using the PI-MIMO control method were increased more as $t \to \infty$.

6.2 Future works

- ❖ Application of the propose control methods to real industrial AC induction motor.
- ❖ Development of BLDC motor drive system for electric vehicle.

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Publications and Conferences

A. Publication Journal, Accepted and Submitted

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B. Conferences

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Appendix

Appendix A

A1. Proof of Eqs. (2.17) and (2.18)

Linearization by Taylor series expansion for Eqs. (2.7)~(2.9) is described as follows :

Eqs. (2.7)-(2.9) can be defined by the simplified forms :

$$\frac{dx(t)}{dt} = f(x(t), u(t)), x(t_0) = x_0 \tag{A.1}$$

$$y(t) = g(x(t), u(t)) \tag{A.2}$$

where

$$\begin{aligned} &x = [e_d, e_g, \omega]^T = [x_1, x_2, x_3]^T, u = i_g, y = \Omega \\ &f(x,u) = [f_{ed}, f_{eg}, f_\omega]^T = [f_1, f_2, f_3]^T \\ &y = g(x,u) = \Omega \end{aligned} \tag{A.3}$$

If the nominal state vector is defined by $\bar{x}$ and the corresponding control input is defined by $\bar{u}$, the state vector $\bar{x}(t)$ and the output vector $\bar{y}(t)$ are satisfied with

$$\frac{d\bar{x}(t)}{dt} = f(\bar{x}(t), \bar{u}(t)) \tag{A.4}$$

$$\bar{y}(t) = g(\bar{x}(t), \bar{u}(t)) \tag{A.5}$$

We now consider the deviation of the state vector and the output vecotor from their nominal trajectory owing to deviation of the input vector from $\bar{u}(t)$. These deviations are defined by

$$\begin{cases} \delta u(t) = u(t) - \bar{u}(t) \\ \delta x(t) = x(t) - \bar{x}(t) \\ \delta y(t) = y(t) - \bar{y}(t) \end{cases} \tag{A.6}$$

If these variations are assumed to be small, Eqs. (A.1)-(A.2) can be extended by Taylor series around the nominal values as follows :

$$\frac{dx_i(t)}{dt}+\frac{d\delta x_i(t)}{dt}=f_i(\overline{x}(t),\overline{u}(t))+ \frac{\partial f_i(x,y)}{\partial x}\bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}}^{T}\delta x(t)+\frac{\partial f_i(x_1 u)}{\partial u}\bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}}^{T}\delta u+O(\delta x,\delta u) \tag{A.7}$$

$$\overline{y}(t)+\delta y(t)=g(\overline{x},\overline{u})+ \frac{\partial g(x,u)}{\partial x}\bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}}^{T}\delta x(t)+\frac{\partial g(x,u)}{\partial y}\bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}}^{T}\delta u+O(\delta x,\delta u) \tag{A.8}$$

where T means "transposed" and $O(\delta x,\delta u)$ denotes higher order terms by introducing the Jacobian matrices as follows:

$$A(t)=\frac{\partial f(x,u)}{\partial x^T}\bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}}=\begin{pmatrix}\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3}\\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3}\\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3}\end{pmatrix}\Bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}} \tag{A.9}$$

$$B(t)=\frac{\partial f(x,u)}{\partial u^T}\bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}}=\begin{pmatrix}\frac{\partial f_1}{\partial u}\\ \frac{\partial f_2}{\partial u}\\ \frac{\partial f_3}{\partial u}\end{pmatrix}\Bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}} \tag{A.10}$$

$$C(t)=\frac{\partial g(x,u)}{\partial x^T}\bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}}=\begin{pmatrix}\frac{\partial g}{\partial x_1} & \frac{\partial g}{\partial x_2} & \frac{\partial g}{\partial x_3}\end{pmatrix}\bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}} \tag{A.11}$$

$$D(t)=\frac{\partial g(x,u)}{\partial u}\bigg|_{\substack{x=\overline{x}\\ u=\overline{u}}} \tag{A.12}$$

$\overline{x}(t),\overline{u}(t)$ are chosen as constants as the equilibrium or steady states such that

$$\begin{cases} f(\bar{x}(t),\bar{u}(t)) = 0 \\ \bar{y}(t) = g(\bar{x}(t),\bar{u}(t)) \end{cases} \tag{A.13}$$

From Eqs. (A.7)~(A.13), Eqs. (A.1)~(A.2) become linearized by ignoring the high order terms in the equilibrium states as follows:

$$\frac{d\delta x(t)}{dt} = A(t)\delta x(t) + B(t)\delta u(t) \ \ for \ \delta x(t_0) = \delta x_0 \tag{A.14}$$

$$\delta y(t) = C(t)\delta x(t) + D(t)\delta u(t) \tag{A.15}$$

Eqs. (A.14)~(A.15) describe the variations around the nominal trajectory and has at least the first order accuracy.

The linearized equations corresponding to Eqs. (A.14)-(A.15) can be described as the time -invariant linear system as follows :

$$\frac{d\delta x}{dt} = A\delta x + B\delta u \tag{A.16}$$

$$\delta y = C\delta x + D\delta u \tag{A.17}$$

From Eqs. (2.7)~(2.9) of the modeling of an AC induction motor, $f(x,u)$ in Eq. (A.3) is given and $A(t), B(t), C(t), D(t)$ from can be obtained as follows :

$$f_1(t) = f_{e_d}(t) = -\alpha_r e_d + (p\omega_{ref} - p\omega + \frac{\hat{\alpha}_r L_m}{\lambda_{ref}} i_q)e_q \tag{A.18}$$

① $\frac{\partial f_1}{\partial x_1} = \frac{\partial f_1}{\partial e_d} = -\alpha_r$

② $\frac{\partial f_1}{\partial x_2} = \frac{\partial f_1}{\partial e_q} = p\omega_{ref} - p\omega + \frac{\hat{\alpha}_r L_m}{\lambda_{ref}} i_q$

③ $\frac{\partial f_1}{\partial x_3} = \frac{\partial f_1}{\partial \omega} = -pe_q$

④ $\frac{\partial f_1}{\partial u} = \frac{\partial f_1}{\partial i_q} = \frac{\hat{\alpha}_r L_m}{\lambda_{ref}} e_q$

$$f_2(t) = f_{e_q}(t) = -\alpha_r e_q - (p\omega_{ref} - p\omega + \frac{\hat{\alpha}_r L_m}{\lambda_{ref}} i_q)e_d$$
$$+(\hat{\alpha}_r - \alpha_r)L_m i_q + p(\omega_{ref} - \omega)\lambda_{ref} \tag{A.19}$$

① $\dfrac{\partial f_2}{\partial x_1} = \dfrac{\partial f_2}{\partial e_d} = -(p\omega_{ref} - p\omega + \dfrac{\hat{\alpha}_r L_m}{\lambda_{ref}} i_q)$

② $\dfrac{\partial f_2}{\partial x_2} = \dfrac{\partial f_2}{\partial e_q} = -\alpha_r$

③ $\dfrac{\partial f_2}{\partial x_3} = \dfrac{\partial f_2}{\partial \omega} = pe_d - p\lambda_{ref}$

④ $\dfrac{\partial f_2}{\partial u} = \dfrac{\partial f_2}{\partial i_q} = -\dfrac{\hat{\alpha}_r L_m}{\lambda_{ref}} e_d + (\hat{\alpha}_r - \alpha_r)L_m$

$$f_3(t) = f_\omega(t) = \mu[i_q(\lambda_{ref} - e_d) + \frac{e_q \lambda_{ref}}{L_m}] - b\omega - \frac{T_L}{J} \tag{A.20}$$

① $\dfrac{\partial f_3}{\partial x_1} = \dfrac{\partial f_3}{\partial e_d} = -\mu i_q$

② $\dfrac{\partial f_3}{\partial x_2} = \dfrac{\partial f_3}{\partial e_q} = \mu \dfrac{\lambda_{ref}}{L_m}$

③ $\dfrac{\partial f_3}{\partial x_3} = \dfrac{\partial f_3}{\partial \omega} = -b$

④ $\dfrac{\partial f_3}{\partial u} = \dfrac{\partial f_3}{\partial i_q} = \mu(\lambda_{ref} - e_d)$

The output is obtained as follows :

$$y(t) = \Omega = g(x,u) = \left(\frac{\lambda_{ref} - e_d}{\lambda_{ref}}\right)\omega + \frac{\partial_r e_q}{p\lambda_{ref}} - ai_q \tag{A.21}$$

① $\dfrac{\partial \Omega}{\partial e_d} = \dfrac{\partial g}{\partial x_1} = \dfrac{-\omega}{\lambda_{ref}}$

② $\dfrac{\partial \Omega}{\partial e_q} = \dfrac{\partial g}{\partial x_2} = \dfrac{\alpha_r}{p\lambda_{ref}}$

③ $\dfrac{\partial \Omega}{\partial \omega} = \dfrac{\partial g}{\partial x_3} = \dfrac{\lambda_{ref} - e_d}{\lambda_{ref}}$

④ $\dfrac{\partial \Omega}{\partial i_q} = \dfrac{\partial g}{\partial u} = -a = -\dfrac{(\hat{\alpha}_s - \alpha_s)\eta + (\hat{\alpha}_r - \alpha_r)\beta L_m}{\beta p \lambda_{ref}}$

(A.9)~(A.12) can be obtained as the following matrices :

$$A(t)=\begin{bmatrix} -\alpha_r & p\omega_{ref}-p\omega+\frac{\hat{\alpha}_r L_m}{\lambda_{ref}}i_q & -pe_q \\ -(p\omega_{ref}-p\omega+\frac{\hat{\alpha}_r L_m}{\lambda_{ref}}i_q) & -\alpha_r & pe_d-p\lambda_{ref} \\ -\mu i_q & \mu\frac{\lambda_{ref}}{L_m} & -b \end{bmatrix} \quad (A.22)$$

$$B(t)=\begin{bmatrix} \frac{\hat{\alpha}_r L_m}{\lambda_{ref}}e_q \\ -\frac{\hat{\alpha}_r L_m}{\lambda_{ref}}e_d+(\hat{\alpha}_r-\alpha_r)L_m \\ \mu(\lambda_{ref}-e_d) \end{bmatrix} \quad (A.23)$$

$$C(t)=[\frac{\partial g}{\partial x_1}\frac{\partial g}{\partial x_2}\frac{\partial g}{\partial x_3}]=\begin{bmatrix} \frac{-\omega}{\lambda_{ref}} & \frac{\alpha_r}{p\lambda_{ref}} & \frac{\lambda_{ref}-e_d}{\lambda_{ref}} \end{bmatrix} \quad (A.24)$$

$$D(t)=\frac{\partial g}{\partial u}=\frac{\partial g}{\partial i_q}=-a=-\frac{(\hat{\alpha}_s-\alpha_s)\eta+(\hat{\alpha}_r-\alpha_r)\beta L_m}{\beta p\lambda_{ref}} \quad (A.25)$$

The equilibrium point($\overline{e}_d,\overline{e}_q,\overline{\omega},\overline{i}_q$) in $\hat{\alpha}_s=\alpha_s$ can be obtained from Eq. (A.13) as follows :

$$\overline{e}_d=\overline{e}_q=0, \overline{i}_q=\frac{b\omega_{ref}+T_L/J}{\mu\lambda_{ref}-\frac{b(\hat{\alpha}_r-\alpha_r)L_m}{p\lambda_{ref}}}, \overline{\omega}=\omega_{ref}+\frac{(\hat{\alpha}_r-\alpha_r)L_m}{p\lambda_{ref}}\overline{i}_q \quad (A.26)$$

At the equilibrium point of Eq. (A.26) in $\hat{\alpha}_s=\alpha_s$, the following matrices from Eqs. (A.22)~(A.25) can be obtained

$$A\big|_{e_d^*,e_q^*,\omega^*,i_q^*}=\begin{bmatrix} -\alpha_r & \frac{\alpha_r L_m}{\lambda_{ref}}i_q^* & 0 \\ -\frac{\alpha_r L_m}{\lambda_{ref}}i_q^* & -\alpha_r & -p\lambda_{ref} \\ -\mu i_q^* & \mu\frac{\lambda_{ref}}{L_m} & -b \end{bmatrix} \quad (A.27)$$

$$B\big|_{e_d^*,e_q^*,\omega^*,i_q^*} = \begin{bmatrix} 0 \\ (\hat{\alpha}_r - \alpha_r)L_m \\ \mu\lambda_{ref} \end{bmatrix} \tag{A.28}$$

$$C\big|_{e_d^*,e_q^*,\omega^*,i_q^*} = \begin{bmatrix} \dfrac{-\omega}{\lambda_{ref}} & \dfrac{\alpha_r}{p\lambda_{ref}} & 1 \end{bmatrix} \tag{A.29}$$

$$D\big|_{e_d^*,e_q^*,\omega^*,i_q^*} = -a = -\frac{(\hat{\alpha}_r - \alpha_r)L_m}{p\lambda_{ref}} \tag{A.30}$$

The equilibrium point $\bar{\Omega}$ of Ω Eq. (A.21) with $\hat{\alpha}_s = \alpha_s$ and $\hat{\alpha}_r = \alpha_r$ can be obtained as follows ;

$$\bar{\Omega} = \left(\frac{\lambda_d - e_d}{\lambda_d}\right)\omega + \frac{\alpha_r e_g}{p\lambda_{ref}} - ai_q = \bar{\omega} - a\bar{i}_q = \omega_{ref} \tag{A.31}$$

Transfer function of Eqs. (2.17)~(2.18) can be obtained

$$G(s) = \frac{N(s)}{P(s)} = C(sI - A)^{-1}B + D \tag{A.32}$$

$$N(s) = \mu\lambda_{ref}\left[s^2 + \alpha_r s + \frac{\omega_c \alpha_r L_m}{\lambda_{ref}}\bar{i}_q\right] \times \left[1 - \frac{(\hat{\alpha}_r - \alpha_r)L_m}{\mu p \lambda_{ref}^2}(s+b)\right]$$

$$P(s) = (s+b)\left[(s+\alpha_r)^2 + \left(\frac{\alpha_r L_m}{\lambda_{ref}}\bar{i}_q\right)^2\right] + \frac{p\mu\lambda_{ref}^2}{L_m}\left(s + \alpha_r - \frac{\alpha_r L_m^2}{\lambda_{ref}^2}\bar{i}_q^2\right)$$

Appendix B

B1. [Proof of theorem 2]

The controllability matrix of the extended system of Eq. (3.36) can be written b

$$V_e = \begin{bmatrix} B_e & A_e B_e & A_e^2 B & \cdots & A_e^{n+pq-1} B_e \end{bmatrix} \tag{B.1}$$

$$A_e = \begin{bmatrix} A & 0 & \cdots & 0 & 0 \\ \begin{bmatrix} \mathbf{0} \\ C_1^T \end{bmatrix} & N & 0 & \vdots & 0 \\ \begin{bmatrix} \mathbf{0} \\ C_2^T \end{bmatrix} & 0 & N & 0 & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \begin{bmatrix} \mathbf{0} \\ C_p^T \end{bmatrix} & 0 & \cdots & 0 & N \end{bmatrix} \in R^{(n+pq)\times(n+pq)}, \quad B_e = \begin{bmatrix} B \\ \mathbf{0} \\ \vdots \\ \vdots \\ \mathbf{0} \end{bmatrix} \in R^{(n+pq)\times m}$$

The controllability matrix can be arranged as

$$\tilde{V}_e = \begin{bmatrix} B_e & \bar{A}_e B_e & \bar{A}_e^2 B & \cdots & \bar{A}_e^{n+pq-1} B_e \end{bmatrix}$$

$$\tilde{V}_e = \begin{bmatrix} B_e & \bar{A}_e B_e & \bar{A}_e^2 B & \cdots & \bar{A}_e^{n+pq-1} B_e \end{bmatrix}$$

$$= \left[\begin{matrix} B & AB & A^2B & A^3B & A^4B \\ 0 & CB & CAB-\alpha_{q-1}CB & CA^2B-\alpha_{q-1}(CAB-\alpha_{q-1}CB)-\alpha_{q-2}CB & CA^3B-\alpha_{q-1}[CA^2B-\alpha_{q-1}(CAB-\alpha_{q-1}CB)-\alpha_{q-2}CB]-\alpha_{q-2}[CAB-\alpha_0 CB]-\alpha_{q-3}CB \\ 0 & 0 & CB & CAB-\alpha_{q-1}CB & CA^2B-\alpha_{q-1}(CAB-\alpha_{q-1}CB)-\alpha_{q-2}CB \\ 0 & 0 & 0 & CB & CAB-\alpha_{q-1}CB \\ 0 & 0 & 0 & 0 & CB \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \right.$$

$$\left. \begin{matrix} \cdots & A^{n+pq-2}B & A^{n+pq-1}B \\ & \vdots & CA^{n+pq-2}B-\cdots \\ & & \\ & \vdots & \vdots \\ & CA^2B-\alpha_{q-1}(CAB-\alpha_{q-1}CB)-\alpha_{q-2}CB & CA^3B-\alpha_{q-1}[CA^2B-\alpha_{q-1}(CAB-\alpha_{q-1}CB)-\alpha_{q-2}CB]-\alpha_{q-2}[CAB-\alpha_0 CB]-\alpha_{q-3}CB \\ & CAB-\alpha_{q-1}C & CA^2B-\alpha_{q-1}(CAB-\alpha_{q-1}CB)-\alpha_{q-2}CB \\ & CB & CAB-\alpha_{q-1}CB \\ \cdots & 0 & CB \end{matrix} \right] \tag{B.2}$$

where the matrix $\bar{A}_e$ is a permutated matrix of A_e of Eq. (3.36) arranged by a permutation transformation without loss of its rank property as the following:

$$\bar{A}_e = \begin{bmatrix} A & 0 & 0 & \cdots & \cdots & 0 & 0 \\ C & -\alpha_{q-1} I & -\alpha_{q-2} I & \cdots & \cdots & -\alpha_0 I & -\alpha_0 I \\ 0 & I & 0 & \cdots & \vdots & 0 & 0 \\ 0 & 0 & I & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & & I & 0 \end{bmatrix} \tag{B.3}$$

Proof of Eq. (B.3) is written in **Appendix C**.

[Proofs of Eq. (B.2)]

The second item of the controllability matrix of Eq. (B.2) is obtained as follows:

$$\bar{A}_e B_e = \begin{bmatrix} A & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ C & -\alpha_{q-1} I_p & -\alpha_{q-2} I_p & \cdots & -\alpha_1 I_p & -\alpha_0 I_p \\ \mathbf{0} & I_p & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_p & \vdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & I_p & \mathbf{0} \end{bmatrix} \begin{bmatrix} B \\ \mathbf{0} \\ \vdots \\ \vdots \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} AB \\ CB \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \tag{B.4}$$

The third item of the controllability matrix of Eq. (B.2) is obtained as follows:

$$\bar{A}_e^2 B_e = \bar{A}_e (\bar{A}_e B_e) = \begin{bmatrix} A & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ C & -\alpha_{q-1} I_p & -\alpha_{q-2} I_p & \cdots & -\alpha_1 I_p & -\alpha_0 I_p \\ \mathbf{0} & I_p & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_p & \vdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & I_p & \mathbf{0} \end{bmatrix} \begin{bmatrix} AB \\ CB \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} A^2 B \\ CAB - \alpha_{q-1} CB \\ CB \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \tag{B.5}$$

The fourth item of the controllability matrix of Eq. (B.2) is obtained as follows:

$$\bar{A}_e^3 B_e = \bar{A}_e(\bar{A}_e^2 B_e) = \begin{bmatrix} A & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ C & -\alpha_{q-1}I_p & -\alpha_{q-2}I_p & \cdots & -\alpha_1 I_p & -\alpha_0 I_p \\ \mathbf{0} & I_p & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_p & \vdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & I_p & \mathbf{0} \end{bmatrix} \begin{bmatrix} A^2B \\ CAB-\alpha_{q-1}CB \\ CB \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} A^3B \\ CA^2B-\alpha_{q-1}(CAB-\alpha_{q-1}CB)-\alpha_{q-2}CB \\ CAB-\alpha_{q-1}CB \\ CB \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}$$

(B.6)

In case of $n=4,\ p=1\ and\ q=2$, the controllability matrix of Eq. (B.2) is obtained as follows:

$$\tilde{V}_e = \begin{bmatrix} B_e & \bar{A}_e B_e & \bar{A}_e^2 B & \cdots & \bar{A}_e^{n+pq-1} B_e \end{bmatrix} = \begin{bmatrix} B_e & \bar{A}_e B_e & \bar{A}_e^2 B & \cdots & \bar{A}^3 B_e \end{bmatrix}$$

$$= \begin{bmatrix} B & AB & A^2B & A^3B \\ 0 & CB & CAB-\alpha_{q-1}CB & CA^2B-\alpha_{q-1}(CAB-\alpha_{q-1}CB)-\alpha_{q-2}CB \\ 0 & 0 & CB & CAB-\alpha_{q-1}CB \\ 0 & 0 & 0 & CB \end{bmatrix} \quad \text{(B.7)}$$

By linear combination of the columns of Eq. (B.2), the following permutated controllability matrix can be are obtained:

$$\hat{V}_e = \begin{bmatrix} B & AB+\alpha_{q-1}B & A^2B+\alpha_{q-1}AB+\alpha_{q-2}B & A^3B+\alpha_{q-1}A^2B+\alpha_{q-2}AB+\alpha_{q-3}B & \cdots & A^{n+pq-1}B+\sum_{i=0}^{q-1}\alpha_{q-1-i}A^{n+pq-2-i}B \\ 0 & CB & CAB & CA^2B & \cdots & CA^{n+pq-2}B \\ 0 & 0 & CB & CAB & \cdots & CA^{n+pq-3}B \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & & 0 & 0 & \cdots & CA^{n+pq-p-1}B \end{bmatrix}$$

(B.8)

Proof of Eq. (B.8)

The columns of Eq. (B.8) are obtained by linear combination of columns of Eq. (B.2) as follows:

The 2 nd column of Eq. (B.8) =(2nd column + α_{q-1} 1st column) of Eq. (B.2)= $\begin{bmatrix} AB+\alpha_{q-1}B \\ CB \\ 0 \\ \vdots \\ \mathbf{0} \end{bmatrix}$ (B.9)

The 3rd column of Eq. (B.8) = (3rd column $+\alpha_{q-1}$ 2nd column $+\alpha_{q-2}$ 1st column) of Eq.(B.2)

$$=\begin{bmatrix} A^2B+\alpha_{q-1}AB+\alpha_{q-2}B \\ CAB \\ CB \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{B.10}$$

The 4th column of Eq. (B.8)= (4th column $+\alpha_{q-1}$ 3rd column $+\alpha_{q-2}$ 2nd column $+\alpha_{q-3}$ 1st column) of Eq. (B.2)

$$=\begin{bmatrix} A^3B-\alpha_{q-1}A^2B-\alpha_{q-2}AB-\alpha_{q-3}B \\ CA^2B \\ CAB \\ CB \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \tag{B.11}$$

The final $(n+pq)^{th}$ column of the controllability matrix of Eq. (B.8) can be obtained by taking a combination procedure of $(n+pq-1)^{th}$ column $+\alpha_{q-1}\times(n+pq-2)^{th}$ column + $\alpha_{q-2}\times$ $(n+pq-3)^{th}$ column $+\cdots+\alpha_2\times$ 3rd column $+\alpha_1$ $\times$2nd column $+\alpha_0$ $\times$1st column for Eq. (B.2) as follows:

$$\bar{A}_e^{n+pq-1}B_e = \begin{bmatrix} A^{n+pq-1}B + \sum_{i=0}^{q-1}\alpha_{q-1-i}A^{n+pq-2-i}B \\ CA^{n+pq-2}B \\ CA^{n+pq-3}B \\ \vdots \\ \vdots \\ CA^{n+pq-p-1}B \end{bmatrix} \tag{B.12}$$

By the property of the rank, the following rank condition can be held:

$$rank(V_e) = rank(\hat{V}_e) \tag{B.13}$$

Eq. (B.8) can be described by linear combination procedure of the columns of $\bar{V}_e$ that is by the product form of two matrices of $\bar{V}_e$ in Eq. (3.40) and a new matrix V_I as follows:

$$\hat{V}_e = \bar{V}_e V_I \tag{B.14}$$

where

$$V_I = \begin{bmatrix} I_{pq} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & V_c \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & \cdots & 0 & \vdots & 0 & 0 & \cdots & 0 \\ 0 & I & 0 & \cdots & \vdots & \vdots & 0 & 0 & \cdots & \vdots \\ \vdots & 0 & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & I & \vdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \vdots & B & AB & \cdots & A^{n-1}B \end{bmatrix} \tag{B.15}$$

and $V_c = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$ is the controllability matrix of Eq. (1).

From $rank(\bar{V}_e) = n + pq$ by the condition (2) of Theorem 2 and $rank\,(V_I) = n + pq$ due to Eq. (B.15) and $rank\,(V_c) = n$ by condition (1) of Theorem 2, the rank condition for the controllability of the extended system is satisfied as follows:

$$rank(V_e) = rank(\hat{V}_e) = rank(\bar{V}_e\, V_I) = rank(V_I) = n + pq\ .$$

Therefore, the extended system of Eq. (3.36) is controllable under two assumed conditions of theorem 2.

Appendix C

C1. The proof of Eq. (B.3)

A_e in Eq. (3.36) is given as

$$A_e = \begin{bmatrix} A & 0 & \cdots & 0 & 0 \\ \begin{bmatrix} \mathbf{0} \\ C_1^T \end{bmatrix} & N & 0 & \vdots & 0 \\ \begin{bmatrix} \mathbf{0} \\ C_2^T \end{bmatrix} & 0 & N & 0 & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \begin{bmatrix} \mathbf{0} \\ C_p^T \end{bmatrix} & 0 & \cdots & 0 & N \end{bmatrix} \in R^{(n+pq)\times(n+pq)} \tag{C.1}$$

$$A_e = \begin{bmatrix} A & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \begin{bmatrix} \mathbf{0} \\ C_1^T \end{bmatrix} & \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \; \vdots & \vdots \\ 0 & \vdots & 0 & \ddots \; \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & 1 \\ -\alpha_0 & -\alpha_1 & & \cdots & -\alpha_{q-1} \end{bmatrix} & \mathbf{0} & \vdots & \mathbf{0} \\ \begin{bmatrix} \mathbf{0} \\ C_2^T \end{bmatrix} & \mathbf{0} & \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \; \vdots & \vdots \\ 0 & \vdots & 0 & \ddots \; \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & 1 \\ -\alpha_0 & -\alpha_1 & & \cdots & -\alpha_{q-1} \end{bmatrix} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \begin{bmatrix} \mathbf{0} \\ C_p^T \end{bmatrix} & \mathbf{0} & \mathbf{0} & \cdots & \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \; \vdots & \vdots \\ 0 & \vdots & 0 & \ddots \; \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & 1 \\ -\alpha_0 & -\alpha_1 & & \cdots & -\alpha_{q-1} \end{bmatrix} \end{bmatrix}$$

$$
=\begin{bmatrix}
A & \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\end{bmatrix} & \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\end{bmatrix} & \cdots & \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\end{bmatrix} \\
\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \vdots\\ C_1^T\end{bmatrix} & \begin{bmatrix}0 & 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & 1 & 0 & \vdots & \vdots\\ 0 & \vdots & 0 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & 1\\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & \cdots & -\alpha_{q-1}\end{bmatrix} & \begin{bmatrix}0 & 0 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & 0 & \vdots & \vdots\\ 0 & \vdots & 0 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & 0\\ 0 & 0 & 0 & \cdots & \cdots & 0\end{bmatrix} & \vdots & \begin{bmatrix}0 & 0 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & 0 & \vdots & \vdots\\ 0 & \vdots & 0 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & 0\\ 0 & 0 & 0 & \cdots & \cdots & 0\end{bmatrix} \\
\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \vdots\\ C_2^T\end{bmatrix} & \begin{bmatrix}0 & 0 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & 0 & \vdots & \vdots\\ 0 & \vdots & 0 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & 0\\ 0 & 0 & 0 & \cdots & \cdots & 0\end{bmatrix} & \begin{bmatrix}0 & 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & 1 & 0 & \vdots & \vdots\\ 0 & \vdots & 0 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & 1\\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & \cdots & -\alpha_{q-1}\end{bmatrix} & \cdots & \begin{bmatrix}0 & 0 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & 0 & \vdots & \vdots\\ 0 & \vdots & 0 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & 0\\ 0 & 0 & 0 & \cdots & \cdots & 0\end{bmatrix} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \vdots\\ C_p^T\end{bmatrix} & \begin{bmatrix}0 & 0 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & 0 & \vdots & \vdots\\ 0 & \vdots & 0 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & 0\\ 0 & 0 & 0 & \cdots & \cdots & 0\end{bmatrix} & \begin{bmatrix}0 & 0 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & 0 & \vdots & \vdots\\ 0 & \vdots & 0 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & 0\\ 0 & 0 & 0 & \cdots & \cdots & 0\end{bmatrix} & \cdots & \begin{bmatrix}0 & 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & 1 & 0 & \vdots & \vdots\\ 0 & \vdots & 0 & \ddots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & 1\\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & \cdots & -\alpha_{q-1}\end{bmatrix}
\end{bmatrix} \qquad \text{(C.2)}
$$

By linear combination of Eq. (C.2), permutation transformation of (C.2) is expressed into

$$
\Rightarrow\begin{bmatrix}
A & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\
C_1^T & -\alpha_0 & -\alpha_1 & \cdots & -\alpha_{q-2} & -\alpha_{q-1} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
C_2^T & 0 & 0 & \cdots & 0 & 0 & -\alpha_0 & -\alpha_1 & \cdots & -\alpha_{q-2} & -\alpha_{q-1} & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
C_p^T & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & -\alpha_0 & -\alpha_1 & \cdots & -\alpha_{q-2} & -\alpha_{q-1} \\
\mathbf{0} & 0 & 1 & \ddots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\mathbf{0} & 0 & 0 & \ddots & \vdots & 0 & 0 & 0 & \cdots & \vdots & \vdots & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & \ddots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \cdots & 1 & 0 & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & \ddots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 1 & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 1
\end{bmatrix}
$$

$$
\Rightarrow \begin{bmatrix}
A & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
C_1^T & -\alpha_{q-1} & 0 & \cdots & 0 & -\alpha_{q-2} & 0 & \cdots & 0 & \cdots & -\alpha_1 & 0 & \cdots & 0 & -\alpha_0 & 0 & \cdots & 0 \\
C_2^T & 0 & -\alpha_{q-1} & \ddots & \vdots & 0 & -\alpha_{q-2} & \ddots & \vdots & \vdots & 0 & -\alpha_1 & \ddots & 0 & 0 & -\alpha_0 & \ddots & \vdots \\
\vdots & 0 & 0 & \ddots & 0 & 0 & 0 & \ddots & 0 & \cdots & 0 & 0 & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\
C_p^T & 0 & 0 & \cdots & -\alpha_{q-1} & 0 & 0 & \cdots & -\alpha_{q-2} & \cdots & 0 & 0 & \cdots & -\alpha_1 & 0 & 0 & \cdots & -\alpha_0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \vdots & 1 & 0 & \vdots & 0 & 0 & 0 & \cdots & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 & \cdots & 0 & 0 & 0 & \vdots & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \vdots \\
\mathbf{0} & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & & 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & & 0 & 0 & \ddots & 0 & 0 & 0 & \cdots & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 & & \vdots & \vdots & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\mathbf{0} & 0 & 1 & \ddots & 0 & 0 & 0 & \ddots & 0 & & & & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & & & \cdots & 1 & 0 & 0 & \cdots & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & & & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \vdots & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0
\end{bmatrix}
$$

$$
\Rightarrow \bar{A}_e = \begin{bmatrix}
A & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
C_1^T & -\alpha_{q-1} & 0 & \cdots & 0 & -\alpha_{q-2} & 0 & \cdots & 0 & \cdots & -\alpha_1 & 0 & \cdots & 0 & -\alpha_0 & 0 & \cdots & 0 \\
C_2^T & 0 & -\alpha_{q-1} & \cdots & 0 & 0 & -\alpha_{q-2} & \cdots & 0 & \cdots & 0 & -\alpha_1 & \cdots & 0 & 0 & -\alpha_0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & 0 & 0 & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
C_p^T & 0 & 0 & \cdots & -\alpha_{q-1} & 0 & 0 & \cdots & -\alpha_{q-2} & \cdots & 0 & 0 & \cdots & -\alpha_1 & 0 & 0 & \cdots & -\alpha_0 \\
0 & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \vdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 1 & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & \ddots & 0 & \cdots & 0 & 0 & \cdots & 0 & & & & \\
 & & & & \vdots & \vdots & \vdots & \ddots & 1 & \cdots & 0 & 0 & \cdots & 0 & & & & \\
 & & & & & & & \cdots & 0 & \ddots & \vdots & \vdots & \ddots & \vdots & & & & \\
 & & & & & & & \vdots & 0 & \ddots & 1 & 0 & \cdots & 0 & & & & \\
 & & & & & & & \cdots & 0 & \ddots & 0 & 1 & \ddots & 0 & & & & \\
 & & & & & & & \cdots & \vdots & \ddots & 0 & 0 & \ddots & \vdots & & & & \\
0 & 0 & 0 & & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0
\end{bmatrix}
$$

$$
= \begin{bmatrix}
A & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\
C & \alpha_{q-1} I_p & \alpha_{q-2} I_p & \cdots & \alpha_1 I_p & \alpha_0 I_p \\
\mathbf{0} & I_p & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & I_p & \vdots & \mathbf{0} & \mathbf{0} \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & I_p & \mathbf{0}
\end{bmatrix} \in R^{(n+pq)\times(n+pq)}
$$

Appendix D

D1. Finding N

N of Eq. (3.36) in Table 4.3 and Table 5.2 can be obtained as follows :

1) For a step reference($q=1$), $\dot{y}_r = sy_r = L(s)y_r = 0$
2) $L(D) = D^{(q)} = \alpha_{q-1}D^{(q-1)} + \cdots + \alpha_0$, $L(s) = s = s + \alpha_0 \Rightarrow \alpha_0 = 0$,

$$N = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & -\alpha_3 & \cdots & -\alpha_{q-1} \end{bmatrix} \in R^{q \times q}$$

By using Eq. (3.30), N of Eq. (3.35) is obtained as follows: $N = \left[-\alpha_0\right] = \left[0\right]$

3) For a ramp reference($q = 2$),

$$\ddot{y}_r = s^2 y_r = L(s)y_r = 0$$

$$L(s) = s^2 = s^2 + \alpha_1 s + \alpha_0 \Rightarrow \alpha_1 = 0, \alpha_0 = 0,$$

By using Eq. (3.30), N of Eq. (3.35) is obtained as follows:

$$N = \begin{bmatrix} 0 & 0 \\ -\alpha_0 & -\alpha_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

4) For a parabola reference($q = 3$)

$$\dddot{y}_r = s^3 y_r = L(s)y_r = 0$$

$$L(s) = s^3 = s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 \Rightarrow \alpha_2 = 0, \alpha_1 = 0, \alpha_0 = 0,$$

By using Eq. (3.30), N of Eq. (3.35) is obtained as follows:

$$N = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_2 & -\alpha_1 & -\alpha_0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

For an AC Induction motor ($n = 3, m = 1, p = 1$) as a SISO system, In chapter 4

System matrices of a given system Eqs. (4.5) and (4.6) are given as

$$A = \begin{bmatrix} -701.33 & 25151.9 & 0 \\ -25151.9 & -701.33 & -0.98 \\ -2.2931 & 0.0639 & -0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0.0239 \end{bmatrix},$$

$$C = \begin{bmatrix} -2040.8 & 715.6514 & 1 \end{bmatrix}$$

System matrices of an extended system of Eq. (3.36) can be obtained from Eq. (4.5) as

(1) For a step reference($q = 1$)

$$A_e = \begin{bmatrix} -701.33 & 25151.9 & 0 & 0 \\ -25151.9 & -701.33 & -0.98 & 0 \\ -2.2931 & 0.0639 & 0 & 0 \\ -0.2048 & 715.6514 & 1 & 0 \end{bmatrix}, B_e = \begin{bmatrix} 0 \\ 0 \\ 0.0239 \\ 0 \end{bmatrix}$$

(2) For a ramp reference($q = 2$)

$$A_e = \begin{bmatrix} -701.33 & 25151.9 & 0 & 0 & 0 \\ -25151.9 & -701.33 & -0.98 & 0 & 0 \\ -2.2931 & 0.0639 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -0.048 & 715.6514 & 1 & 0 & 0 \end{bmatrix}, B_e = \begin{bmatrix} 0 \\ 0 \\ 0.0239 \\ 0 \\ 0 \end{bmatrix}$$

(3) For a parabola reference($q = 3$)

$$A_e = \begin{bmatrix} -701.33 & 25151.9 & 0 & 0 & 0 & 0 \\ -25151.9 & -701.33 & -0.98 & 0 & 0 & 0 \\ -2.2931 & 0.0639 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -0.048 & 715.6514 & 1 & 0 & 0 & 0 \end{bmatrix}, B_e = \begin{bmatrix} 0 \\ 0 \\ 0.0239 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Servo compensator of Eq. (3.44) can be obtained as follows :

(1) For a step reference($q = 1$)

$$N_z = N = [0],\ I_\zeta = [1 \quad 0 \quad 0],$$

(2) For a ramp reference($q = 2$)

$$N_z = N = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ I_\zeta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(3) For a parabola reference($q = 3$)

$$N_z = N = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\ I_\zeta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For a 4 wheel steering vehicle ($n = 2, m = 2, p = 2$) as a MIMO system in Chapter 5, System matrices of Eq. (5.4) are given as

$$A = \begin{bmatrix} -3.41 & -0.9045 \\ 46.5451 & 3.173 \end{bmatrix}, B = \begin{bmatrix} 1000 & 2069 \\ 18.046 & -37.3367 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

System matrices of an extended system

(1) For a step reference($q = 1$)

$$A_e = \begin{bmatrix} -3.41 & -0.9045 & 0 & 0 \\ 46.5451 & 3.173 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ B_e = \begin{bmatrix} 1000 & 2069 \\ 18.046 & -37.3367 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(2) For a ramp reference($q = 2$)

$$A_e = \begin{bmatrix} -3.41 & -0.9045 & 0 & 0 & 0 & 0 \\ 46.5451 & 3.173 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},\ B_e = \begin{bmatrix} 1000 & 2029 \\ 18.049 & -37.3367 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(3) For a parabola reference($q = 3$)

$$A_e = \begin{bmatrix} -3.41 & -0.9045 & 0 & 0 & 0 & 0 & 0 & 0 \\ 46.5451 & 3.173 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B_e = \begin{bmatrix} 1000 & 2029 \\ 18.049 & -37.3367 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

System matrices in servo compensator of Eq. (3.44) can be obtained as follows :

(1) For a step reference($q = 1$)

$$N_z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, I_\zeta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

(2) For a ramp reference($q = 2$)

$$N_z = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, I_\zeta = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

(3) For a parabola reference($q = 3$)

$$\mathrm{N_z} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathrm{I_z} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Appendix E

If (A_e, B_e) is not controllable, then if has an equivalent system of the form

$$\overline{\mathbf{A}}_{\mathbf{e}} = T^{-1}A_eT = \begin{bmatrix} \overline{A}_{ce} & \overline{A}_{e12} \\ 0 & \overline{A}_{e22} \end{bmatrix}$$

$$\overline{\mathbf{B}}_{\mathbf{e}} = T^{-1}B = \begin{bmatrix} \overline{B}_{ce} \\ 0 \end{bmatrix}$$

where T us nonsingular matrix and the matrix pair ($\overline{A}_{ce}, \overline{B}_{ce}$) is controllable.

The closed loop characteristic determinant of this system is given by

$$\det\left[SI - \overline{A}_e - \overline{B}_e\overline{F}\right] = \det\begin{bmatrix} SI - \overline{A}_{ce} - \overline{B}_{ce}\overline{F}_e & -\overline{A}_{e12} - \overline{B}_{ce}\overline{F}_2 \\ 0 & SI - \overline{A}_{e22} \end{bmatrix}$$

$$= \det\left[SI - \overline{A}_{ce} - \overline{B}_{ce}\overline{F}_c\right] \det\left[SI - \overline{A}_{e22}\right]$$

where $\overline{F} = FT^{-1} = \left[\overline{F}_c, \overline{F}_2\right]$ and F is a feedback control law designed for the extended system of Eq. (3.36).

The above determinant result shows that all the characteristic roots of $\overline{A}_{e22}$ cannot be changed by state feedback controller design.

Thus, the controllability of (A_e, B_e) is a necessary condition for all the poles to be assigned arbitrarily.

The sufficiency condition can be proved by determining the feedback control law F for the controllable extended system (A_e, B_e) so that the resulting closed loop system has a preassigned characteristic equation.

Let the controllability indices of the system $\{\sigma_i\}$, then one of the equivalent system can be represented by the controllable cannonical Form.

$$\overline{\mathbf{A}}_{\mathrm{e}} = \begin{bmatrix} \mathrm{E}_1 \\ \mathrm{a}_1^{\mathrm{T}} \\ \mathrm{E}_2 \\ \mathrm{a}_2^{\mathrm{T}} \\ \vdots \\ \mathrm{E}_m \\ \mathrm{a}^{\mathrm{T}}{}_m \end{bmatrix} \qquad \overline{\mathrm{B}}_{\mathrm{e}} = \begin{bmatrix} 0 \\ \mathrm{b}_1^{\mathrm{T}} \\ 0 \\ \mathrm{b}_2^{\mathrm{T}} \\ \vdots \\ 0 \\ \mathrm{b}^{\mathrm{T}}{}_m \end{bmatrix}$$

$$F_i = \left[\underbrace{0 \cdots 0}_{\sum_{j=1}^{i-1} \sigma_{i+1}}, I_{\sigma-1}, 0, \cdots, \underbrace{0}_{\sigma_m} \right] \sigma_{i-1}$$

$$a_i^T = [\alpha_{i0}, \alpha_{i1}, \cdots, \alpha_{im-1}]$$

$$b_i^T = [0, \cdots, 0, 1, \beta_{ii+1}, \cdots, \beta_{im}]$$

$$i = 1, \cdots, m$$

Let

$$\hat{B}_m = \begin{bmatrix} b_1^T \\ \vdots \\ b_m^T \end{bmatrix}$$ and define $\bar{F} = \hat{B}_m^{-1} \hat{F}$ where $\hat{F} = \begin{bmatrix} f_1^T \\ f_2^T \\ \vdots \\ f_m^T \end{bmatrix}$

Then $\bar{A}_e + \bar{B}_e \bar{F} = \begin{bmatrix} E_1 \\ a_1^T + f_1^T \\ E_2 \\ \vdots \\ a_m^T + f_m^T \end{bmatrix}$

If $\hat{F}$ is determined so that $\det(SI - A_e - B_e F) = S^n + \alpha_{n-1} S^{n-1} + \cdots + \alpha_0$

Then the f's are given for $i = 1, 2, \cdots, m-1$ as follows

$$f_i^T = -a_i^T + \left[\underbrace{0 \cdots 0}_{\sum_{j=1}^{i} \sigma_i}, 1, 0, \cdots, 0 \right]$$

$$f_m^T = -a_m^T + [-\alpha_0, \alpha_1, \cdots, -\alpha_{m-1}]$$

This $\bar{F}$ gives

$$\bar{A}_e + \bar{B}_e\bar{F} = \begin{bmatrix} 0 & 1 & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & \ddots & \vdots & \vdots \\ & \vdots & \vdots & \vdots & \ddots & 1 \\ -\alpha_0 & \cdots & \cdots & \cdots & \cdots & -\alpha_{m-1} \end{bmatrix}$$

Whose characteristic equation is specified a priori. This indicates that the feedback matrix

$$F = \hat{B}_m^{-1}\hat{F}T^{-1}$$

Makes the characteristic equation of $\bar{A}_e + \bar{B}_e\bar{F}$ of the extended system