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Thesis for the Degree of Master of Engineering

A Study on the Detection of Circles and Arcs Using the Least Squares Method

by

Armel Nkurunziza

Department of IT Convergence and Applications Engineering

The Graduate School

Pukyong National University

August 2016

A Study on the Detection of Circles and Arcs Using the Least Squares Method

최소자승법를 이용한 원과호 검출에 대한 연구

Advisor: Prof. Jong-Nam Kim

Co-Advisor: Prof. Bong Kee Sin

by

Armel Nkurunziza

A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Engineering

in the Department of IT Convergence and Applications Engineering,

The Graduate School,

Pukyong National University

August 2016

A Study on the Detection of Circles and arcs Using the Least Squares Method

A dissertation

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(Chairman) **Ki-Ryong Kwon**

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August 26, 2016

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최소자승법를 이용한 원과호 검출에 대한 연구

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요 약

산업 또는 연구에서 디지털 이미지와 비디오의 육안검사는 쉽지 않은 작업이며 때로는 특별한 기술과 신중한 판단을 필요로 한다. 요즘 사회에서는 육안검사의 중요성으로 인해 이 작업을 수행하는 많은 알고리즘들이 개발되고 있다.

일반적인 형상검출 알고리즘과 특별한 원과 원호 검출 알고리즘은 계속 개발되고 있지만 더 나은 최적의 정확성과 효율성을 목표로 더 개선되어야 함은 말할 필요가 없다.

본 논문에서는 일반적은 원호 검출에 대해 간단히 설명한다. 또한 현재 자주 사용되고 있는 원과 원호의 검출방법을 설명하고 이를 더욱 개선할 수 있는 검출 알고리즘을 제안한다.

제안한 방법은 호의 크기에 맞게 맞춘 원의 중심과 원호의 반경을 찾기 위해 최소자승법을 적용하여 원호의 가장 중요한 부분을 선택하는데 초점을 맞춘다. 호의 두 끝 지점은 반지름과 호의 점 사이의 거리를 비교하여 찾는다.

우리의 작업은 다음에 따라 수행한다. 1 장에서는 일반적인 형상검출과 이미지에서 특별한 원호 검출방법을 설명하고 2 장에서는 일반적인 원호 검출방법, 3 장에서는 제안하는 검출 알고리즘을 설명한다. 4 장은 일반적인 방법을 이용한 검출방법과 제안한 검출방법을 비교 분석하고 5 장에서 결론으로 마무리한다.

A Study on the Detection of Circles and Arcs Using the Least Squares Method

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Abstract

Visual inspection tasks in digital images and videos whether industrial or academic is not easy and require sometimes special skills and careful judgments. Due to the importance taken by the visual inspection in nowadays society, many algorithms have been developed to help accomplishing that task. The shape detection algorithms in general and the circles and arcs detection algorithms in particular, participate in this task but continue to be improved in order to achieve the optimal accuracy and the efficiency. In this thesis we have discussed briefly about the arcs detection. We gave a description of the main conventional methods of circles and arcs detection and we proposed another approach of how we can improve the detection.

In our proposed method we focused on selecting the best part of the arc which will be applied to the least squares method to find the center and the radius of the circle able to fit the best that arc. The two endings points of the arc are then found by comparing the distance between the radius and the points of the arc which vary from its circular shape.

Our work is done as follows: chapter1 introduces the shape detection in general and the arc detection in images particularly. Chapter 2 gives the conventional methods of arc detection. Chapter 3 presents our proposed method. Chapter 4 gives the results of our experiments and the proposed method is compared to the conventional method. Chapter 5 concludes the thesis.

Ⅰ. Introduction

1. Background

Computer vision seeks to enhance the ability of machines to understand the visual world through the development of algorithms for tasks such as object recognition, tracking, and 3D reconstruction from image and video data [1].

Object recognition in general and shape detection in particular is a fundamental and important theme in computer vision. Shape is one of the main sources of information that can be used for object recognition. For example, In medical image analysis, geometrical models of anatomical structures play an important role in automatic tissue segmentation. The shape of an organ can also be used to diagnose diseases [2]. In electronics industry, automated visual inspection are applied to products as printed circuit boards.

Generally, we can say that visual inspection tasks whether industrial or academic, are often tedious and very intensive but nonetheless require specialist skills and careful judgments. Advances in computer technology, sensing devices, image processing and pattern recognition have resulted, in recent years, in better and cheaper visual inspection equipment and automated visual inspection for a wide variety of applications that is now in use. [3]. However, many techniques and researches continue to be made in order to improve the efficacy and the accuracy of those visual inspections algorithms.

The arc and circle detection is one of the techniques which has been extremely studied in the last decades and has a lot of applications in the real world. Many algorithms have been developed but most of them require a lot of computation and memory space or need a large number and correct choice of parameters.

The main contribution of this Thesis, is to propose a simple, efficient and more accurate algorithm for arc and circle detection. The method concentrates on the selection of the best part of the arc (maximally circular arc) which will be used to detect the arc. In addition, we propose the detection of the two ending points of the arcs. We emphasize in detecting arcs which may contain outlier's points, have some occlusions or have a portion of his shape which varies from his general circular shape.

2. Purpose and structure of the thesis

In this thesis, we proposed a new method for arcs detection. Regarding important progresses that have been made in arcs and circles detection, our goal is to propose a new method for the arcs detection which is more simple, lightweight and more accurate. The proposed method finds, after some preprocessing operations, the best circular part of the arc and ignores the rest which may contain outliers or points which do not lie on the circular arc. Only the points lying on the best part of the arc will be applied to the least squares method to find the parameters (center, radius) of the arc. This will ensure a high accuracy in finding those parameters from which the 2 ending points will be calculated afterwards. This method is proved to produce more accurate results in arcs detection and is simple comparatively to the others methods which need mostly a correct choice of parameter in order to get good quality in arc detection.

This thesis is organized in five chapters:

Chapter 1 introduces the background and the purpose of the thesis.

Chapter 2 examines the conventional method for arcs detection. The Hough Transform as the commonly used technique in circles and arcs detection will be discussed in details.

Chapter 3 discusses the proposed method. A detailed description and explanation for each step of the method is presented.

Chapter 4 shows and discusses the arc detection results in many images using the proposed method.

Chapter 5 concludes the thesis

Ⅱ. Conventional methods of arcs detection

The arcs and circles detection gives important information for detecting specific objects in digital images or for shape recognition. This area has been intensively studied in the field of computer vision and pattern recognition due to its broad targeting applications from industrial manufacturing, scientific research, biomedical study, etc. Many Algorithms such as the least squares method [15] and the Hough Transform have been proposed to detect arcs and circles . The Hough Transform method is the most commonly used algorithms for the detection of different shapes in an image. In this chapter , we will be discussing how The Hough Transform is used to detect the circles and arcs in an image.

1. Hough Transform

The Hough transform is a method for extracting features and often used in images analysis and computer vision. The original Hough transform was a line transform and was a quick method for searching a binary image for straight lines. Later, the transform has been extended in 1972 by Richard Duda and Peter Hart under the name generalized Hough Transform. As a special form of General Hough Transform, the Circle Hough Transform (CHT), is used to detect circles and arcs.

1.1. Circle Hough Transform

The Circle Hough transform is a parametric transform algorithm developed to find circles in imperfect image inputs. In the two dimensional (x, y) plane, a circle can be described by

$$(x - a)^2 + (y - b)^2 = r^2 \quad (2.1)$$

where (a, b) is the center point of the circle and r its radius. The Hough Transform applies this equation to find all the points of the image satisfying (x, y). Those points are then lying on the surface of an inverted right-angle cone whose apex is at (x, y, 0). In the 3D space, it is the intersection of many conic that helps to identify the circle parameters. First the radius can be fixed and then find the optimal center of circles in 2D parameter space or finding the optimal radius in one dimensional parameter space. If the radius is known, the parameter space would be reduced to 2D. For each point (x, y) on the original circle, it defines a circle centered at (x, y) with radius r. The intersection point of all such circles in the parameter space would be corresponding to the center point of the original circle and can be found with a Hough accumulation array.

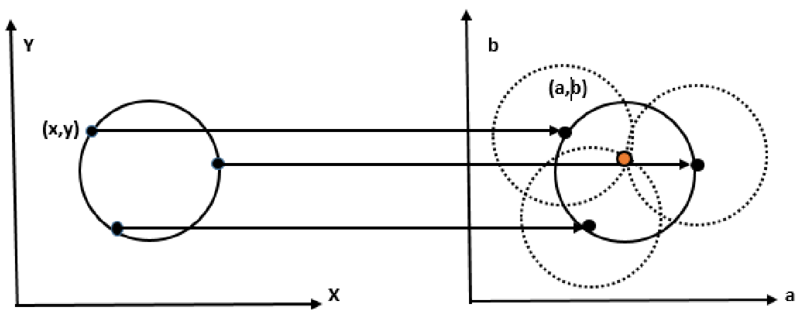


Figure 1 Illustration of The Circular Hough Transform when the radius is known

Each point in geometric space (left) generates a circle in parameter space (right). The circles in parameter space intersect at the (a, b) that is the center in geometric space. Multiple circles can be found also with the same technique if the radius is known.

If the radius is not known, then the locus of points in parameter space will fall on the surface of a cone. Each point (x, y) on the perimeter of a circle will produce a cone surface in parameter space. The triplet (a, b, r) will correspond to the accumulation cell where the largest number of cone surfaces intersect. The circle candidates are produced by voting in the Hough parameter space and select the local maxima in the accumulator matrix. For each edge point on the image plane, the curves passing through the point are computed, and the accumulators corresponding to these curves are incremented by 1. The accumulator with a peak value demonstrates the existence of a curve, and is specified by the accumulator's coordinates, on the image plane [5].

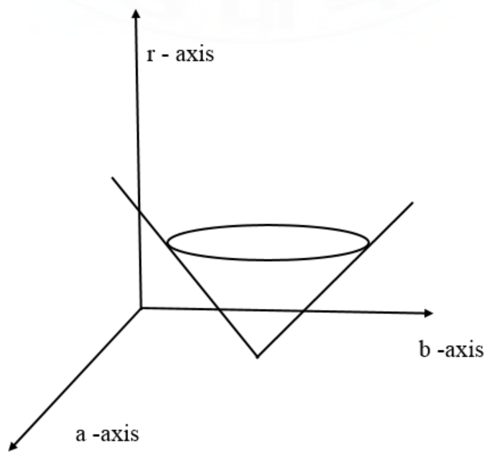


Figure 2 Circular Hough transform voted by an edge point

Figure 2 illustrates the generation of a conical surface in parameter space for one (x, y) point. A circle with a different radius will be constructed at each level, r. The search for circles with unknown radius can be conducted by using a three dimensional accumulation matrix.

The Circle Hough transform is capable of detecting partial shapes. Treating a circular arc as a partial shape of a circle, the Hough transform for detecting circles can be applied to detect circular arcs [5].

1.2. Disadvantages of the Hough Transform

The Hough Transform has the following main disadvantages:

- The difficulty to choose an appropriate size of accumulator array
- A high computational cost: A high dimensional parameter space can slow the Hough Transform process.
- The need of lots of storage: As the accumulator is used in random-accessed fashion, it can easily overrun the available memory.
- Spurious circles can be detected: The advantage of Hough Transform is that it connects edge points across the image to some form of parametric curve. However, this is also its weakness, because it is possible that spurious circles might also be detected with a reasonably large voting array.

2. Variations of the Circle Hough Transform

Many ideas have been proposed to improve the Hough Transform, based mostly on various parameter reduction or different parameter accumulator structures, all that in order to reduce the memory required and the computation cost. We can describe some of them.

Tsuji and Matsumoto decomposed the parameter space and used the parallel property of circles [17]. Several methods utilize randomized selection of edge points and geometrical properties of circle instead of using the information of edge pixels in the parameter space. Xu *et al* [18] presented an approach that randomly selects three pixels. The method selects three non-collinear edge pixels and votes for the circle parameters which are found by using the circle equation. Chen and Chung [19] improved Xu *et al*.'s method by using the randomized selection of four pixels. However, the randomized selection method has its own problems such as probability estimation, accuracy and speed that are dependent on the number of edge pixels. Ho and Chen [20] used the global geometrical symmetry of circles to reduce the dimension of the parameter space. All those methods tried to reduce the amount of storage and the computing power required by the CHT. A comparison of those ideas has been made [21], and no solution has been chosen among them to be better than the CHT itself in term of accuracy.

3. Least squares method

The least squares fit (LSF) is based on minimizing the mean square distance from the fitting curve to data points. Given n points (xi, yi), $1 \leq i \leq n$, the objective function is defined by

$$F = \sum_{i=1}^{n} di^2 \quad (3.1)$$

Where d_i is the orthogonal geometric distance from the measurement (xi, yi) to the curve. When fitting circles, it will be necessary to draw the circle

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3.2)$$

minimizing the mean square distance from given point to the circle. (a, b) is the center and R the radius, and then

$$d_i = \sqrt{(Xi - a)^2 + (yi - b)^2} - r. \quad (3.3)$$

This is a nonlinear least squares problem and has no closed form solution because there is no direct algorithm for the least squares fitting for curves (other than straight lines). All known algorithms are either iterative and costly or approximative by nature. This nonlinear least squares problem may be solved using a Gauss-Newton method [16].

An alternative to the geometric fit, called algebraic fit minimizes the sum of squares of algebraic distances.

$$F = \sum_{k=1}^{n} dk^2 \quad (3.4)$$

where d_k is the algebraic formula.

$$d_k = (x_k - a)^2 + (y_k - b)^2 - r^2 \quad (3.5)$$

a change of parameter gives the following equation

$$d_k = (z_k + Bx_k + Cy_k + D)^2 \tag{3.6}$$

Where $z_k = x_k^2 + y_k^2$, B = -2a, C= -2b and D = $a^2 + b^2 - r^2$. Differentiating *F* with respect to B, C, D yields a system of linear equations which may be solved recursively in closed form. This algorithm is fast but inaccurate when data are sampled along small circular arcs.

In general those 2 methods are all sensitive to noises and to outliers even though the geometric fit is considered as more accurate than the algebraic fit especially when data are sampled along small arcs.

III. Proposed Approach of circular arc detection using the least squares method

1. Introduction

Circular arcs are important features on the image of industrial parts or tools [7, 8, 9], in the field of computer vision and pattern recognition [10], etc. Many approaches have been proposed to estimate the arc parameters and most of them are still sensitive to noise and to outlier points.

The new approach for arcs detection proposed in this thesis focuses on selecting the best part of the arc by removing the noises and ignoring the outlier points or the additional points which vary from the circular arc. The best part of the arc is applied to the least squares method in order to find the center and radius of the best circle which can fit the arc. The two end points of the arc are also detected. The algorithm for selecting the best part of the arc is applied after some pre-processing operations on the image like the smoothing, binarization and the edge detection.

With this new algorithm, the whole process of the arc detection can be divided as follows: A color image is converted in gray-scale image, we extract in a new image, the region of interest (ROI) containing the circular shape, the median filtering [11] is applied to the new image containing the circular shape in order to remove noises but a small window (3*3) is used in order to keep the original size of the circular shape. After the thresholding of the image, the canny edge detection algorithm [12] is applied to have a clear boundary of the circular shape which is very important to accurately analyze the arc shape. At that time we select the best part of the arc which will be applied to the least squares method in order to find the parameter (center and the radius) of the circle which fits the circular arc. The two ending points are then found by comparing the radius and each point of the circular arc.

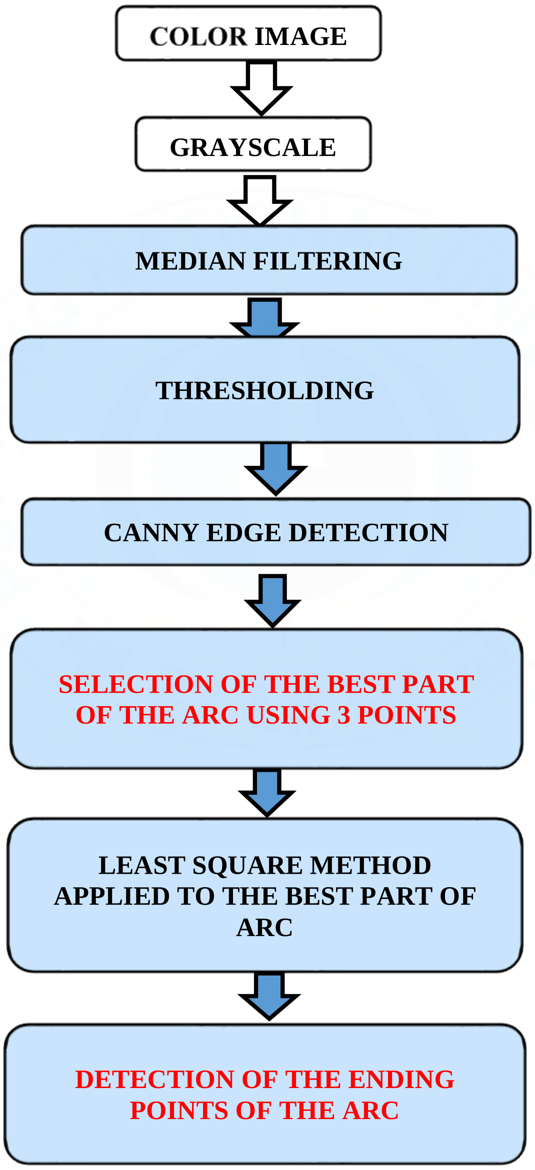


Figure 3 the architecture of the process of a circular arc detection

2. Pre-processing operations

2.1. Median Filtering

The Median filter is a pre-processing operation usually used to remove noise on an image in order to improve the results of later processing such as binarization or edge detection.

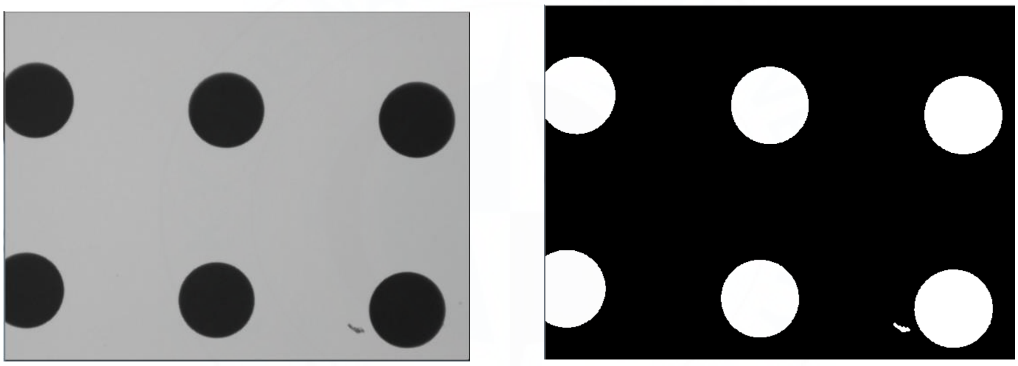
In terms of image processing, median filtering is defined as follows: Let $[x_{ij}]$ be the matrix representing a digitized image. Then the result of the median filtering with an m * n (where m, n = odd integers) window is an image $[y_{ij}]$ where y_{ij} is equal to the median of the gray levels of the picture elements lying in an m * n window centered at the picture element x_{ij} in the input image [13].

The median filter replaces each pixel by the median or "middle" pixel value in a square neighborhood around the center pixel [14].One of the main advantage of using the median filtering is his ability to preserve edges while removing noise comparatively to the others filtering techniques such as the Gaussian filtering.

Figure 4. Example of an image with 'salt and pepper' noise and its result after median filtering

2.2. Thresholding

Thresholding is the simplest form of image segmentation. It is a widely used pre-processing operation for image segmentation and can also be used to create binary images. The Thresholding method helps to make a decision about the pixels in an image or to categorically reject those pixels below or above some value while keeping the others [14]. The simplest thresholding method replaces every pixel in an image with a black pixel if the pixel value is less than a fixed value or a white pixel if the pixel value is greater than that value or vice versa.



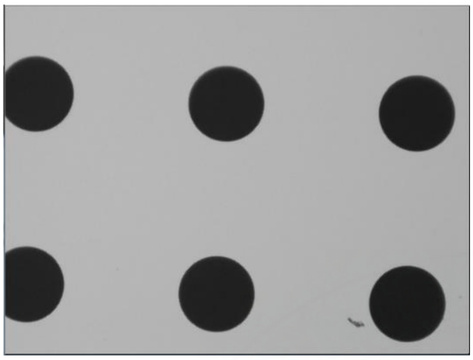
(a) (b)

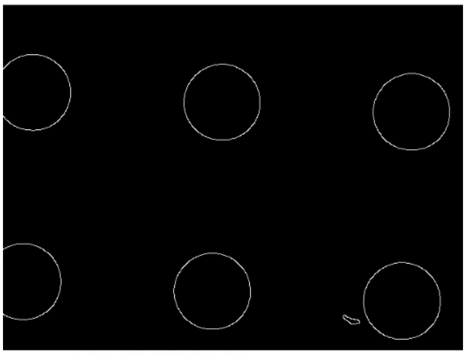
Figure 5. Example of a grayscale image and his binary image. (a) Grayscale image, (b) Binary image

2.3. Canny Edge detection

Edges characterize boundaries and therefore carry important information of an image. The edge detection process serves to simplify the analysis of images by drastically reducing the amount of data to be processed, while at the same time preserving useful structural information about object boundaries [12].

Canny edge detector is one of the basic algorithms used in shape recognition and determines edges by an optimization process and proposed an approximation to the optimal detector as the maxima of gradient magnitude of a Gaussian-smoothed image [12].





(a) (b)

Figure 6. Example of a grayscale image and the edge image from it. (a) Grayscale image,(b) Binary image

3. Selection of the best Part of an arc using 3 points

The best part of the arc means the part in which all the points are lying on the circular shape. It excludes the outlier points or points which vary from the circular shape. The best part of the arc is obtained using 3 points designated by the user along the arc. The 3 points must be designated in a such a way that they show automatically the orientation of the arc.

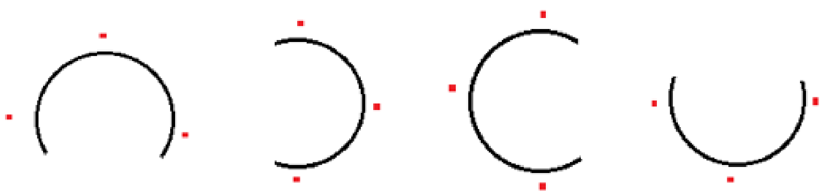
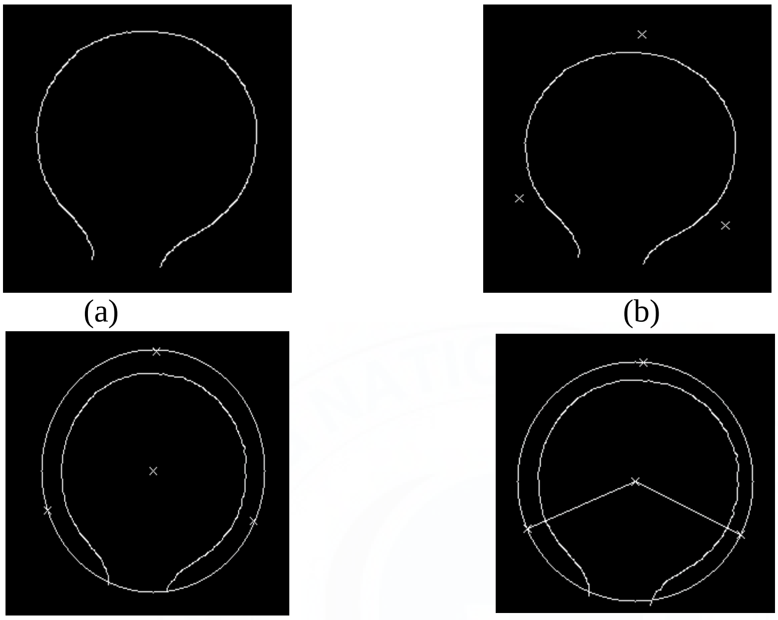


Figure 7. Illustration how the 3 points indicate the orientation of the arc

From the 3 points , we made an algorithm for calculating a circle ROI passing through them. With the 3 points, the circle ROI and its center we can have the space containing the best part of the arc



(a) (b)

(c) (d)

Figure 8. Description step by step, the selection of the best part of the arc

(a) Result of the preprocessing operations

(b) Designation of the 3 points along the arc

(c) Circle ROI passing through the 3 points

(d) Selection on the best part of the arc

From this example , the space containing the best part of the arc can be shown in the following image

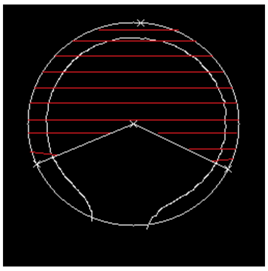


Figure 9. The space containing the best part of the arc

4. Least squares circle fitting

The least square is applied to all the points of the "best part" of the circular arc, and finds the circle which fits the best the circular arc's points. The noise and the outlier points being ignored in the selection of the best part of the arc, the result of the least square circle fitting will be more accurate.

5. Detection of the two ending points of the arc

The two ending points of the arc are calculated by the difference between the radius of the circle fitting the arc and all the points of the arc which are not in the best part of it. If the difference is equal to a certain value fixed by the user, then the ending points are detected.

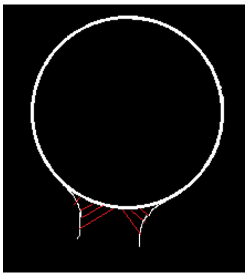


Figure 10. Illustration of the detection of the 2 ending points of the arc

Ⅳ. Experimental Results

1. Dataset

This study presents experiments for the evaluation of the proposed method. A set of 10 images with almost a hundred of circular arcs objects has been used to test the effectiveness of the proposed method. The arcs vary from 100 to 15 pixels of radius. All the images have a size of 640×480 pixels, and they are saved in TIF format.

2. Evaluation Environment

This study is implemented using Visual Studio, openCV, c++ as programming language and Windows 7 – 64bit Operating system. The hardware specification is as follows:

- Intel (R) Core(TM) i7 CPU 860 2.80GHZ
- 4GB of memory (RAM)
- 500 GB of hard disk drive

3. Experiments

The following figure shows an example of two experimental images. Many arcs objects with different size are on the two images but only 5 arcs are chosen to show the results of the proposed method.

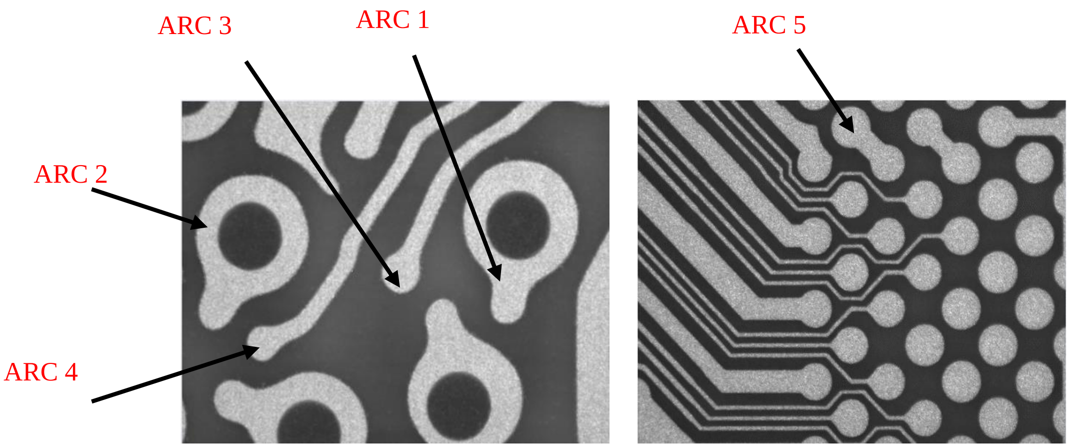
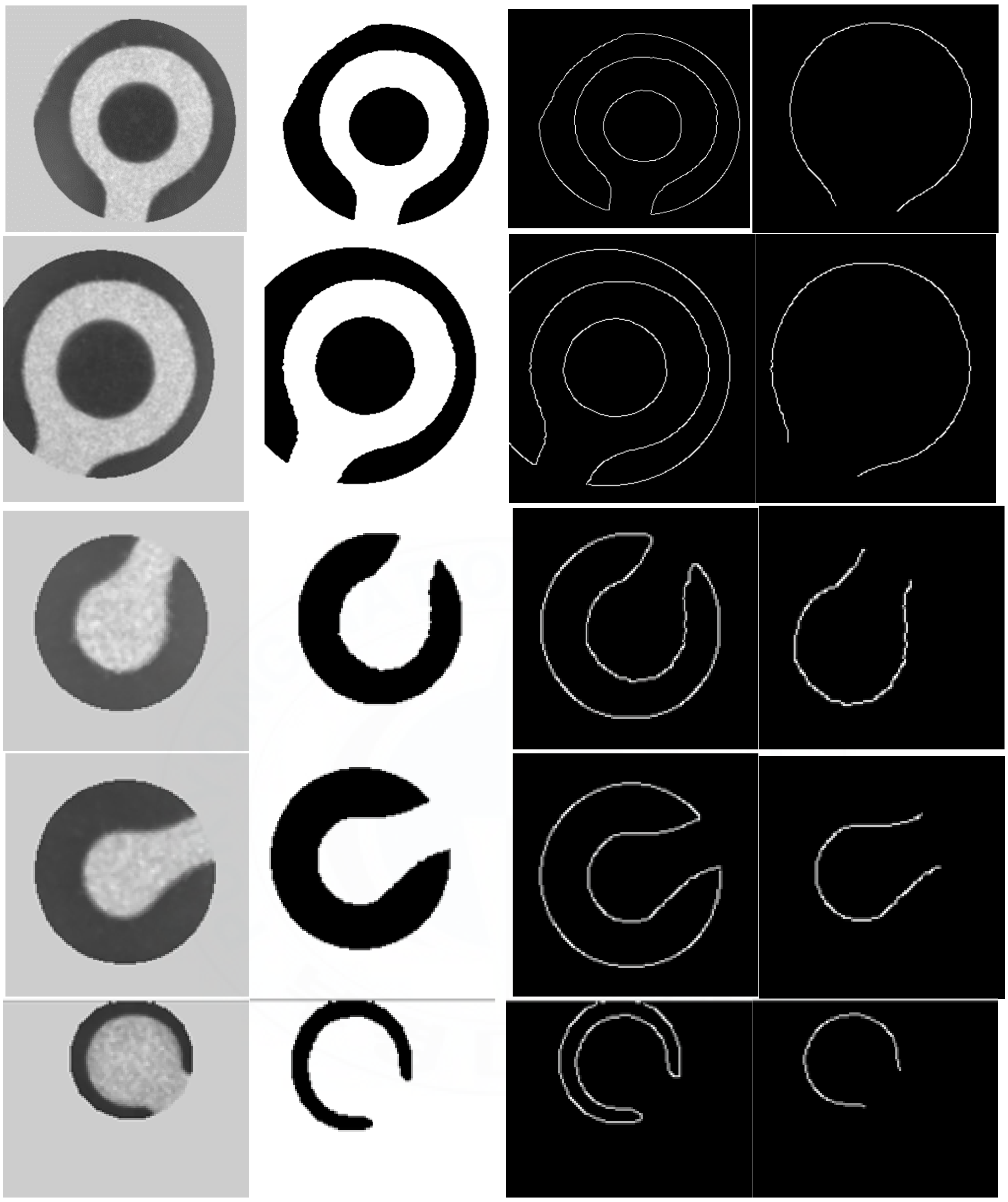


Figure 11. Samples of Original images used to test the proposed method

After extraction of a given arc, the pre-processing operations (Median filtering, Binarization, Canny edge detection, unnecessary shape removal) are applied to it in order to have the real edge of the arc to be detected.



(a) **(b)** **(c)** **(d)**

Figure 12. Examples of results obtained by the preprocessing operations.

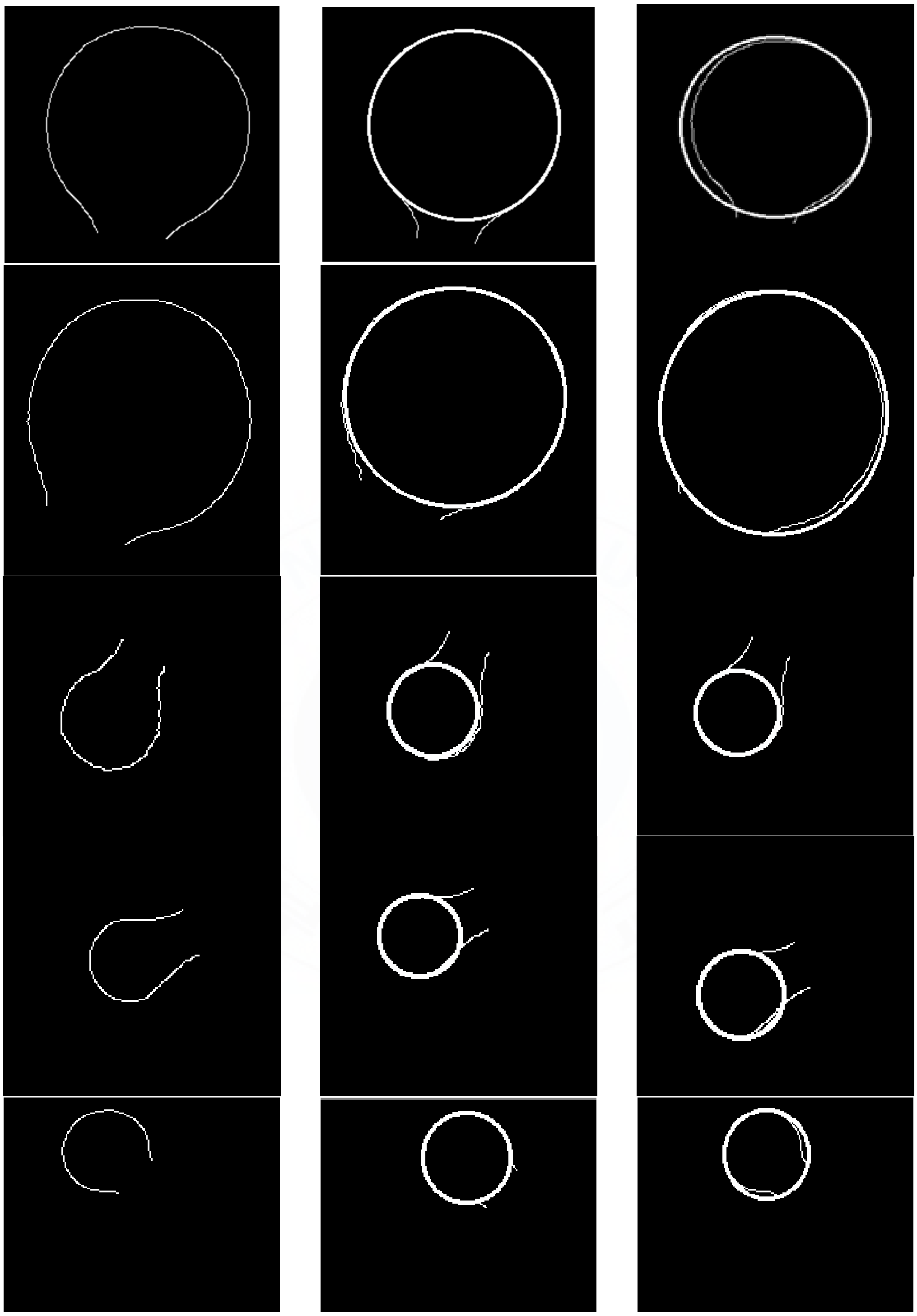
(a) Column of the results of the Median filtering

(b) Column of the results of binarization

(c) Column of the results of Canny edge detection

(d) Results after the removal of unnecessary shapes

After the result of the preprocessing operations, the proposed method is used to select first the best part of the arc which will be applied the least square method in order to find the center point and the radius of a circle fitting the best the arc. The result of the proposed method will then be compared with the result of Hough Transform method.



(a) **(b)** **(c)**

Figure 13 Examples of results obtained by the pre-processing operations. First column: result of pre-processing, 2nd column: Result of the proposed method, 3rd column: results of Hough transform

(a) the results of the pre-processing operations

(b) Results of the proposed method

(c) Results of the Hough Transform

Table 1 Center position and radius for the different arcs by 2 methods

	Proposed Method		Hough Transform	
	Radius	Center point	Radius	Center point
Arc 1	85	(505,167)	89	(500,172)
Arc 2	82	(108,186)	85	(106,190)
Arc 3	28	(325,239)	28	(325,241)
Arc 4	25	(122,338)	25	(123,339)
Arc 5	29	(318,37)	31	(321,39)

The proposed method finds also the 2 ending points of the arc by calculating the difference between the radius and the distance between outlier points and the center of the arc. If that difference is equal to a certain fixed number, the ending points are found.

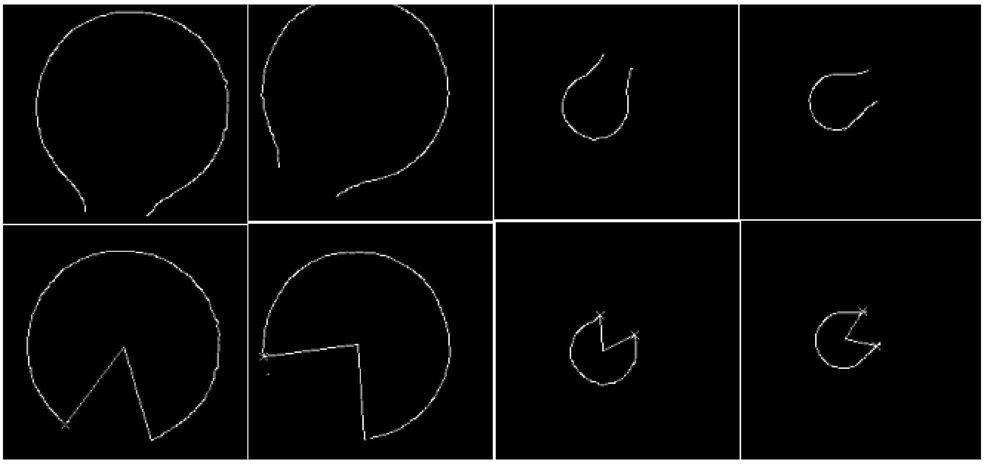


Figure 14. Examples of the ending point's results on the arcs. Top: result of pre-processing operations. Bottom: result of proposed method + ending points detection

4. Performance Evaluation

The performance of the proposed method is given in terms of accuracy. First , The percentages of the arc's points which are perfectly matching with the detected circle using the proposed method and the Hough Transform are calculated and compared. Second , the distances between the curve's points and the circle fitting the arc are computed for the 2 methods and for each arc.

Table 2. Total points of the arc and the average matching rate of each method

	Proposed Method	Hough Transform
Total points of the arc	Average Matching ratio (%)	Average Matching ratio (%)
129	69.77	44.19
139	63.31	48.92
139	84.17	51.08
142	78.17	61.97
149	95.97	95.30
152	84.87	61.84
154	91.56	54.55
155	80.65	87.74
156	86.54	55.13
157	63.06	61.15
162	88.89	40.12
163	89.57	77.30
164	84.76	47.56
166	52.41	39.16
167	61.08	51.50
168	67.26	48.21
169	77.51	69.23
171	91.81	75.44
172	91.28	66.28
173	85.55	50.29
176	89.20	32.39
179	89.39	37.43
179	84.92	46.93
180	70.56	70.00
181	84.53	78.45
183	77.05	48.63
184	91.30	49.46
184	95.11	88.59
186	83.87	45.70
188	72.34	53.72
192	80.73	51.56
193	89.12	78.24
193	86.01	77.72
200	80.00	57.50
202	85.15	40.59
504	70.00	35.52
522	94.06	31.80
564	86.17	27.84

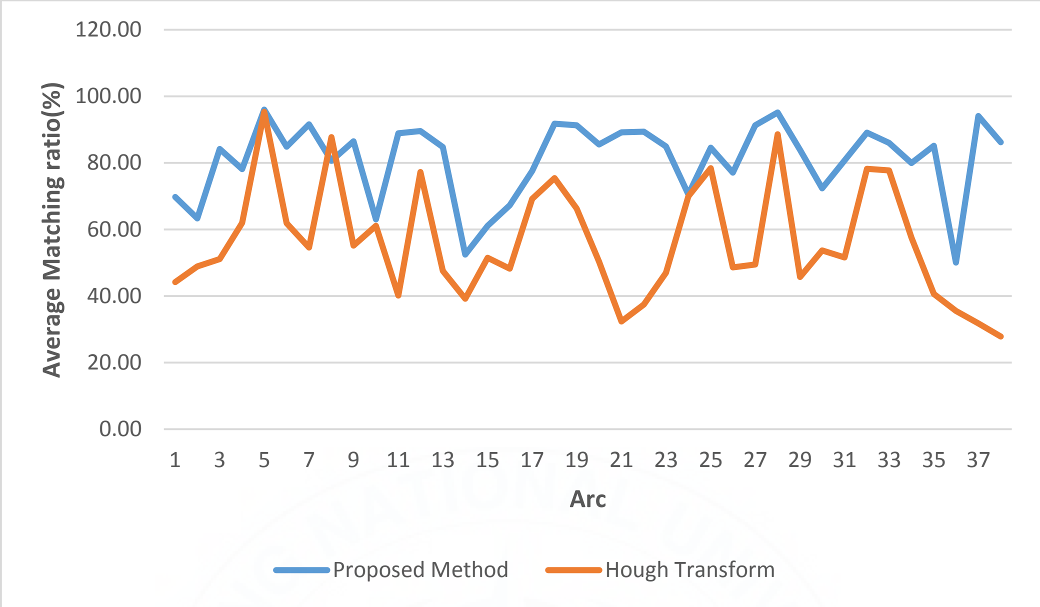


Figure 15 Detection accuracy of the proposed method for different arcs

Table 3Sum of distances between the curve's points and the fitted circle

ARCS	Proposed Method sum of distances	Hough Transform sum of distances
Arc 1	845	3231
Arc 2	1242	1942
Arc 3	447	824
Arc 4	399	494
Arc 5	93	409
Arc 6	209	33
Arc 7	71	114
Arc 8	367	287
Arc 9	67	266
Arc 10	208	189
Arc 11	175	633
Arc 12	143	370
Arc 13	189	324
Arc 14	105	421
Arc 15	257	757
Arc 16	141	451
Arc 17	242	522
Arc 18	121	249
Arc 19	236	455
Arc 20	194	404
Arc 21	165	222
Arc 22	122	535
Arc 23	125	641
Arc 24	260	324
Arc 25	57	207
Arc 26	107	485
Arc 27	179	475
Arc 28	93	188
Arc 29	122	157
Arc 30	162	339
Arc 31	121	165
Arc 32	188	197
Arc 33	79	102
Arc 34	56	250
Arc 35	62	156
Arc 36	152	284
Arc 37	1751	2847

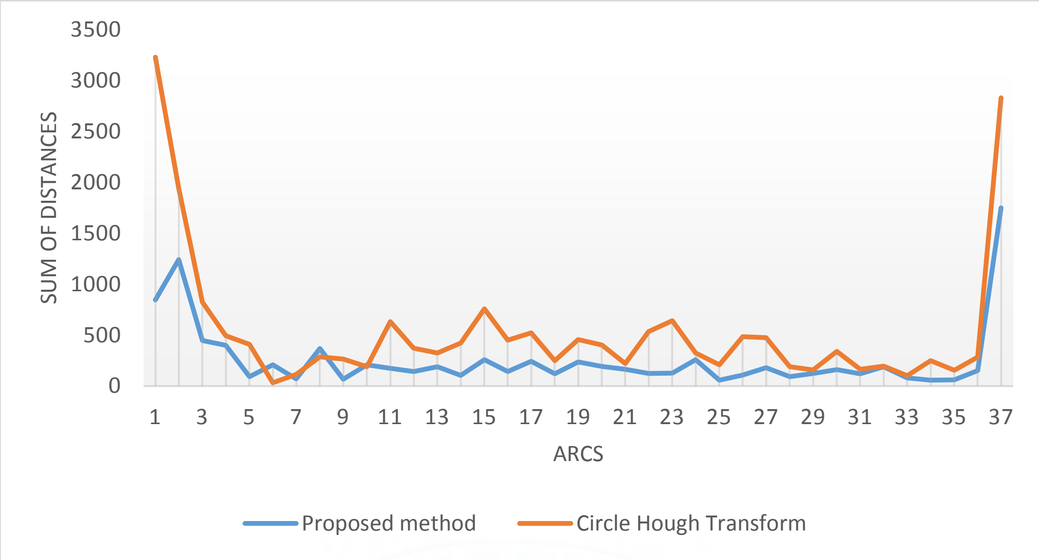


Figure 16 Comparison of the Sum of distances between the curves' points and the fitted circle for the 2 methods

5. Analysis and Comparison

The 2 methods used for the evaluation of the performance show that the proposed method give better results in term of the accuracy of the arcs and circles detection.

For the first method, the number of the arc's points which are perfectly matching with the detected circle is significantly higher for the proposed method than for the Hough Transform and so, for most of the arcs used in this analysis.

For the second method, Second , the distances between the curve's points and the circle fitting the arc are longer for the Hough Transform' results than the proposed method' results, meaning that the proposed method is more accurate than the Hough Transform.

6. Error analysis

The performance of the proposed method can be very poor for very small arcs where the selection of the "best part " of the arc using the 3 points is quite difficult for the user. This proposed method is then recommended for arcs which can allow the designate 3 points along it and then select the best part.

Ⅴ. **Conclusion**

In this thesis , we proposed a new approach for circles and arcs detection. The proposed method focused on the selection of the best part of the arc, before applying it to the least squares method which helps to obtain the center and the radius of the circle that can fit the best that arc. The selection of the part of the arc is operated after some pre-processing operations to remove noises and others unnecessary shapes and then get a clear edge of the arc.

The proposed method has been compared to Hough Transform technique and by the experimental results, the improvement in term of accuracy is quite considerable, especially in case of big arc which can have more variation points or outliers. The proposed algorithm performs also an additional task by allowing the detection of the 2 ending points of the arc. This task is operated by calculating the distance between the radius and the variation points of the arc.

Based on the good performance of the proposed method , our future research will focus on finding how to full automatize this algorithm in order to detect the arcs in real time without human intervention.

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