Thesis for the Degree of Master of Physics

Dynamical Analyses of Communicability Structures in Complex Networks

by

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The Graduate School

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복잡 네트워크에서 집단성의 동역학적 분석에 관한 연구

Advisor: Prof. Kyungsik Kim

by

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A dissertation

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Abstract

본 연구는 한국물리학회(Korean physical society)의 새물리, 한국기상학회(Korean meteorological society)의 발표회 저자들을 추출하여 미시적 및 중시적 사회구조에 관하여 수행한 연구이다. 이론적으로 고전, 양자적 진동자를 도입하여 사회 네트워크에서 온도가 높아짐에 따라 사회성 함수 (communicability function) 을 시뮬레이션하여 분석하였다. 양자적 진동자 네트워크에 대한 사회구조는 온도에 비례하지 않는 반면, 온도의 변화는 고전적 진동자 네트워크의 구조에 선형적으로 영향을 준다. 특히 중시적 사회구조에서 집단들 사이의 사회성등을 비교분석하였다.

Contents

1. Introduction

Over the two decades, the study of networks has generated crucial interest, as the researchers have treated various applications in scientific areas. This has particularly been emerged many topics as one of the important frameworks when each researcher analyzed the method and its technique in complex systems [1-4]. An important property of networks is included the existence of communities, and the communicability between a pair of nodes in a network is concerned with the shortest path connecting both nodes. Estrada et. al. [5] have proposed generally the community structure by elucidating for the shortest paths and all the other walks. The communicability allows one to determine potentially the hidden and unknown relations between nodes. Furthermore, we may describe to reduce and analyze a large group into smaller and smaller groups, and to relate between one large group and other large groups. In the issues of the macroscopic community, several researchers have studied statistical metrics such as the entropy, natural connectivity, total energy, free energy, and bipartivity in order to present the overall society structure [5]. The macroscopic, mesoscopic, and microscopic community structures within the investigation of networks will be an open subject of great interest in the future.

Complex networks are also ubiquitous in many biological, ecological, technological, informational, and infrastructural systems [6–12]. It is clear that the atomic, oscillating, and social systems display network-like structures using the tools of statistical mechanics. These methods and techniques were contributed to shed light on the structure and dynamics of applied systems [13-15]. It is actually recognized that the analogy functions describing the properties depend mainly on the structural properties of the system in networks. An important issue in regards to networks has recently been the cascade effect in both the ecological network and the multiplex network. The former propagates well beyond the nearest neighbors of the extinguished species [16-18] with protein–protein interactions. The latter describes the fact that multiplex structures with different strength of coevolution respond differently to the cascade process, exemplifying the dynamical signature that coevolution can imprint [19].

The viewpoint of community structure is the advantage that it allows the effect of a selective temperature under study. The community structure for network of quantum oscillators changes non-trivially with temperature, while the change of temperature affects linearly the structure of quantum network. We can provide some important results giving evolutionary information, as the communicability functions in the community structure are investigated. The difference between intra-cluster and inter-cluster communicability is also calculated in mesoscopic community structure, and this is the statistical quantity estimated the community structure between two societies.

In this paper, we study the macroscopic and mesoscopic communicability in author networks. Data is extracted from papers in the talk and poster sessions of the Korean meteorological society (KMS) from March 2008 to November 2013 and in Saemulli of the Korean physical society (KPS) from January 2003 to December 2014. In Section II, we treat in detail the theoretical methods of mesoscopic and microscopic communicability in networks. We perform the numerical calculation and give its result in Section III. Our main results are summarized in Section IV.

II. Communicabilities in networks

In this section, we mainly consider the theoretical method of microscopic and mesoscopic communicabilities in networks. First of all, let us introduce the concept of communicability in networks by describing a community structure. Stating point of a network is a sequence of nodes n_0, $n_1, \ldots, n_{k-1}, n_k$ such there is a link from n_{i-1} to n_i for $i = 1, 2, \ldots, k$[20]. Using the concept of link, we can define the communicability between two nodes, p and q. The communicability function [4] is represented in terms of $G_{pq} = \sum_{k=0}^{\infty} c_k (A^k)_{pq}$. Here, A is the adjacency matrix, which has unity if the nodes p and q are linked to each other, and has zero otherwise. The adjacency matrix $(A^k)_{pq}$ gives the number of length k starting at the node p and ending at the node q [17,18]. The communicability function [19] is calculated as

$$G_{pq}^{EA} = \sum_{k=0}^{\infty} \frac{(A^k)_{pq}}{k!} = (e^A)_{pq} \quad . \tag{1}$$

The communicability function G_{pq} is obtained by using the weighted adjacency matrix $W=(W_{ij})_{n\times n}$. The centrality measures were originally introduced in social sciences [20,21] and widely used in the whole field of complex network analysis [9]. We can derive the communicability function as

$$G_{pq}^{RA} = \beta K m \omega^2 G_{pq}(\beta) \tag{2}$$

with the identification $\alpha = 1/K$. Here, the correlation between two nodes in a network structure is given by

$$G_{pq}(\beta) = \frac{1}{\beta K m \omega^2} [1 - (A/K)^{-1}]_{pq} . \tag{3}$$

From the fact that the Laplacian matrix of a connected network has a non-zero eigenvalue, we can calculate another correlation function as

$$G_{pq}^{D}(\beta) = \frac{1}{\beta K m \omega^2}(L^{+})_{pq}, \tag{4}$$

Where L^{+} is the Moore–Penrose generalized inverse of the Laplacian.

In a quantum oscillator, we consider the quantum-mechanical counterpart of the Hamiltonian H_A. After arranging several equations, we can see that

$$G_{pq}^{EA} = \exp(\beta\hbar\Omega) G_{pq}^{A}(\beta). \tag{5}$$

The diagonal thermal Green's function is given in the framework of quantum mechanics, and we can compute the thermal Green's function as

$$G_{pq}^{A}(\beta) = \exp(-\beta\hbar\Omega)(\exp[\frac{\beta\hbar\omega^2}{2\Omega}A])_{pq}. \tag{6}$$

Note that when the temperature tends to infinity or $\beta \to 0$, there is no communicability between any pair of nodes. That is, $G_{pq}^{EA}(\beta \to 0) = 0$. If we consider the case when the temperature tends to zero or $\beta \to \infty$, then there is an infinite communicability between every pair of nodes, i.e., $G_{pq}^{A}(\beta \to \infty) = \infty$. Furthermore, the communicability function is represented in terms of

$$G_{pq}^{EL}(\beta) = G_{pq}^{L}(\beta) - 1, \tag{7}$$

where the quantum-mechanical calculation by using the Hamiltonian H_L in Eq. (5) is calculated as

$$G_{pq}^{L}(\beta) = (\exp[-\frac{\beta\hbar\omega^2}{2\Omega}L])_{pq}. \tag{8}$$

From Eqs. (7) and (8), the communicability function G_{p}^{EL} gives $G_{pq}^{L}(\beta) - 1$ upon setting $\beta h \omega^2 = 2\Omega$ [4]. Lastly, we simulate and analyze the averaged communicability function for a given node defined as

$$G_p = \frac{1}{n-1}\sum_{q \neq p}^{n} G_{pq} \, . \tag{9}$$

Consequently, the communicability functions G_{pq}^{RA} and G_{pq}^{D} become the types of the thermal Green's function of classical harmonic oscillators in networks of the community structure. The communicability functions $G_{pq}^{EA}(\beta)$ and $G_{pq}^{EL}(\beta)$ also become the types of the thermal Green's function in quantum harmonic oscillators. From Eq. (9), G_p^{RA}, $1/G_p^{D}$, G_p^{EA}, and G_p^{EL} are, respectively, the averaged communicability function of G_{pq}^{RA}, $1/G_{pq}^{D}$, G_{pq}^{EA}, and G_{pq}^{EL}.

In mesoscopic communicability of networks, the community structure has become one of the most intensive areas of interdisciplinary research in this field [22,23]. In order to analyze the communicability of mesoscopic level, we recall the community structure of complex networks from the sign separation of the communicability function. The community function used by Estrada and Hatano [5] is represented in terms of

$$G_{pq}^{EA}(\beta) = \phi_{1,A}(p)\phi_{1,A}(q)e^{\lambda_{1,A}} + [\sum_{2\le j\le n}^{++} \phi_{j,A}(p)\phi_{j,A}(q)e^{\lambda_{j,A}} + \sum_{2\le j\le n}^{--} \phi_{j,A}(p)\phi_{j,A}(q)e^{\lambda_{j,A}}]$$
$$+ [\sum_{2\le j\le n}^{+-} \phi_{j,A}(p)\phi_{j,A}(q)e^{\lambda_{j,A}} + \sum_{2\le j\le n}^{-+} \phi_{j,A}(p)\phi_{j,A}(q)e^{\lambda_{j,A}}] \tag{10}$$

where $\sum^{++}$ represents the summation over the terms with both $\phi_{j,A}(p)$ and $\phi_{j,A}(q)$ positive, $\sum^{+-}$ represents the summation over the terms with $\phi_{j,A}(p)$ positive and $\phi_{j,A}(q)$ negative, and so on.

The network consists of several clusters of connected nodes forming distinguishable communities which are relatively poorly connected to each other. The main difference between intra-cluster and inter-cluster communicabilities can be calculated as follows:

$$\Delta G = G_{pq}^{EA} - \phi_{1,A}(p)\phi_{1,A}(q)e^{\lambda_{1,A}} + \sum_{j>1}^{\text{intra}} \phi_{1,A}(p)\phi_{1,A}(q)e^{\lambda_{1,A}} - \left| \sum_{j>1}^{\text{inter}} \phi_{1,A}(p)\phi_{1,A}(q)e^{\lambda_{1,A}} \right|, \quad (11)$$

where intra (inter) denotes intracluster (intercluster).

Next, the modularity is a value of a community structure that is the number of edges falling within groups minus the expected number in an equivalent network with edges placed at random [24]. This is represented in terms of

$$Q = \frac{1}{4m}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(A_{ij} - \frac{k_i k_j}{2m}\right) s_i s_j, \quad (12)$$

where $s_i = 1$ if the node i exists in a group, and $s_i = -1$ if the node I does not exist in another group. The quantity A_{ij} is the number of edges between vertices i and j and will normally be 0 or 1, although larger values are possible in networks where multiple edges are allowed. The expected number of edges between i and j when edges are placed at random is $k_i k_j/2m$, where k_i and k_j are the degrees of the vertices, and $m = \frac{1}{2}\sum_{i=1}^{N} k_i$ is the total number of edges in the network. As the modularity goes to a larger value, the system extends to a network that is more modularized or become a richer community structure.

We will make use of averaged communicability functions in a microscopic community and other metrics to compute the measures of a mesoscopic community structure. These statistical quantities will lead us to more general results and predictions in the future. From calculated results, we consider that the analysis of communicability functions and other metrics is very useful and rewarding in the community structure.

III. Numerical calculations and results

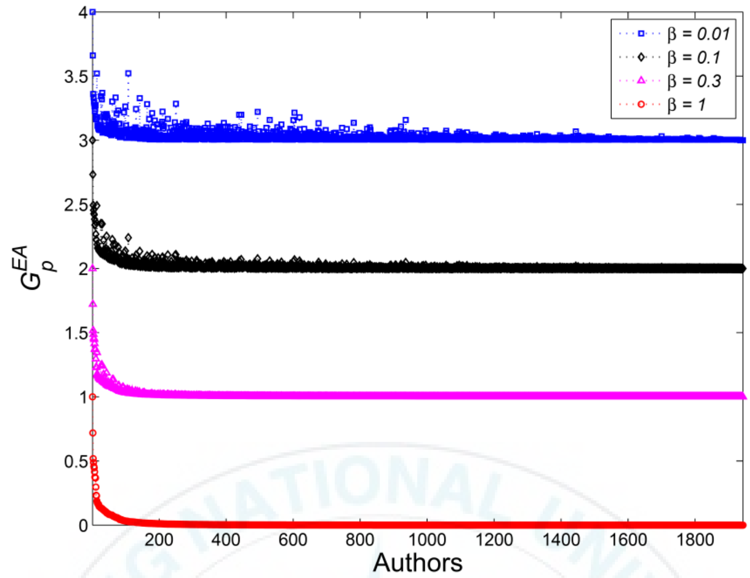

Fig.1: Averaged communicability function G_p^{EA} from β=1 to 0.01 in the KMS. These values are displaced upward by 1.0 unit each in order to provide better visibility.

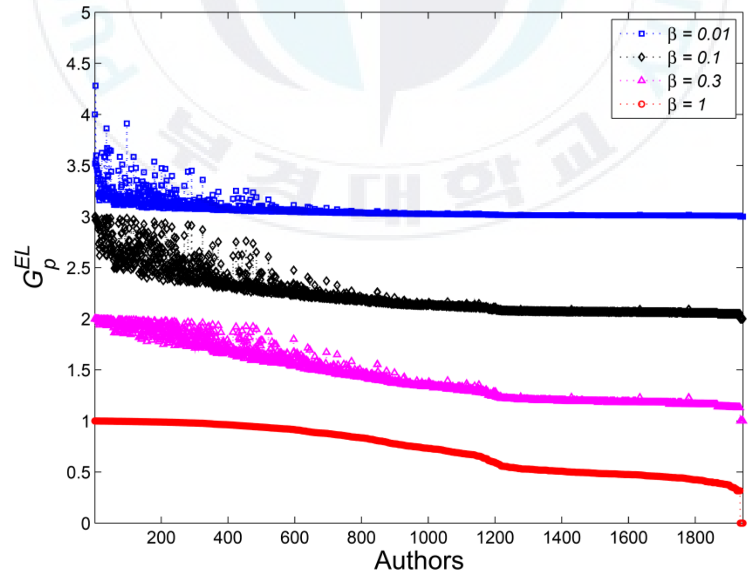

Fig.2: Averaged communicability function G_p^{EL} from β=1 to 0.01 in the KMS. These values are displaced upward by 1.0 unit each in order to provide better visibility.

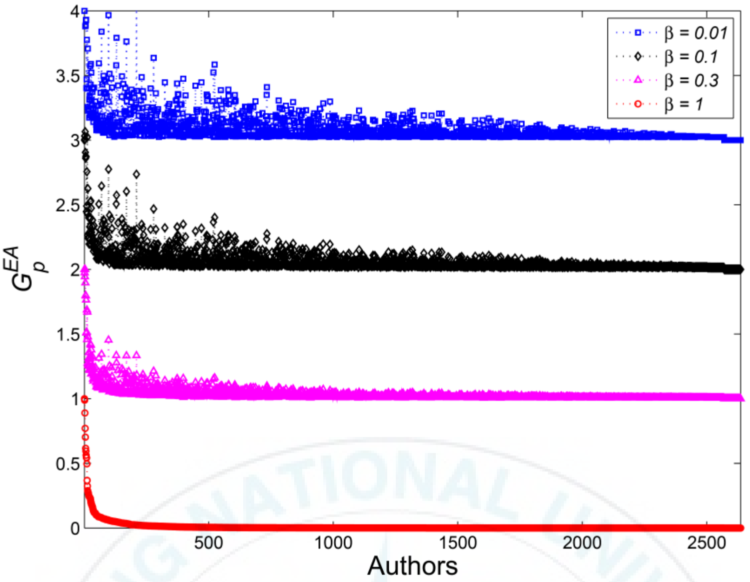

Fig.3: Averaged communicability function G_p^{EA} from β=1 to 0.01 in the Saemulli of the KPS. These values are displaced upward by 1.0 unit each in order to provide better visibility.

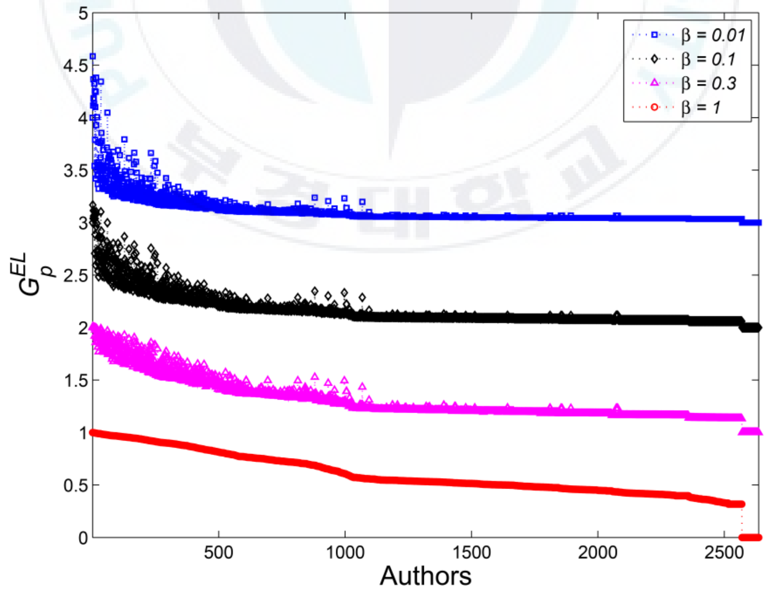

Fig.4: Averaged communicability function G_p^{EL} from β=1 to 0.01 in the Saemulli of the KPS. These values are displaced upward by 1.0 unit each in order to provide better visibility.

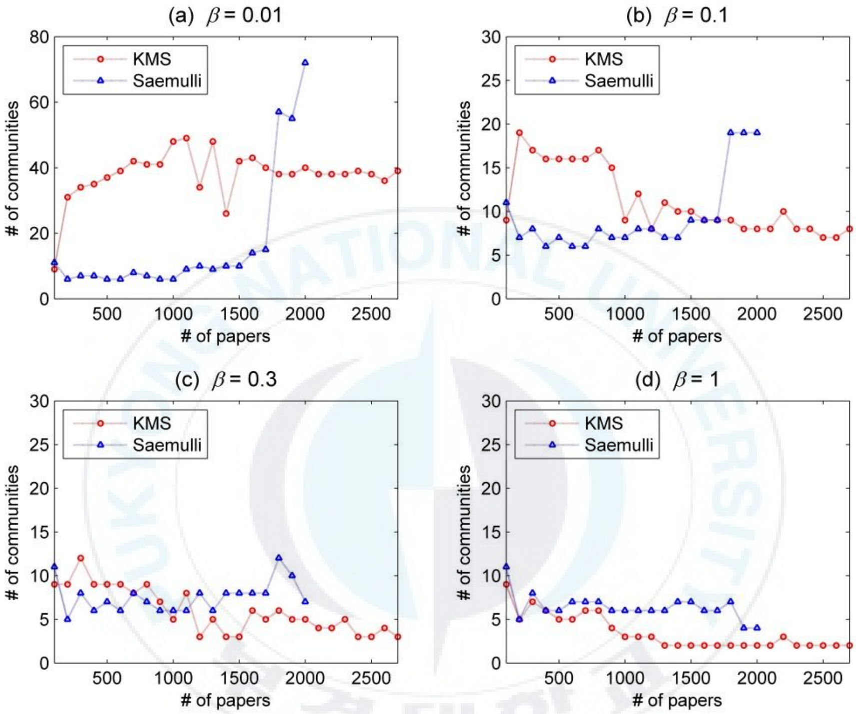

Fig. 5: Number of communities as a function of the number of papers from β=1 to 0.01 in the KMS and the Saemulli of the KPS.

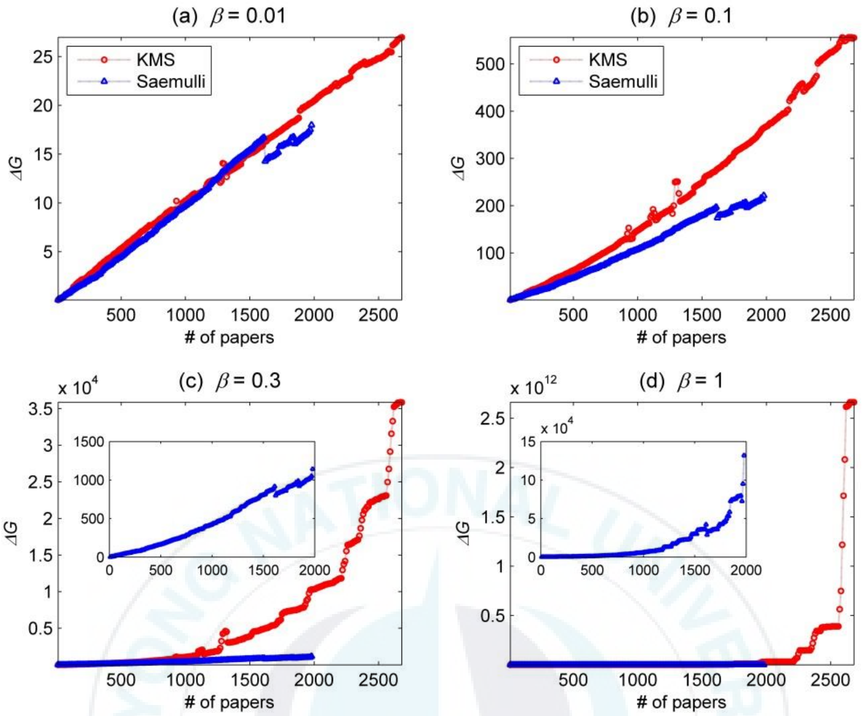

Fig. 6:ΔG as a function of the number of papers from β=1 to 0.01 in the KMS and the Saemulli of the KPS.

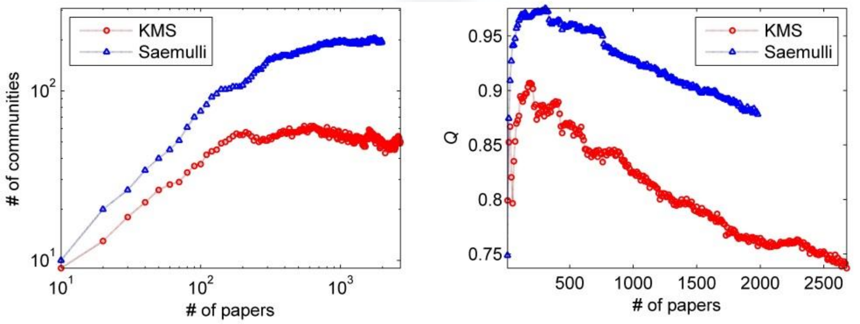

Fig. 7: Number of communities (left) and modularity (right) as a function of the number of papers in the KMS and the Saemulli of the KPS.

Table 1: Values of the sum of degrees D_s, the weight of community W_c, and the number of published papers N_c, and the averaged communicability functions. Here, the averaged communicability functions have the maximum value (one) for first rank author, respectively. This is the data for authors of (a) the Korean meteorological society and (b) Saemulli of KPS publications in the author network at β=1.

(a) KMS

Sequential order of authors	D_s	W_c	N_c	G_p^{EA}	G_p^{RA}	G_p^{EL}	$1/G_p^{D}$
1	135.9767	32.5395	190	1	1	1	1
100	13.366	4.931	20	0.1448	0.112	0.9995	5.6197
200	8.481	1.2177	12	0.0851	0.0703	0.9967	8.6968
300	6.1595	2.65	9	0.0022	0.0425	0.9806	13.9951
400	4.5265	3.2	7	0.0014	0.0296	0.9671	18.2785
500	3.6682	1.7333	5	0.0004	0.0246	0.953	21.5809
600	3.0173	0.5667	4	0.0049	0.0201	0.9248	27.2853
700	2.4364	0.7429	4	0.0001	0.0157	0.8811	33.449
800	2.0921	0.5833	3	0.0075	0.0146	0.838	61.1628
900	1.6667	0.2679	3	0.0001	0.0109	0.7561	56.0115
1000	1.4643	0.3333	2	0.0019	0.0102	0.757	46.8707

(b) Saemulli of KPS

Sequential order of authors	D_s	W_c	N_c	G_p^{EA}	G_p^{RA}	G_p^{EL}	$1/G_p^{D}$
1	26.6222	11.1667	37	1	1	1	1
100	6.1239	2.1389	10	0.3572	0.274	0.9817	3.1066
200	4.0762	1.2468	7	0.0389	0.162	0.9443	7.0265
300	3.1682	1.1333	5	0.0347	0.11	0.9377	6.9127
400	2.5333	0.7774	4	0.0084	0.0949	0.868	8.8308
500	2.131	1.3167	3	0.0034	0.0745	0.8146	12.3472
600	1.7574	0.9167	3	0.1708	0.0805	0.8154	7.6484
700	1.5556	0.5833	2	0.0005	0.0498	0.727	7.1131
800	1.4643	0.9167	2	0.0003	0.0458	0.6905	5.7709
900	1.3333	0.7333	2	0.0015	0.0443	0.6634	16.1744
1000	1.1667	0.3667	2	0.0004	0.0367	0.6172	32.6687

In order to simulate and analyze the communicability structure, the first type of data is extracted 2684 papers for 1943 authors in the talk and poster sessions of Korean meteorological society from March 2008 to November 2013. Second type of data is extracted 1983 papers for 2636 authors published of Saemulli in the Korean physical society from January 2003 to December 2014. We assume that it only takes an equally contributed weight between all authors in one published paper.

Figures 1 and 2 plot the averaged communicability functions G_p^{EA} and G_p^{EL} in the KMS, respectively, as the temperature increases to β = 0.1from β=1. While Fig. 1 has the higher communicability function achieved by authors within 200 ranks, Fig. 2 has the higher communicability function achieved by authors within 1,200 ranks. We find that the gap change of G_p^{EL} shows more difference than that of G_p^{EL}. It is however noted that our result has the better communicated persons in community structures only surpassed by the higher persons at the level of lower temperature.

In Figs. 3 and 4, the averaged communicability functions G_p^{EA} and G_p^{EL} in the Saemulli of KPS are, respectively, showed at four different temperatures. The averaged communicability functions G_p^{EA} and G_p^{EL} in Saemulli of KPS is appeared similar to those of the KMA (Figs. 1 and 2). The gap change of G_p^{EL} in Saemulli of KPS shows notoriously larger difference rather than that of KMA.

In Fig. 5, we show the number of communities as a function of the number of papers from β=1 to 0.01 in the KMS and the Saemulli of the KPS. We expect that the number of communities increases relatively as the number of papers increases, but the number of communities appear no change except the Saemulli at β=0.01 and 0.1. That is, the number of communities increases notoriously near 1,500 papers in the Saemulli of KPS at β=0.01 and 0.1.

Figure 6 plots ΔG as a function of the number of papers in the KMS and the Saemulli of the KPS. From the viewpoint of mesoscopic structure, the Saemulli of KPS the ΔG appears no linearity at β=0.3, but the ΔG increases linearly at β=1.0.

Figure 7 plots the number of communities and modularity as a function of the number of papers in the KMS and the Saemulli of the KPS. To compare two societies, we find the more number of communities in the Saemulli rather than in the KMS. The modularity has larger values in the Saemulli of the KPS as well. As shown in Fig. 7, the values of modularity over 200 (number of papers) has a decreasing trend, and the modularity of two societies has a larger value rather than the generic value of random data.

In the Korean meteorological society and (b) Saemulli of KPS publications of the author network at β=1, Table 1 summarizes values of the sum of degrees D_s, the weight of community W_c, and the number of published papers N_c, and the averaged communicability functions. The averaged communicability functions of each author have relative values from normalized maximum value (one) of first ranked author.

IV. Summary

We have studied the microscopic and mesoscopic community structures in the two Korean scientific societies. As well-known, the community structure for the network of quantum oscillators changes non-trivially with temperature, while the change of temperature affects linearly the structure of classical network. Particularly, the difference between intra-cluster and inter-cluster communicability is calculated in mesoscopic community structure.

It is expected that our result may be cultivated the better communicated persons having potential ability surpassed by the excellent and brilliant persons of higher rank at the level of lower temperature. We may provide some important results giving evolutionary information, as the microscopic and mesoscopic community structures are investigated.

We emphasize that our result can be very useful for studying phase transitions as our numerical results is compared with others. In the future, the universal and irregular properties would like to find from our and other networks. As future works, we are planning to investigate extensively the dynamical behavior of systems in other community.

In our community structures of two societies on the basis on the communicability functions, it is not simple to make the novel community structure as the number of authors (or papers) increases more and more, and we may consider our structure as the analytical method of a conservative viewpoint. In next time, we hope to discuss the phase transition of the averaged communicability functions, with network systems of other societies. The scientific field of community structure in complex systems is small but still growing. It opens up mesoscopic community structure to a vast range of brain and neuron systems. Therefore, further work is needed for the case with societies of more than the author and citation networks [25-27]. In the next time, we hope to discuss the phase transition of the averaged communicability functions, with network systems of other societies. The result of our analysis can be extended to both the discrimination and the characterization of communicability functions in other various societies.

References

[1] M.J.E. Newman, *Networks,* Oxford University Press, Oxford, 2010.
[2] C. Castellano, S. Fortunato, and V. Loreto, Rev. Modern Phys. 81 (2009) 591.
[3] Y.S. Cho, S. Hwang, H.J. Herrmann, and B. Kahng, Science 339 (2013) 1185.
[4]E. Estrada, N. Hatano, M. Benzi, Phys. Rep. 514 (2012) 89.
[5] E. Estrada and N. Hatano, Phys. Rev. E 77 (2008) 036111.
[6] G. Caldarelli, *Scale-Free Networks*, *Complex Webs in Nature and Technology,* Oxford University Press, Oxford, 2007.
[7] L. da Fontoura Costa, O.N. Oliveira Jr., G. Travieso, F.A. Rodrigues, P.R. Villas Boas, L. Antiqueira, M.P. Viana, and L.E. Correa Rocha, Adv. Phys. 60 (2011) 329.
[8] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U.Hwang, Phys. Rep.

424 (2006) 175.
[9] M.E.J. Newman, SIAM Rev. 45 (2003) 167.
[10] M.J.E. Newman, Networks, Oxford University Press, Oxford, 2010.
[11] S.H. Strogatz, Nature 419 (2001) 268.
[12] D.J. Watts, *Small Worlds: The Dynamics of Networks Between Order and Randomness*, Princeton University Press, Princeton, 2003.
[13] M. Buchanan, The Social Atom: Why the Rich Get Richer, Cheats Get Caught and Your Neighbor Usually Looks Like You, Cyan Books and MarshallCavendish, 2007.
[14] R. N. Mantegna and E. H. Stanley, *Introduction to Econophysics: Correlations and Complexity in Finance*, Cambridge University Press, Cambridge, 1999.
[15] B.K. Chakrabarti, A. Chakraborti, A. Chatterjee, *Econophysics and Sociophysics: Trends and Perspectives*, Wiley VCH, Berlin, 2006.
[16] J.A. Dunne, R.J. Williams, N.D. Martinez, Ecol. Lett. 5 (2002) 558.
[17] E. Estrada, J.A. Rodriguez-Velazquez, Phys. Rev. E 71 (2005) 056103.
[18] F. Harary, A.J. Schwenk, Pacific J. Math. 80 (1979) 443.
[19] N. Higham, *Function of Matrices: Theory and Computation, Society for Industrial and Applied Mathematics*, Philadelphia, PA, 2008.
[20] L.C. Freeman, *Centrality in networks: I. Conceptual clarification*, Social Netw. 1 (1979) 215–239.
[21] S. Wasserman, K. Faust, *Social Network Analysis*, Cambridge University Press, Cambridge, 1994.
[22] S. Fortunato, Phys. Rep. 486 (2010) 75.
[23] S. Fortunato, M. Barthelemy, Proc. Natl. Acad. Sci. USA 104 (2007) 36.
[24] S. Min, K.-H. Chang, K. Kim, Y.-S.Lee,Physica A 442 (2016) 268.
[25] S. Redner, Eur. Phys.J. B 4 (1998) 131.
[26] F. Radicchi and C. Castellano, PloS One 7 (2012) e33833.
[27] F.N. Silva, F.A. Rodrigues, O.N. de Oliveira Jr, L.da F. Costa, J. of Informetrics 7 (2003) 469.