Thesis for the Degree of Master of Science

REVISIT TO THE 18-FOLD COVERING KNOT FOR p(-2,3,7)

by

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February 2013

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p(-2,3,7)매듭의 18겹 피복매듭에 대한 재 구현

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A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science

in Department of Applied Mathematics, The Graduate School, Pukyong National University

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February 22, 2013

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양 인 모

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요　약

버기(Berge)는 덴 수술에 의해 렌즈공간을 만드는 삼차원구상의 매듭의 쌍대매듭들이 특별한 1교 따임매듭의 성질을 갖는다고 1990년 미발표논문에서 주장하였다.

본 논문에서는 이 성질을 이용하여 프레젤 매듭 p(-2,3,7)의 18-덴수술에 대하여 가바이 1교 따임(Gabai 1-bridge braid) K(5,2,1)이 렌즈공간 L(18,5)상의 쌍대매듭임을 보이고 p(-2,3,7) 매듭의 18겹 피복매듭을 만드는 방법을 제시한다.

1.INTRODUCTION

Let K be a knot in S^3. If Dehn surgery on K yields a lens space $L=L(p,q)$ with $0<|q|<p$ and $\gcd(p,q)=1$, then the core K^* of the attatched solid torus becomes a knot in L which is called the dual knot of K.

Let $\pi: S^3 \to L$ be the universal covering projection , that is the $p-$ fold regular cyclic covering projection. Since K^* represents a generator of $H_1(L;\mathbb{Z})$ the lifting $\widetilde{K^*}=\pi^{-1}(K^*)$ of K^* turns out to be a knot in S^3. By choosing regular neihbourhoods $N(K^*)$ and $N(\widetilde{K^*})$ of K^* and $\widetilde{K^*}$ respectively so that $\pi(N(\widetilde{K^*}))=N(K^*)$, we can make $S^3-N(K)^\circ$ the $p-$ fold regular cyclic covering space of $L-N(K^*)^\circ$ which is homeomorphic to $S^3-N(K)^\circ$ for some regular neihbourhood $N(K)$ of K. Thus we call $\widetilde{K^*}$ the $p-$ fold covering knot for K. After Fintushel-Stern's discovery [4] of a pretzel knot $p(-2,3,7)$ with lens space surgery slopes 18 and 19, Berge[2] suggested a systematic method, called doubly primitive construction, of obtaining knots in S^3 with integral lens space surgery slopes. They are referred to as doubly primitive knots or Berge knots in short. Among the Berge knots, the covering knots for torus knots and cabled knots are well understood, for instance, see [8,13]. But regarding the covering knots for hyperbolic Berge knots, to the author's knowledge, only Gabai's work [5] on $p(-2,3,7)$ is known so far. Indeed by executing Kirby moves to the Fintushel-Stern's surgery desctiption of $-p(-2,3,7)(-18)=L(18,7)$, he came up with the dual knot in figure 1(a)(which turns out to be the mirror image of $K(11,8,6)$). Concatenating 18-copies of the braided part of the dual knot and 11 righthand full twists (of 11-strands), he eventually ended up with the covering knot with its minimal genus Seifert surface in figure 1(b) after working for 40 hours!

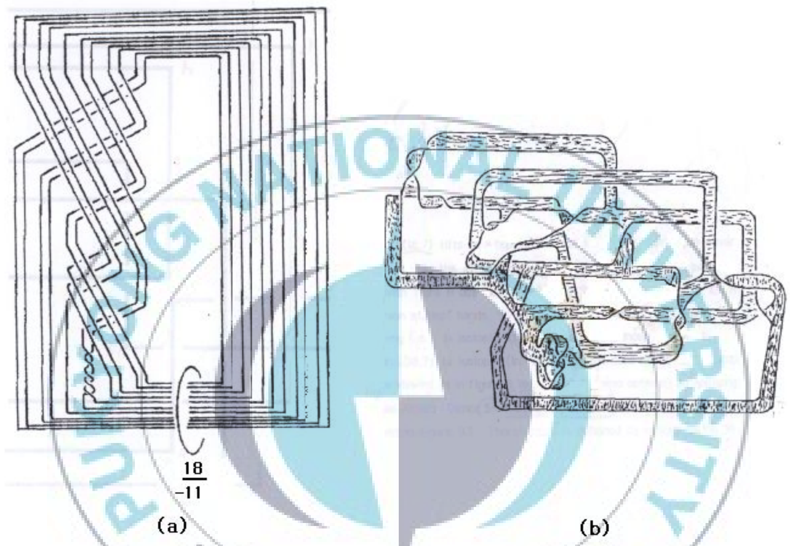


Figure1

Later on he showed that the dual knot of a 1-bridge braid type Berge knot also admits a 1-bridge braid presentation such that the winding numbers of both 1-bridge braids are the same([7,Corollary 3.3]).

For instance it is known that $p(-2,3,7)$ is represented by $K(7,4,2)$ ([1, Fig.11]); the dual knot of $p(-2,3,7)$ in$p(-2,3,7)(18)=L(18,11)$ admits $K(7,2,4)$ as a 1-bridge presentation(cf. figure 7). At last Berge claimed that the dual knot of any doubly primitive knot may be simultaneously braided with respect to both of the solid tori of a genus one heegaard splitting of L ([2, Remark in p.3]). Using this property we show that the dual knot of $p(-2,3,7)$ in $L(18,5)$ can be represented by a simpler 1-bridge braid $K(5,2,1)$ which eventually allows us to present a knot diagram of the 18-fold covering knot without much pains (cf. figure 11(d)). From a 1-bridge position $(V_1,t_1)\cup_F(V_2,t_2)$ for the dual knot of$p(-2,3,7)$ in $L(18,5)$ discovered by Saito([11]), we get a pair of 1-bridge prensentations $t_1\cup t'_2$ in V_1 and $t'_1\cup t_2$ in V_2 where t'_i are projections of the trivial arcs t_i on the Heegaard torus F $(i=1,2)$. Then the former yields $K(5,2,1)$ whereas the latter does $K(7,2,4)$. It may be compared with Wu's treatment of 1-bridge braids in [15]. He used them to detect the core knots in $O(\frac{p}{q}),\frac{p}{q}-$ surgery of an unknot which admit braidings in the complement of O. There are only finitely many 1-bridge braids associated with the simultaneous braiding of the dual knot of a given Berge knot whereas there are infinitely many 1-bridge braids representing a given core knot in $O(\frac{p}{q})$ for a fixed slope $\frac{p}{q}$. See [15, Corollary 3.3] or figure 8 perhaps for more instructive view on the Wu's observation.

2. PRELIMINARIES

2.1 The covering knots as freely periodic knots

In this section we briefly recall how to construct the covering knots by means of the spherical geometry of a lens space. Let λ be a $p-$ th root of unity ,i.e., $\lambda = e^{i2\pi/p}$. Then the cyclic group $\mathbb{Z}_p = < \lambda | \lambda^p = 1 >$, acts freely on $S^3 = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 = 1\}$ by

$$\lambda(z_1, z_2) = (\lambda z_1, \lambda^q z_2).$$

Then we have the covering projection $\pi : S^3 \to L = S^3/\mathbb{Z}_P$. Note that S^3 admits a $\mathbb{Z}_p$ - equivariant genus one heegaard splitting $S^3 = W_1 \cup W_2$ where $W_1 = \{(z_1, z_2) \in S^3 : |z_2| \leq |z_1|\}$ and $W_1 = \{(z_1 z_2) \in S^3 : |z_1| \leq |z_2|\}$.
Thus if we take solid tori $V_i = W_i/\mathbb{Z}_p$ for $i = 1,2$, then we have agenus one Heegaard splitting of L and the covering projection respecting heegaard solid tori $\pi : S^3 = W_1 \cup W_2 \to L = V_1 \cup V_2$.
Here $\mathbb{Z}p$ action on W_1 is understood as follows.

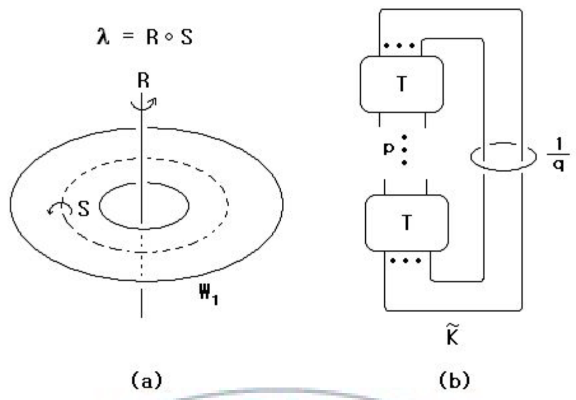


Figure2

In figure 2(a), let R be $2\pi/p$-rotation with respect to the core of the complementary solid torus of W_1 and let S be $2q\pi/p$-rotation with respect to the core of W_1.

Then we can realize λ as composition R ∘ S of the two rotations. Let K be a knot in L. Then without loss of generality we may assume that K may lie in one of the heegaard solid tori, say V_1.

In general the lifting of K ,$\widetilde{K}=\pi^{-1}(K)$ is a link in W_1 whose component is given by $\gcd(p,w)$ where w is the winding number of K in V_1, i.e., the algebraic intersection number of K with a meridian disk of V_1. In particular, if K represents a generator of $H_1(L;\mathbb{Z})$, then $\widetilde{K}$ is indeed a knot. Let ω be the wrapping number of K in V_1, i.e., the minimal geometric intersection number of K with a meridian disk D_1 of V_1 and let (B,T) be a $\omega-$ strand tangle obtained by cutting $(V_{1,}K)$ along D_1. By concatenating $p-$copies of the tangle T and $q-$full twist (lefthand for $q>0$ and righthand for $q<0$) of ω

-strands we have the covering tangle of T

$$\widetilde{T}= T^p(\sigma_{1,} \cdots,\sigma_{\omega-2}\sigma_{\omega-1})^{q\omega}$$

where σ_i are the standard generators of the braid group B_ω of ω -strands. Then we can get $\widetilde{K}$ by closing $\widetilde{T}$ as illustrated in figure 2(b). Thus $\widetilde{K}$ is a freely periodic link of order p. Chbili[3] called it a (p,q)-lens link. Indeed Thurston's geometrization conjecture implies that any free cyclic action of order p is conjugated to the linear $\mathbb{Z}_P$-action described in the above. In turn it implies that any freely periodic link of order p is isotopic to a (p,q)-lens link.

2.2 The dual knots of Berge knots as special (1,1)-knots

In this section we recall some known results and terms for investigation of dual knots of Berge knots. A properly embedded arc t in a solid torus V is said to be trivial if we have an arc t' on ∂V with $\partial t = \partial t'$ and a disk C in V such that $\partial \Delta = t \cup t'$. Such a disk C and arc t' are called a spanning disk and projection of t respectively. Let F be a genus one Heegaard torus of a lens space L. For a knot K in L, we say that K admits a 1-bridge poistion (with respect to F) or it is called a (1,1)-knot in short if (L,K) can be decomposed into a union $(V_{1,}t_1) \cup_F (V_{2,}t_2)$ where t_i is a trivial arc in a solid torus V_i $(i=1,2)$. Given a 1-bridge position of K,choosing meridian disks D_i of V_i so that $D_i \cap t_i = \varnothing$ $(i=1,2)$ and taking a pair of points $\{P,Q\}$ on $F - \partial D_1 \cup \partial D_2$ such that $K \cap F = \partial t_1 = \partial t_2$, we have a triple $(F,\{\partial D_1,\partial \mathrm{D}_2\},\{\mathrm{P},\mathrm{Q}\})$ called a 1-bridge diagram of K. Note that for a meridian disk D of a solid torus V any two trivial arcs in V with the same end points and disjoint from D are relatively isotopic in V. Thus from a given genus one Heegaard diagram $(F,\{\partial D_1,\partial D_2\})$ choosing a pair of points $\{P,Q\}$ on $F-\partial D_1\cup\partial D_2$, we have triple representing a 1-bridge diagram of a (1,1)-knot in L. On the other hand, given a 1-bridge position of K, we may view K as a knot in one of the Heegaard solid tori, say V_1 by taking a projection t'_2 of t_2 on F so that $K=t_1\cup t'_2$. Such a representation of K is called a 1-bridge presentation of K. Now we recall a family of knots in a standard solid torus $D^2\times S^1$ introduced by Gabai[6] which admit rather simple 1-bridge presentations. A braid σ is represented by a set of ω strings in $D^2\times I$. Thus by gluing $D^2\times 0$ to $D^2\times 1$ we have the closure of σ which is a knot or link in the solid torus $D^2\times S^1$. A 1-bridge braid is a knot $K(\omega,b,t)$ in V which is the closure of the braid

$$\sigma=\sigma_b\cdots\sigma_2\sigma_1(\sigma_{\omega-1}\cdots\sigma_2\sigma_1)^t.$$

Call t the twist number and b the bridge width.

REMARK 1.

In [10], Menasco and Zhang utilized the mirror image versions of the 1-bridge braids defined in the above, which are denoted by $K_{MZ}(\omega,b,t)$. For example the two 1-bridge braids $K(5,2,1)$ and $K_{MZ}(5,2,3)$ in figure 3 are isotopic to each other in $D^2\times S^1$.

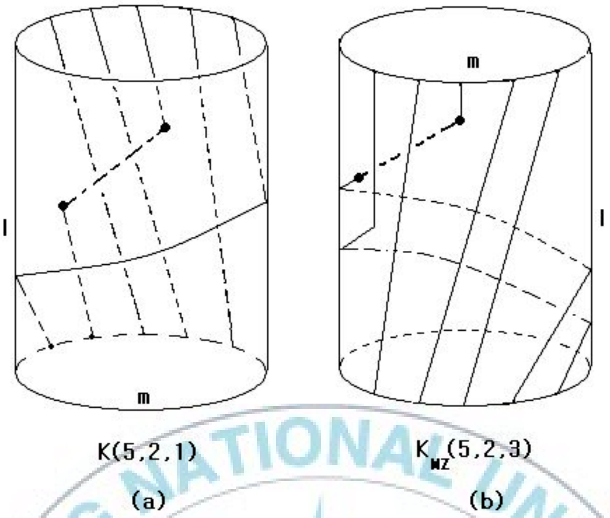


Figure3

Berge showed that the dual knot K in a lens space $L=L(p,q)$ of a doubly primitive knot admits a very pleasant 1-bridge position ([2, Theorem 2]). Namely, (L,K) can be decomposed into $(V_{1,}t_1)\cup_F(V_{2,}t_2)$ so that for each solid torus V_i we may take a meridian disk Δ_i with following properties;

(i) an oriented Heegaard diagram $(F,\{\partial\Delta_{1,}\partial\Delta_2\})$ is normaized; $\partial\Delta_1$ meets $\partial\Delta_2$ at exactly p-points in the same direction up to isotopy in F and

(ii) each trivial arc t_i in V_i lies in Δ_i $(i=1,2)$.

We call a (1,1)-knot or its 1-bridge position with properties (i) and (ii) special.

REMARK 2.

For example of a (1,1)-knot in a lens space which is not special, see [4, Fig.8]. Moreover there are no nontrivial (1,1)-knots in S^3 admitting special 1-bridge positions. Here we reproduce Wu's explanation ([13]) on trivial arcs of a general (1,1)-knot in a lens space;

Let $(V_{1,}t_1)\cup_F(V_{2,}t_2)$ be a 1-bridge position of an arbitrary (1,1)-knot in $L(p,q)$. Since t_1 is trivial in V_1, we have a meridian disk D_1 of V_1 such that $t_1 \subset D_1$. On the other hand, we have another meridian disk D'_1 of V_1 such that $\partial D'_1$ meets ∂D_2 at exactly $p-$ points in the same direction. However, in general one cannot choose D_1 and D'_1 to be the same disk up to isotopy in V_1 relative to t_1.

For the trivial arc t_i of V_i in a special 1-bridge position, we choose its spanning disk C_i lying in the meridian disk Δ_i of V_i $(i=1,2)$. We call $\partial C_i \cap F$ a special projection of t_i, denoted by $\mathrm{Pr}oj(t_i)$. Note that each trivial arc t_i admits two special projection s a union of which constitutes $\partial\Delta_i$.

Then taking a meridian disk D_i of V_i from a small regular neighbourhood of Δ_i in V_i so that $D_i \cap t_i = \varnothing$ $(i=1,2)$, we have a following straightfoward characterization of a 1-bridge diagram of a special (1,1)-knot.

Lemma 3.

Let a triple $(F,\{\partial D_1,\partial D_2\},\{P,Q\})$ be a 1-bridge diagram of an irreducible (1,1)-knot K in a lens space $L=L(p,q), p\geq 2$. Then K is special if and only if the associated oriented Heegaard diagram $(F,\{\partial D_1,\partial D_2\})$ is normalized and P, Q lie in mutually distinct components of $F-\partial D_1\cup\partial D_2$.

For the two special projections of t_1, we consider their intersection numbers with ∂D_2 the minimum of which is called the jumping number of t_1 by adopting the definition 2.1 in [13], denoted by $k=k(P,Q)$. Then position of P and Q in $F-\partial D_1\cup\partial D_2$ can be uniquely described by the jumping number $k=k(P,Q)$ of t_1. Indeed Hempel([8]) has delt with such 1-bridge diagrams under the name of $OBL^+(\frac{q}{p};k)$ knots. By means of the universal abelian covering projection of a torus F $\pi:\mathbb{R}^2\to\mathbb{R}^2/\mathbb{Z}^2\cong F$, he constructed 1-bridge diagrams

$$OBL^+(\frac{q}{p};k)=(F,\left\{\partial D_1=\pi(the\ \ y-axis),\partial D_2=\pi(y=\frac{q}{p}x)\right\},k)$$

where the symbol + indicates that the meridians ∂D_i $(i=1,2)$ are oriented so that they may meet positively at each point. Thus we see that Saito's (1,1)-knot $K(L(p,q);k)$, which is referred to as a monotone (1,1)-knot in [12], is equivalent to $OBL^+(\frac{q}{p};k)$ However it is more convenient to describe 1-bridge diagrams of special (1,1)-

knots by using the standard convention on the genus one Heegaard diagrams of the lens spaces. We identify the solid tori V_i with $D^2 \times S^1$ with the standard orientation and the meridian and longitude pair $\{m_i, l_i\}$ of V_i being taken so that $l \cong x \times S^1$ and $m_i = \partial D_1 \cong \partial D_2 \times y$ for some $(x,y) \in \partial D^2 \times S^1$. Then a meridian m_2 of the complementary solid torus V_2 is represented by a simple closed curve on F corresponding to $p[l_1] + q[m_1] \in H_1(F;\mathbb{Z})$. For example figure 4 shows $OBL^+(\frac{5}{18};7)$.

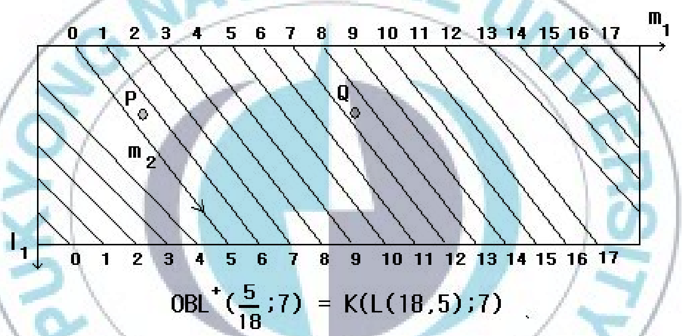


Figure4

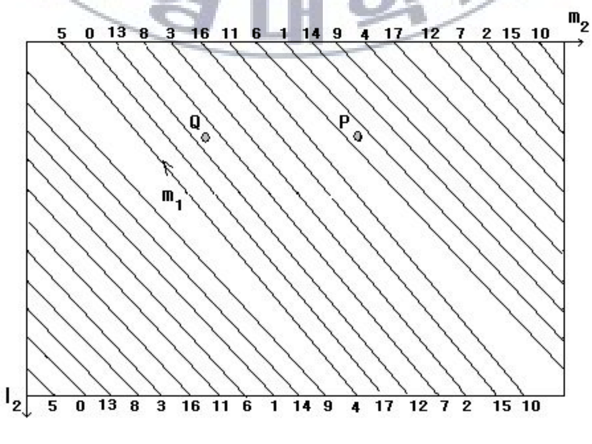


Figure5

Note that there is an orientation preserving self-homeomorphism h of a lens space $L(p,q)$ swapping the side of the Heegaard splitting $V_1 \cup_F V_2$, that is $h(V_1)=V_2$ and $h(V_2)=V_1$. Thus h induces swapping of the meridians m_i of V_i $(i=1,2)$. There are two types of swapping of the oriented meridians. One keeps the given orientations of the meridians, that is $h(m_1)=m_2$ and $h(m_2)=m_1$, which is called the positive swapping. The other reverses the given orientations of both meridians, that is $h(m_1)=-m_2$ and $h(m_2)=-m_1$, which is called the negative swapping. Then for the mulitivative inverse q^{-1} of q in $\mathbb{Z}/p\mathbb{Z}$ we may identify $V_2 \cup_F V_1$ with a Heegaard splitting of $L(p,q^{-1})$ such that m_1 is reprented by a simple closed curve on F corresponding to either $p[l_2]+q^{-1}[m_2]$ for the postive swapping or $-(p[l_2]+q^{-1}[m_2])$ for the negative swapping. Thus for the 1-bridge position $(V_1,t_1)\cup_F(V_2,t_2)$ associated with $OBL^+(\frac{p}{q};k)$ we have a 1-bridge diagram $OBL^+(\frac{q^{-1}}{p};l)$ representing the swapped 1-bridge position $(V_2,t_2)\cup_F(V_1,t_1)$. In figure 5 we have description of the negative swapping in a 1-bridge diagram $OBL^+(\frac{11}{18};5)$.

2.3 The 1-bridge braid presentations of the special (1,1)-knots

For a knot K with a special 1-bridge position $(V_{1,}t_1)\cup_F(V_{2,}t_2)$ defined by $OBL^+(\frac{q}{p};k)$, we try to get its 1-bridge presentations in one of the Heegaard tori of L. From the special projections $\mathrm{Pr}oj(t_i)$ of t_i $(i=1,2)$, we have a closed curve $\mathrm{Pr}oj(t_1)\cup\mathrm{Pr}oj(t_2)$ on F (with self intersection points in general). It is called a special projection diagram of K . A given special projection diagram of K, we have a 1-bridge presentation either $t_1\cup\mathrm{Pr}oj(t_2)$ in V_1 or $t_2\cup\mathrm{Pr}oj(t_1)$ in V_2 which form 0- or 1-bridge braids with respect to the Heegaard torus of the lens space because t_i lies in the meridian disk $\varDelta_i$ of V_i (i=1,2) and $\mathrm{Pr}oj(t_1)$, $\mathrm{Pr}oj(t_2)$ meet $\partial\varDelta_2$, $\partial\varDelta_1$ at each pont in the same direction respectively.Thus we have

Theorem 4.
([2, Remark in p.3]). Any special (1,1)-knot K in a lens space L is simultaneously braided with respect to both of the solid tori of a genus one Heegaard splitting of L.

The special projection diagram of $OBL^+(\frac{q}{p};k)$ is always oriented so that $\mathrm{Pr}oj(t_2)$ may meet $m_1=\partial D_1$ postively at each point which assures that a 1-bridge braid presentation $t_1\cup\mathrm{Pr}oj(t_2)$ in V_1 has a positive winding number. Figure 6 shows that four possible choices of special projection diagrams of $OBL^{+}(\frac{5}{18};7)$

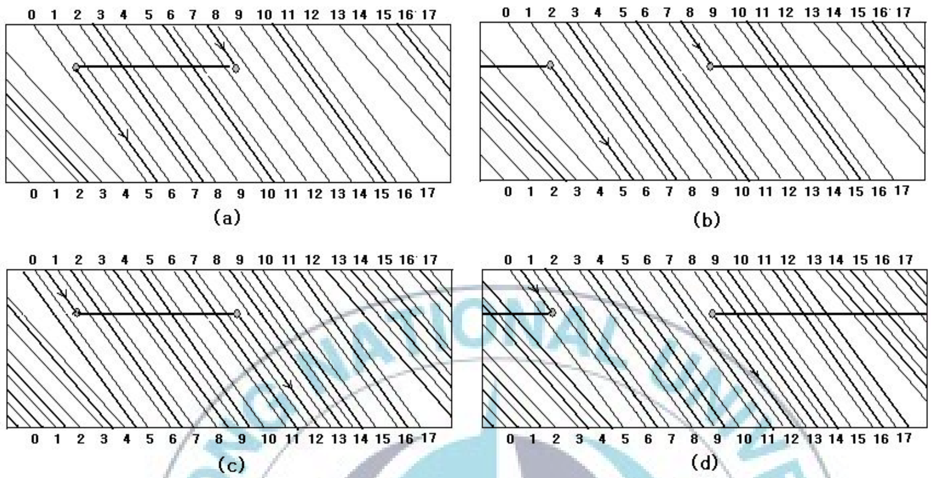


Figure6

We immediately notice that figure 6(a) and 6(d) respectively yield $K(5,2,1)$ and $K(13,8,3)$ for 1-bridge presentations $t_1 \cup \mathrm{Pr}oj(t_2)$ (cf. Figure 3). On the other hand we see that the 1-bridge braids in-duced by figure (b) and (c) are isotopic to $K(5,2,1)$ and $K(13,8,3)$ respectively. Considering the negatively swapped 1-bridge diagram of $OBL^{+}(\frac{5}{18};7)$ we can obtain 1-bridge braids corresponding to $t_2 \cup \mathrm{Pr}oj(t_1)$. Here we take the negative swapping which assures that the oriented special projection diagrams carried by the swapping yield 1-bridge braids with positive winding numbers.

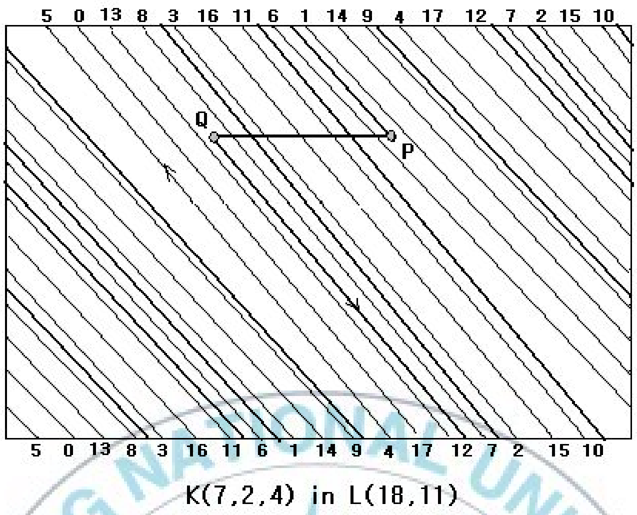


Figure7

In figure 7 we have the special projection diagram of Figure 6(a) in the swapped 1-bridge diagram of $OBL^{+}(\frac{5}{18};7)$ which yields $K(7,2,4)$ for a 1-bridge presentation $t_2 \cup \mathrm{Pr}oj(t_1)$. Considering the special projection diagrams of Figure 6(b), 6(c) and 6(d) in the swapped 1-bridge diagram, we have 1-bridge braids $K(11,8,6)$, $K(7,2,4)$ and $K(11,8,6)$ respectively.

For a genus one Heegaard splitting $V_1 \cup_F V_2$ of a lens space $L(p,q)$, let C_i be the core of V_i $(i=1,2)$ and take $g=[C_2]$ as a generator of $\pi_1(L(p,q))$, i.e., $\pi_1(L(p,q)) = < g | g^p >$. Then we have

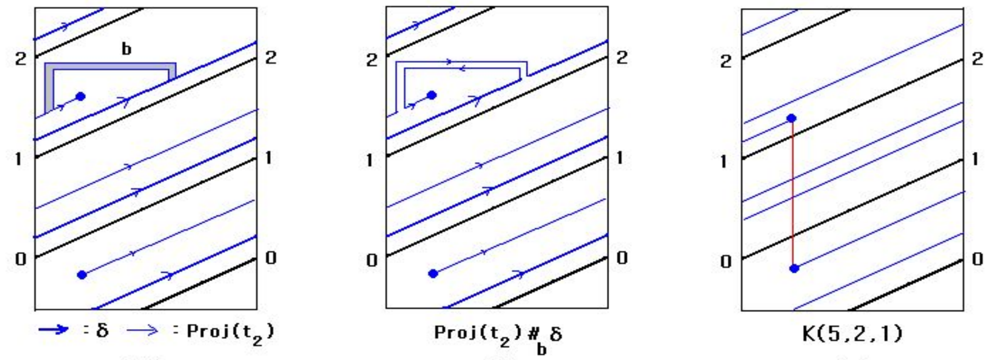
Lemma 5.

For a special (1,1)-knot K with a 1-bridge diagram $OBL^{+}(\frac{q}{p};k)$, k represents the order of $[K]$ in $\pi_1(L(p,q)) = < g|g^p >$, i.e., $[K] = g^k$.

Proof. Note that K admits a 1-bridge braid presentation $t_2 \cup \mathrm{Pr}oj(t_1)$ in V_2 such that $|\mathrm{Pr}oj(t_1) \cap \partial D_2| = k$. Thus $[K] = g^k$ because k represents the winding number of K in V_2.

Although 1-bridge braids arising from a special projection diagram serve our purpose of constructing the covering knot of a special (1,1)-knot K, it is pointed out that we have more projections of t_2 on F which induce 1-bridge braid presentations of K other than $t_1 \cup \mathrm{Pr}oj(t_2)$. Take a parallel copy δ of ∂D_2 on F so that it may be disjoint from $\mathrm{Pr}oj(t_2)$. Further we assume that both $\mathrm{Pr}oj(t_2)$ and δ meet ∂D_1 at at each point in the same direction. Then consider components of $\mathrm{Pr}oj(t_2)$ and δ lying in $p-$ rectangular blocks in $F - \partial D_1 \cup \partial D_2$. Note that each bock contains at most one component of $\mathrm{Pr}oj(t_2)$ and one component of δ.

Choosing a rectangular block which contains a component of $\mathrm{Pr}oj(t_2)$ with one of its two endpoints and taking a band sum of $\mathrm{Pr}oj(t_2)$ and δ along b for a band b in this block , we have another projection $t'_2 = \mathrm{Pr}oj(t_2) \#_b \delta$ of t_2 disjoint from ∂D_2. Then a newly obtained 1-bridge presentation $t_1 \cup t'_2$ of K in V_1 constitutes a 1-bridge braid (eventhough $t_1 \cup \mathrm{Pr}oj(t_2)$ is a 0-bridge braid). But it cannot lie in the boundary of any meridian disk of V_2 because it meets ∂D_1 at more than $p-$ times in the same direction.



(a) (b) (c)

Figure8

Here we illustrate the above idea with an example in figure 8 which shows that a 1-bridge braid $K(5,2,1)$ represents a core knot in $L(3,1)$. For a 1-bridge diagram $OBL^{+}(\frac{1}{3};1)$ which represents a core knot in $L(3,1)$, we have a special projection $\mathrm{Pr}oj(t_2)$ of t_2 which induces a 0-bridge braid $K(2,1,1)$ as a 1-bridge presentation in V_1. Then taking band sum of $\mathrm{Pr}oj(t_2)$ and δ, a parallel copy of $m_2 = \partial D_2$, we have a 1-bridge braid $K(5,2,1)$. Taking one more band sum of another parallel copy δ' of m_2 and the projection of t_2 in figure 8(c), we have a 1-bridge braid $K(8,3,2)$ representing the same core knot ([13, TABLE 1]). Thus iterating the above band sum operation with arbitray many parallel copies of m_2, we can have infinitely many 1-bridge braids occurring as 1-bridge representation of the given core knot or special (1,1)-knot in general.

REMARK 6.

We take this opportunity to briefly review on Wu's work [15] for its comparison with ours. The first part of his work(Theorem 2.2 and Corollary 2.6) may be thought of as identification of the core knots among (1,1)-knots in lens spaces.It was also delt by Hempel([8, Lemma 6.2]). Indeed he identified all core knots among special (1,1)-knots;

A special (1,1)-knot $K = OBL^{+}(\frac{q}{p};k)$ is a core knot in $L(p,q)$ if and only if the jumping number k is either 1 or $\min\{q,p-q\}$.

And the second part of his work, which guides us to understand relationship between 1-bridge braids and dual knots of Berge knots, amounts to identifying 1-bridge braids which occur as 1-bridge presentations of the core knots in lens spaces.

3. construction of the 18-fold covering knot for $p(-2,3,7)$

In [11, EXAMPLE 5.2], Saito showed that a special (1,1)-knot $K((18,5);7)$ is the dual knot of $p(-2,3,7)$. Using this fact, we derive the covering knot for $p(-2,3,7)$.

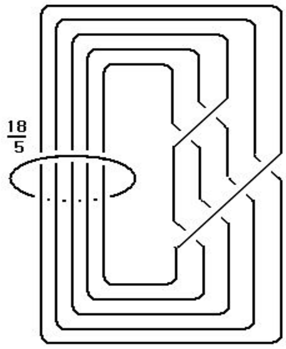


Figure9

With a view of the dual knot in figure 9 where we identify the Heegaard solid torus V_1 of $L(18,5)$ with the exterior of the unknot O, we can confirm that it admits a S^3- surgery slope. Indeed by taking a surgery slope 7 for $K(5,2,1)$ and observing that $O\cup K(5,2,1)$ is a pretzel link $p(-2,3,8)$ which is also known as the Whitehead sister link, we see that $O\cup K(5,2,1)(\frac{18}{5},7)=S^3$ through the Montesinos trick as shown in figure 10. Perhaps for more elegant replacement of the above geometric argument, note that 7 is a $\partial-$reducing slope of $K(5,2,1)$ in $D^2\times S^1$. And a new solid torus has a meridian disk D with ∂D represented by a slope $\frac{25}{7}$ in $\partial E(O)(\cong\partial D^2\times S^1)$, the boundary torus of the exteror of the unknot O ([9, Lemma 3.3(ii)]). Thus by taking slopes $\frac{p}{q}$ in $\partial E(O)$ so that

$$\Delta(\frac{p}{q},\frac{25}{7})=|\det\begin{pmatrix}p & 25\\ q & 7\end{pmatrix}|=1\ ;\ \frac{p}{q}=\frac{25k+7}{7k+2},k\in Z$$

we have infinitely many dual knots of Berge knots which admits $K(5,2,1)$ as 1-bridge presentations including the one in figure 9 (cf. Table with the dual knots K represented by 5 in [2]) . Finally with a view of $K(5,2,1)$ in V_1, we have the desired covering knot by concatenating 18-copies of its braided part with 5 lefthand full twists (of 5-strands). Note that a full twisting is commutable with any braiding. Thus by concatenating 3-copies of the braided part of $K(5,2,1)$ with a single lefthand full twist we have figure 11(a) which is simplified to a braid in figure 11(b). By concatenating 5-copies of the braid in figure (b) with the remaining 3-copies of the braided part of $K(5,2,1)$, we get a braid in figure 11(c) closing of which yields the desired covering knot in figure 11(d). The knot diagram in figure 11(d) is managed to be inputed in SnapPea[14] which shows that the covering knot has the volume 50.90619759, 18 times the volume of $p(-2,3,7)$ and its symmetry group is D_{18}.

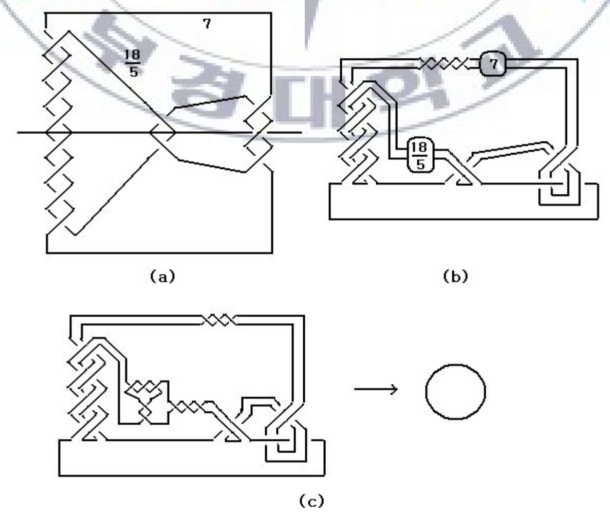


Figure10

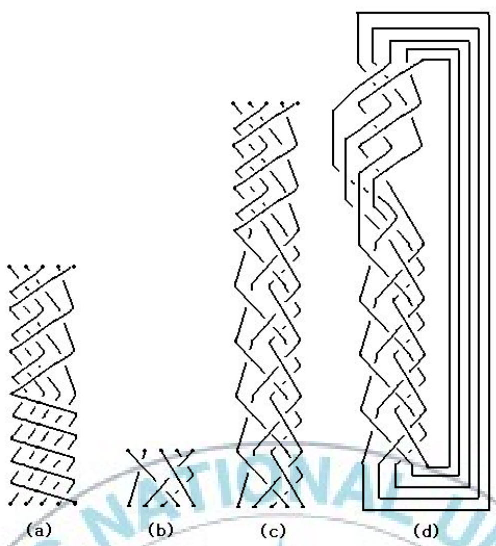


Figure11

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