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Generalized Intuitionistic Fuzzy Topological Spaces

by

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Generalized Intuitionistic Fuzzy Topological Spaces
(일반화된 직관적 퍼지 위상 공간)

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by

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요 약

본 논문에서는 Mondal과 Samanta „ 의해 소개된 일반화된 직관적 퍼지 집합의 연산을 기초로 하여 Coker에 의해 소개된 직관적 퍼지 위상 공간의 확장인 일반화된 직관적 퍼지 위상 공간을 연구한 것으로써 일반 위상 공간에서의 중요한 개념인 연속성, 피복성 및 연결성을 이들 공간으로 확장하였고 특히 일반화된 직관적 퍼지 피복성과 일반화된 직관적 퍼지 C_5−연결성에 대한 보존정리를 얻었다.

1 Introduction

Since Zadeh [24] introduced fuzzy sets, many scholars introduced various notions of higher-order fuzzy sets. Among them, interval-valued fuzzy sets (IVFSs) [23, 25] provides us with a flexible mathematical framework to deal with imperfect and/or imprecise information. Deng [14] called IVFSs as grey sets. Atanassov [3, 5] introduced the notion of intuitionistic fuzzy sets (IFSs), as a generalization of fuzzy sets. Atanassov and Gargov [6] showed that IFSs and IVFSs are equipollent generalizations of fuzzy sets. The equivalence between the structures of IFSs and IVFSs was proved by Deschrijver and Kerre [15]. In 1993, Gau and Buehrer [17] proposed the notion of vague sets. Bustince and Burillo [10] proved that the notion of vague sets identifies with the one of IFSs. The idea of "intuitionistic fuzzy set" was first published by Atanassov [2] and many works by the same author and his colleagues appeared in the literature [3, 4, 7]. Later, this concept was generalized to "intuitionistic L-fuzzy sets" by Atanassov and Stoeva [8]. Çoker [13] introduced the concept of "intuitionistic fuzzy topological spaces" investigated its basic properties. Mondal and Samanta [19] introduced the notion of generalized intuitionistic fuzzy sets (GIFSs) as a generalization of Atanassov's IFSs and studied their basic properties.

The purpose of this thesis is to construct the basic concepts of the so-called "general intuitionistic fuzzy topological spaces". After giving the fundamental definitions and the necessary examples we introduce the definitions of fuzzy continuity, fuzzy compactness and fuzzy connectedness, and obtain several preservation properties and some characterizations concerning fuzzy compactness and fuzzy connectedness.

2 Preliminaries

First we shall present the fundamental definitions given by Mondal and Samanta [19]:

Definition 2.1 (Mondal and Samanta [19]) Let X be a nonempty fixed set. A generalized intuitionistic fuzzy set (GIFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $\min\{\mu_A(x), \gamma_A(x)\} \leq \frac{1}{2}$ for each $x \in X$.

Remark 2.2 A generalized intuitionistic fuzzy set$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ in X can be identified to an ordered pair $\langle \mu_A, \gamma_A\rangle$ in $I^X \times I^X$ or to an element in $(I \times I)^X$.

Remark 2.3 For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A\rangle$ for the GIFS $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Remark 2.4 The concept of triangular norms (t-norms) is known as the axiomatic skeletons that we use for characterizing fuzzy intersection. Several examples for this concept is follows:

- drastic product: $t_w(a,b) = \begin{cases} \min\{a,b\}, & \max\{a,b\} = 1 \\ 0, & \text{otherwise} \end{cases}$,
- bounded difference: $t_1(a,b) = \max\{0, a+b-1\}$,
- Eistein product: $t_{1.5}(a,b) = \dfrac{ab}{2-(a+b-ab)}$,
- algebraic product: $t_2(a,b) = ab$,
- Hamacher product: $t_{2.5}(a,b) = \dfrac{ab}{a+b-ab}$,

- minimum: $t_3(a,b) = \min\{a,b\}$.

Since these operators are ordered as follows:

$$t_w \leq t_1 \leq t_{1.5} \leq t_2 \leq t_{2.5} \leq t_3,$$

it is possible to change the operator min(i.e., t_3) in above definition by another t-norm.

Example 2.5 Every intuitionistic fuzzy set A on a nonempty set X is obviously a GIFS having the form $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$ [4].

One can define several relations and operations between GIFSs as follow:

Definition 2.6 (Mondal and Samanta [19]) Let X be a nonempty set, and the GIFSs A and B be in the forms $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}, B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;
(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(c) $A^c = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;
(f) $[\ ]A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;
(g) $\langle\ \rangle A = \{\langle x, 1 - \gamma_A(x), \gamma_A(x)\rangle : x \in X\}$.

We can easily generalize the operations of intersection and union in above definition to arbitrary family of GIFSs as follows:

Definition 2.7 Let $\{A_i : i \in J\}$ be an arbitrary family of GIFSs in X. Then

(a) $\bigcap A_i = \{\langle x, \bigwedge \mu_{A_i}(x), \bigvee \gamma_{A_i}(x)\rangle : x \in X\}$;
(b) $\bigcup A_i = \{\langle x, \bigvee \mu_{A_i}(x), \bigwedge \gamma_{A_i}(x)\rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing generalized intuitionistic fuzzy topological spaces, we must introduce the GIFSs $0_\sim$ and $1_\sim$ in X as follows:

Definition 2.8 $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$.

Here are the basic properties of inclusion and complementation:

Corollary 2.9 *Let A, B, C be GIFSs in X. Then*

(a) $A \subseteq B$ *and* $C \subseteq D \Rightarrow A \cup C \subseteq B \cup D$ *and* $A \cap C \subseteq B \cap D$,

(b) $A \subseteq B$ *and* $A \subseteq C \Rightarrow A \subseteq B \cap C$,

(c) $A \subseteq C$ *and* $B \subseteq C \Rightarrow A \cup B \subseteq C$,

(d) $A \subseteq B$ *and* $B \subseteq C \Rightarrow A \subseteq C$,

(e) $(A \cup B)^c = A^c \cap B^c$,

(f) $(A \cap B)^c = A^c \cup B^c$,

(g) $A \subseteq B \Rightarrow B^c \subseteq A^c$,

(h) $(A^c)^c = A$,

(i) $1_\sim^c = 0_\sim$,(j) $0_\sim^c = 1_\sim$.

Proof We shall only prove (e).

(e) Let $A = \langle x, \mu_A, \gamma_A\rangle$ and $B = \langle x, \mu_B, \gamma_B\rangle$. Then we have

$$A \cup B = \langle x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B\rangle, \; A^c = \langle x, \gamma_A, \mu_A\rangle, \; B^c = \langle x, \gamma_B, \mu_B\rangle$$

Thus,

$$(A \cup B)^c = \langle x, \gamma_A \wedge \gamma_B, \mu_A \vee \mu_B\rangle = A^c \cap B^c.$$

Now we shall define the image and preimage of GIFSs. Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

Definition 2.10 (a) If $B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is a GIFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the GIFS in X defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}.$$

(b) If $A = \{\langle x, \lambda_A(x), \vartheta_A(x)\rangle : x \in X\}$ is a GIFS in X, then the image of A under f, denoted by $f(A)$, is the GIFS in Y defined by

$$f(A) = \{\langle y, f(\lambda_A)(y), (1 - f(1 - \vartheta_A))(y)\rangle : y \in Y\}$$

where

$$f(\lambda_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \lambda_A(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$(1 - f(1 - \vartheta_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \vartheta_A(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

For the sake of simplicity, let us use the symbol $f_-(\vartheta_A)$ for $1 - f(1 - \vartheta_A)$. Now we list the properties of images and preimages, some of which we shall frequently use in Sections 4-6.

Corollary 2.11 *Let A, A_i $(i \in J)$ be GIFSs in X, B, B_j $(j \in K)$ be GIFSs in Y and $f : X \to Y$ a function. Then*

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,
(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,
(c) $A \subseteq f^{-1}(f(A))$ [*If f is injective, then* $A = f^{-1}(f(A))$],
(d) $f(f^{-1}(B)) \subseteq B$ [*If f is surjective, then* $f(f^{-1}(B)) = B$],
(e) $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$,
(f) $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$,
(g) $f(\bigcup A_i) = \bigcup f(A_i)$,
(h) $f(\bigcap A_i) \subseteq \bigcap f(A_i)$ [*If f is injective, then* $f(\bigcap A_i) = \bigcap f(A_i)$],
(i) $f^{-1}(1_\sim) = 1_\sim$, (j) $f^{-1}(0_\sim) = 0_\sim$,

(k) $f(1_\sim) = 1_\sim$, *if* f *is surjective,*

(l) $f(0_\sim) = 0_\sim$,

(m) $f(A)^c \subseteq f(A^c)$, *if* f *is surjective,*

(n) $f^{-1}(B^c) = f^{-1}(B)^c$,

Proof Let $B_j = \langle y, \mu_{B_j}, \gamma_{B_j}\rangle$, $A_i = \langle x, \lambda_{A_i}, \vartheta_{A_i}\rangle$ $(i \in J, j \in K)$, $B = \langle y, \mu_B, \gamma_B\rangle$ and $A = \langle x, \lambda_A, \vartheta_A\rangle$.

(a) Let $A_1 \subseteq A_2$. Since $\lambda_{A_1} \leq \lambda_{A_2}$ and $\vartheta_{A_1} \geq \vartheta_{A_2}$, we obtain $f(\lambda_{A_1}) \leq f(\lambda_{A_2})$ and

$$\begin{aligned} 1 - \vartheta_{A_1} \leq 1 - \vartheta_{A_2} &\Rightarrow f(1 - \vartheta_{A_1}) \leq f(1 - \vartheta_{A_2}) \\ &\Rightarrow 1 - f(1 - \vartheta_{A_1}) \geq 1 - f(1 - \vartheta_{A_2}) \\ &\Rightarrow f_-(\vartheta_{A_1}) \geq f_-(\vartheta_{A_2}) \end{aligned}$$

from which we easily obtain the result $f(A_1) \subseteq f(A_2)$.

(b) This is similar to (a).

(c)

$$\begin{aligned} f^{-1}(f(A)) = f^{-1}(f(\langle x, \lambda_A, \vartheta_A\rangle) &= f^{-1}(\langle y, f(\lambda_A), f_-(\vartheta_A)\rangle) \\ &= \langle x, f^{-1}(f(\lambda_A)), f^{-1}(f_-(\vartheta_A))\rangle \\ &\supseteq \langle x, \lambda_A, \vartheta_A\rangle = A. \end{aligned}$$

[Notice that $f^{-1}(f(\lambda_A)) \geq \lambda_A$ and $f^{-1}(f_-(\vartheta_A)) = f^{-1}(1 - f(1 - \vartheta_A)) = 1 - f^{-1}(f(1 - \vartheta_A)) \leq 1 - (1 - \vartheta_A) = \vartheta_A$.]

(d)

$$\begin{aligned} f(f^{-1}(B)) = f(f^{-1}(\langle y, \mu_B, \gamma_B\rangle)) &= f(\langle x, f^{-1}(\mu_B), f^{-1}(\gamma_B)\rangle) \\ &= \langle y, f(f^{-1}(\mu_B)), f_-(f^{-1}(\gamma_B))\rangle \\ &\subseteq \langle y, \mu_B, \gamma_B\rangle = B. \end{aligned}$$

[Notice that $f(f^{-1}(\mu_B)) \leq \mu_B$. On the other hand, we have $f_-(f^{-1}(\gamma_B)) = 1 - f(1 - f^{-1}(\gamma_B)) = 1 - f(f^{-1}(1 - \gamma_B)) \geq 1 - (1 - \gamma_B) = \gamma_B$, i.e., $f_-(f^{-1}(\gamma_B)) \geq \gamma_B$.]

(e)

$$\begin{aligned} f^{-1}(\bigcup B_j) &= f^{-1}(\langle y, \bigvee \mu_{B_j}, \bigwedge \gamma_{B_j}\rangle) = \langle x, f^{-1}(\bigvee \mu_{B_j}), f^{-1}(\bigwedge \gamma_{B_j})\rangle \\ &= \langle x, \bigvee f^{-1}(\mu_{B_j}), \bigwedge f^{-1}(\gamma_{B_j})\rangle = \langle x, \bigvee \mu_{f^{-1}(B_j)}, \bigwedge \gamma_{f^{-1}(B_j)}\rangle \\ &= \bigcup f^{-1}(B_j). \end{aligned}$$

(f) It is similar to (e).

(g)

$$\begin{aligned} f(\bigcup A_i) &= f(\langle x, \bigvee \lambda_{A_i}, \bigwedge \vartheta_{A_i}\rangle) = \langle y, f(\bigvee \lambda_{A_i}), f_-(\bigwedge \vartheta_{A_i})\rangle \\ &= \langle y, \bigvee f(\lambda_{A_i}), \bigwedge f_-(\vartheta_{A_i})\rangle = \bigcup \langle y, f(\lambda_{A_i}), f_-(\vartheta_{A_i})\rangle = \bigcup f(A_i). \end{aligned}$$

[Notice that $f(\bigvee \lambda_{A_i}) = \bigvee f(\lambda_{A_i})$ and $f_-(\bigwedge \vartheta_{A_i}) = 1 - f(1 - \bigwedge \vartheta_{A_i}) = 1 - f(\bigvee(1 - \vartheta_{A_i})) = 1 - \bigvee f(1 - \vartheta_{A_i}) = \bigwedge(1 - f(1 - \vartheta_{A_i})) = \bigwedge f_-(\vartheta_{A_i})$.]

(h)

$$\begin{aligned} f(\bigcap A_i) &= f(\langle x, \bigwedge \lambda_{A_i}, \bigvee \vartheta_{A_i}\rangle) = \langle y, f(\bigwedge \lambda_{A_i}), f_-(\bigvee \vartheta_{A_i})\rangle \\ &\subseteq \langle y, \bigwedge f(\lambda_{A_i}), \bigvee f_-(\vartheta_{A_i})\rangle = \langle y, \bigwedge f(\lambda_{A_i}), \bigvee f_-(\vartheta_{A_i})\rangle = \bigcap f(A_i). \end{aligned}$$

[Notice that $f(\bigwedge \lambda_{A_i}) \leq \bigwedge f(\lambda_{A_i})$ and $f_-(\bigvee \vartheta_{A_i}) = 1 - f(1 - \bigvee \vartheta_{A_i}) = 1 - f(\bigwedge(1 - \vartheta_{A_i})) \geq 1 - \bigwedge f(1 - \vartheta_{A_i}) = \bigvee(1 - f(1 - \vartheta_{A_i})) = \bigvee f_-(\vartheta_{A_i})$.]

(i) $f^{-1}(1_\sim) = f^{-1}(\langle y, 1, 0\rangle) = \langle x, f^{-1}(1), f^{-1}(0)\rangle = \langle x, 1, 0\rangle = 1_\sim$.

(j),(k),(l) are similar to (i).

(m) Since $f(A^c) = f(\langle x, \lambda_A, \vartheta_A\rangle^c) = f(\langle x, \vartheta_A, \lambda_A\rangle) = \langle y, f(\vartheta_A), f_-(\lambda_A)\rangle$ and $f(A)^c = f(\langle x, \lambda_A, \vartheta_A\rangle)^c = \langle y, f(\lambda_A), f_-(\vartheta_A)\rangle^c = \langle y, f_-(\vartheta_A), f(\lambda_A)\rangle$, we obtain the required result immediately from the fact that f is surjective.

(n) It is similar to (m). □

3 Generalized intuitionistic fuzzy topological spaces

Here we generalize the concept of intuitionistic fuzzy topological space, first initiated by Çoker [13], to the case of generalized intuitionistic fuzzy sets.

Definition 3.1 A generalized intuitionistic fuzzy topology (GIFT for short) on a nonempty set X is a family τ of GIFSs in X satisfying the following axioms:

(T_1) $0_\sim,\ 1_\sim \in \tau$;

(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(T_3) $\bigcup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a generalized intuitionistic fuzzy topological space (GIFTS for short) and any GIFS in τ is known as a generalized intuitionistic fuzzy open set (GIFOS for short) in X.

Example 3.2 Any fuzzy topological space (X, τ_0) in the sense of Chang is obviously a GIFTS in the form $\tau = \{A : \mu_A \in \tau_0\}$ whenever we identify a fuzzy set in X whose membership function is μ_A with its counterpart $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$ as in Example 2.5.

Example 3.3 Let $X = \{a, b, c\}$ and

$$A = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.6}, \frac{b}{0.7}, \frac{c}{0.8}\right)\right\rangle,$$

$$B = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.7}, \frac{b}{0.5}, \frac{c}{0.5}\right)\right\rangle,$$

$$C = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.8}\right)\right\rangle,$$

$$D = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.5}\right)\right\rangle.$$

Then the family $\tau = \{0_\sim, 1_\sim, A, B, C, D\}$ of GIFSs in X is a GIFT on X.

Example 3.4 Let $X = \{1, 2\}$ and define the GIFSs G_n as follows ($n \in \mathbf{N}$):

$$G_n = \left\langle x, \left(\frac{1}{\frac{n}{n+1}}, \frac{2}{\frac{n+1}{n+2}}\right), \left(\frac{1}{\frac{2}{n+2}}, \frac{2}{\frac{2}{n+3}}\right)\right\rangle.$$

In this case the family $\tau = \{0_\sim, 1_\sim\} \cup \{G_n : n \in \mathbf{N}\}$ is a GIFT on X.

Example 3.5 Let (X, τ_0) be a fuzzy topological space in Chang's sense such that τ_0 is not indiscrete. Suppose now that $\tau_0 = \{0, 1\} \cup \{\nu_i : i \in J\}$. Then we can construct two GIFTs on X as follows:

(a) $\tau^1 = \{0_\sim, 1_\sim\} \cup \{\langle x, \nu_i, 0\rangle : i \in J\}$,

(b) $\tau^2 = \{0_\sim, 1_\sim\} \cup \{\langle x, 0, 1 - \nu_i\rangle : i \in J\}$.

Proposition 3.6 *Let (X, τ) be a GIFTs on X. Then, we can also construct several GIFTSs on X in the following way:*

(a) $\tau_{0,1} = \{[\ ]G : G \in \tau\}$,

(b) $\tau_{0,2} = \{\langle\ \rangle G : G \in \tau\}$.

Proof (a) (T_1) and (T_2) are easy.

(T_3) Let $\{[\ ]G_i : i \in J, G_i \in \tau\} \subseteq \tau_{0,1}$. Since $\bigcup G_i = \langle x, \bigvee \mu_{G_i}, \bigwedge \gamma_{G_i}\rangle \in \tau$, we have

$$\begin{aligned}\bigcup([\ ]G_i) &= \left\langle x, \bigvee \mu_{G_i}, \bigwedge(1 - \mu_{G_i})\right\rangle \\ &= \left\langle x, \bigvee \mu_{G_i}, 1 - \bigvee \mu_{G_i}\right\rangle \in \tau_{0,1}\end{aligned}$$

(b) This is similar to (a). □

Remark 3.7 Let (X, τ) be a GIFTS.

(a) $\tau_1 = \{\mu_G : G \in \tau\}$ is a fuzzy topological space on X in Chang's sense.

(b) $\tau_2{}^* = \{\gamma_G : G \in \tau\}$ is the familly of all fuzzy closed sets of the fuzzy topological space $\tau_2 = \{1 - \gamma_G : G \in \tau\}$ on X in Chang's sense.

(c) Since $\min\{\mu_A(x), \gamma_A(x)\} \le \frac{1}{2}$ for each $x \in X$ and for each $G \in \tau$, we obtain $\mu_A(x) \le \frac{1}{2}$ or $\gamma_A(x) \le \frac{1}{2}$.

(d) Using (a) and (b) we may conclude that (X, τ_1, τ_2) is a bifuzzy topological space.

Definition 3.8 Let (X, τ_1), (X, τ_2) be two GIFTSs on X. Then τ_1 is said to be contained in τ_2 (in symbols, $\tau_1 \subseteq \tau_2$) if $G \in \tau_2$ for each $G \in \tau_1$. In this case, we also say that τ_1 is coarser than τ_2.

Proposition 3.9 *Let $\{\tau_i : i \in J\}$ be a family of GIFTs on X. Then $\bigcap \tau_i$ is a GIFT on X. Furthermore, $\bigcap \tau_i$ is the coarsest GIFT on X containing all τ_i's.*

Proof Obvious. $\square$

A GIFTS (X, τ) is, of course, in the sense of Chang. Now we can obtain the definition of a GIFTS in the sense of Lowen [18] in a natural way with one exception:

Definition 3.10 A generalized intuitionistic fuzzy topological space in the sense of Lowen is a pair (X, τ) where (X, τ) is a GIFTS and each GIFS in the form $C_{\alpha,\beta} = \{\langle x, \alpha, \beta\rangle : x \in X\}$, where $\alpha, \beta \in I$ are arbitrary and $\min\{\alpha, \beta\} \leq \frac{1}{2}$, belongs to τ.

Example 3.11 If (X, τ) is a GIFTS in the sense of Lowen, then (X, τ_1) and (X, τ_2) (see Remark 3.7) are also fuzzy topological spaces in the sense of Lowen.

Definition 3.12 The complement A^c of a GIFOS A in a GIFTS (X, τ) is called a generalized intuitionistic fuzzy closed set (GIFCS for short) in X.

Now we define fuzzy closure and interior operations in a GIFTS:

Definition 3.13 Let (X, τ) be a GIFTS and $A = \langle x, \mu_A, \gamma_A\rangle$ be a GIFS in X. Then the fuzzy interior and fuzzy closure of A are defined by

$$cl(A) = \bigcap\{K : K \text{ is a GIFCS in } X \text{ and } A \subseteq K\},$$
$$int(A) = \bigcup\{G : G \text{ is a GIFOS in } X \text{ and } G \subseteq A\}.$$

It can be also shown that $cl(A)$ is a GIFCS and $int(A)$ is a GIFOS in X, and
(a) A is a GIFCS in X iff $cl(A) = A$;
(b) A is a GIFOS in X iff $int(A) = A$.

Example 3.14 Consider the GIFTS (X, τ) in Example 3.3. If

$$F = \left\langle x, \left(\frac{a}{0.55}, \frac{b}{0.55}, \frac{c}{0.45}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.3}\right)\right\rangle,$$

then

$$int(F) = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.6}, \frac{b}{0.7}, \frac{c}{0.8}\right)\right\rangle, \ cl(F) = 1_\sim$$

Proposition 3.15 *For any GIFS A in* (X, τ) *we have*

(a) $cl(A^c) = int(A)^c$,

(b) $int(A^c) = cl(A)^c$.

Proof (a) Let $A = \langle x, \mu_A, \gamma_A\rangle$ and suppose that the family of GIFOS's contained in A are indexed by the family $\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\}$. Then we see that $int(A) = \langle x, \bigvee \mu_{G_i}, \bigwedge \gamma_{G_i}\rangle$ and hence $int(A)^c = \langle x, \bigwedge \gamma_{G_i}, \bigvee \mu_{G_i}\rangle$. Since $A^c = \langle x, \gamma_A, \mu_A\rangle$ and $\mu_{G_i} \le \mu_A$, $\gamma_{G_i} \ge \gamma_A$ for each $i \in J$, we obtain that $\{\langle x, \gamma_{G_i}, \mu_{G_i}\rangle : i \in J\}$ is the family of GIFCS's containing A^c, i.e. $cl(A^c) = \langle x, \bigwedge \gamma_{G_i}, \bigvee \mu_{G_i}\rangle$. Hence $cl(A^c) = int(A)^c$ follows immediately.

(b) This is analogous to (a). □

Proposition 3.16 *Let* (X, τ) *be a GIFTS and A, B be GIFSs in X. Then the following properties hold:*

(a) $int(A) \subseteq A$, (a′) $A \subseteq cl(A)$,

(b) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$, (b′) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$,

(c) $int(int(A)) = int(A)$, (c′) $cl(cl(A)) = cl(A)$,

(d) $int(A \cap B) = int(A) \cap int(B)$, (d′) $cl(A \cup B) = cl(A) \cup cl(B)$,

(e) $int(1_\sim) = 1_\sim$, (e′) $cl(0_\sim) = 0_\sim$.

Proof (a),(b) and (e) are obvious.

(c) It follows from (a) and Definition 3.13.

(d) From $int(A \cap B) \subseteq int(A)$ and $int(A \cap B) \subseteq int(B)$ we obtain $int(A \cap B) \subseteq int(A) \cap int(B)$ by Corollary 2.9. On the other hand, from the facts $int(A) \subseteq A$

and $int(B) \subseteq B \Rightarrow int(A) \cap int(B) \subseteq A \cap B$ and $int(A) \cap int(B) \in \tau$ we see that $int(A) \cap int(B) \subseteq int(A \cap B)$, from which we obtain the required result.

(a$'$)-(e$'$) They can be easily deduced from (a)-(e), Proposition 3.15 and Corollary 2.9. $\square$

Proposition 3.17 *Let (X, τ) be a GIFTS. If $A = \langle x, \mu_A, \gamma_A \rangle$ is a GIFS in X, then we have*

(a) $int(A) \subseteq \langle x, int_{\tau_1}(\mu_A), cl_{\tau_2}(\gamma_A) \rangle \subseteq A$

(b) $A \subseteq \langle x, cl_{\tau_2}(\mu_A), int_{\tau_1}(\gamma_A) \rangle \subseteq cl(A)$,

where τ_1 and τ_2 are the fuzzy topological spaces on X defined in Remark 3.7.

Proof Let $A = \langle x, \mu_A, \gamma_A \rangle$ and suppose that the family of GIFOSs contained in A are indexed by the family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle : i \in J\}$. Then $int(A) = \langle x, \bigvee \mu_{G_i}, \bigwedge \gamma_{G_i} \rangle$. Each member of the family of fuzzy open sets $\{\mu_{G_i} : i \in J\}$ in τ_1 is contained in μ_A, and hence $\bigvee\{\mu_{G_i} : i \in J\} \leq int_{\tau_1}(\mu_A)$.

Similarly we see that $\bigwedge\{\gamma_{G_i} : i \in j\} \geq cl_{\tau_2}(\gamma_A)$. Thus we get

$$int(A) \subseteq \langle x, int_{\tau_1}(\mu_A), cl_{\tau_2}(\gamma_A) \rangle \subseteq A.$$

The other inclusion is obvious. $\square$

Corollary 3.18 *Let $A = \langle x, \mu_A, \gamma_A \rangle$ be a GIFS in (X, τ). Then*

(a) *If A is a GIFCS, then μ_A is fuzzy closed in (X, τ_2) and γ_A is fuzzy open in (X, τ_1),*

(b) *If A is a GIFOS, then μ_A is fuzzy open in (X, τ_1) and γ_A is fuzzy closed in (X, τ_2).*

Example 3.19 Consider again the GIFTS (X, τ) in Examples 3.3 and 3.14. Now we obtain

$$int_{\tau_1}(\mu_F) = \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \; cl_{\tau_2}(\gamma_F) = \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.5}\right)$$

which justifies the nonequality of the inclusion in Proposition 3.17.

4 Generalized intuitionistic fuzzy continuity

Here come the basic definitions first:

Definition 4.1 Let (X, τ) and (Y, Φ) be two GIFTSs and let $f : X \to Y$ be a function. Then f is said to be fuzzy continuous iff the preimage of each GIFS in Φ is a GIFS in τ.

Definition 4.2 Let (X, τ) and (Y, Φ) be two GIFTSs and let $f : X \to Y$ be a function. Then f is said to be fuzzy open iff the image of each GIFS in τ is a GIFS in Φ.

Example 4.3 Let (X, τ_0), (Y, Φ_0) be two fuzzy topological spaces in the sense of Chang.

(a) If $f \to Y$ is fuzzy continuous in the usual sense, then in this case, f is fuzzy continuous in the sense of Definition 4.1, too. Here we consider the GIFTs on X and Y, respectively, as follows:

$$\tau = \{\langle x, \mu_G, 1 - \mu_G \rangle : \mu_G \in \tau_0\}$$

and

$$\Phi = \{\langle y, \lambda_H, 1 - \lambda_H \rangle : \lambda_H \in \Phi_0\}.$$

In this case we have, for each $\langle y, \lambda_H, 1 - \lambda_H \rangle \in \Phi$, $\lambda_H \in \Phi_0$,

$$\begin{aligned} f^{-1}(\langle y, \lambda_H, 1 - \lambda_H \rangle) &= \langle x, f^{-1}(\lambda_H), f^{-1}(1 - \lambda_H) \rangle \\ &= \langle x, f^{-1}(\lambda_H), 1 - f^{-1}(\lambda_H) \rangle \in \tau, \end{aligned}$$

(b) Let $f : X \to Y$ be a fuzzy open function in the usual sense. Then f is also fuzzy open in the sense of Definition 4.2.

Example 4.4 Let (X, τ) be a GIFTS in the sense of Lowen, (Y, Φ) a GIFTS and $c_0 \in Y$. Then the constant function $c : X \to Y$, $c(x) = c_0$ is obviously fuzzy continuous.

Here we obtain some characterizations of fuzzy continuity:

Proposition 4.5 *$f : (X, \tau) \to (Y, \Phi)$ is fuzzy continuous iff the preimage of each GIFCS in Φ is a GIFCS in τ.*

Proof This is obvious if we make use of Corollary 2.11(n). □

Proposition 4.6 *The following are equivalent to each other:*
(a) *$f : (X, \tau) \to (Y, \Phi)$ is fuzzy continuous.*
(b) *$f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for each GIFS B in Y.*
(c) *$cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each GIFS B in Y.*

Proof They can be easily proved using Definitions 4.1 and 2.6, Propositions 4.5 and 3.15 and Corollary 2.11(n). □

Example 4.7 Let (Y, Φ) be a GIFTS, X a nonempty set and $f : X \to Y$ a function. In this case $\tau = \{f^{-1}(H) : H \in \Phi\}$ is a GIFT on X. Indeed, τ is the coarsest GIFT on X which makes the function $f : X \to Y$ fuzzy continuous. One may call the GIFT τ on X the initial generalized intuitionistic fuzzy topology with respect to f.

Proposition 4.8 *Let $f : (X, \tau) \to (Y, \Phi)$ be a fuzzy continuous function. Then the functions*
(a) *$f : (X, \tau_1) \to (Y, \Phi_1)$, (b) $f : (X, \tau_2) \to (Y, \Phi_2)$ are also fuzzy continuous, where $\tau_1, \Phi_1, \tau_2, \Phi_2$ are the fuzzy topological spaces defined in Remark 3.7. [In other words, $\tau_1 = \{\mu_{G_i} : G_i \in \tau\}$, $\Phi_1 = \{\lambda_{H_j} : H_j \in \Phi\}$, $\tau_2 = \{1 - \gamma_{G_i} : G_i \in \tau\}$, $\Phi_2 = \{1 - \vartheta_{H_j} : H_j \in \Phi\}$, if $\tau = \{G_i : i \in J\}$, $\Phi = \{H_j : j \in K\}$, $G_i = \langle x, \mu_{G_i}, \gamma_{G_i} \rangle$ and $H_j = \langle y, \lambda_{H_j}, \vartheta_{H_j} \rangle$.]*

Proof These follow from definitions, immediately. □

5 Generalized intuitionistic fuzzy compactness

First we present the basic concepts:

Definition 5.1 Let (X, τ) be a GIFTS.

(a) If a family $\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\}$ of GIFOSs in X satisfies the condition $\bigcup\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\} = 1_\sim$, then it is called a fuzzy open cover of X. A finite subfamily of a fuzzy open cover $\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\}$ of X, which is also a fuzzy open cover of X, is called a finite subcover of $\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\}$.

(b) A family $\{\langle x, \mu_{K_i}, \gamma_{K_i}\rangle : i \in J\}$ of GIFCSs in X satisfies the finite intersection property (FIP for short) iff every finite subfamily $\{\langle x, \mu_{K_i}, \gamma_{K_i}\rangle : i = 1, 2, \ldots, n\}$ of the family satisfies the condition $\bigcap_{i=1}^{n}\{\langle x, \mu_{K_i}, \gamma_{K_i}\rangle\} \neq 0_\sim$.

Definition 5.2 A GIFTS (X, τ) is called fuzzy compact iff every fuzzy open cover of X has a finite subcover.

Example 5.3 The GIFTS (X, τ) defined in Example 3.4 is not fuzzy compact. This is because the fuzzy open cover $\{G_n : n \in \mathbf{N}\}$ has no finite subcover.

Now we give a proposition stating that fuzzy compactness in (X, τ) is indeed identical to fuzzy compactness in $(X, \tau_{0,1})$:

Proposition 5.4 *Let (X, τ) be a GIFTS on X. If the GIFTS $(X, \tau_{0,1})$ is fuzzy compact then (X, τ) is fuzzy compact (see Proposition 3.6).*

Proof Suppose that $(X, \tau_{0,1})$ is fuzzy compact and consider a fuzzy open cover $\{G_j : j \in K\}$ of X in (X, τ). Since $\bigcup G_j = 1_\sim$, we obtain $\bigwedge \gamma_{G_j}$, $\bigvee \mu_{G_j} = 1$ and $\bigwedge(1 - \mu_{G_j}) = 0$. Since $(X, \tau_{0,1})$ is fuzzy compact, $\exists G_1, G_2, \ldots, G_n$ such that $\bigcup_{i=1}^{n}([\ ]G_i) = 1_\sim$, i.e. $\bigvee_{i=1}^{n} \mu_{G_i} = 1$ and $\bigwedge_{i=1}^{n}(1 - \mu_{G_i}) = 0$. Since $\bigwedge(1 - \mu G_j) = \wedge\gamma_{G_j}$, we obtain

$$1 - \bigvee \mu_{G_j} = \bigwedge \gamma_{G_j} \Rightarrow 1 = \bigvee \mu_{G_j} + \bigwedge \gamma_{G_j}$$
$$\Rightarrow 1 \geq \mu_{G_j} + \bigwedge \gamma_{G_j}$$

$$\Rightarrow \bigwedge \gamma_{G_j} \leq 1 - \mu_{G_j}$$
$$\Rightarrow \bigwedge \gamma_{G_j} \leq \bigwedge (1 - \mu_{G_j})$$
$$\Rightarrow \bigwedge_{i=1}^{n} \gamma_{G_i} \leq \bigwedge_{i=1}^{n} (1 - \mu_{G_i}) = 0$$

Hence $\bigwedge_{i=1}^{n} \gamma_{G_i} = 0$. Hence $\bigcup_{i=1}^{n} G_i = 1_\sim$ follows, i.e. (X, τ) is fuzzy compact.

Proposition 5.5 *Let (X, τ) be a GIFTS on X. If (X, τ) is fuzzy compact and $\mu_{G_j} + \gamma_{G_j} \leq 1$ for all $G_j \in \tau$ then the GIFTS $(X, \tau_{0,1})$ is fuzzy compact (see Proposition 3.6).*

Proof Let (X, τ) be fuzzy compact, and consider a fuzzy open cover $\{[\ ]G_j : j \in K\}$ of X in $(X, \tau_{0,1})$. Since $\bigcup([\ ]G_j) = 1_\sim$, we obtain $\bigvee \mu_G = 1$, and hence, by $\gamma_{G_j} \leq 1 - \mu_{G_j} \Rightarrow \bigwedge \gamma_{G_j} \leq 1 - \bigvee \mu_{G_j} = 1 - 1 = 0 \Rightarrow \bigwedge \gamma_{G_j} = 0$, we deduce $\bigcup G_j = 1_\sim$. Since (X, τ) is fuzzy compact, $\exists G_1, G_2, \ldots, G_n$ such that $\bigcup_{i=1}^{n} G_i = 1_\sim$, from which we obtain $\bigvee_{i=1}^{n} \mu_{G_i} = 1$ and $\bigwedge_{i=1}^{n}(1 - \mu_{G_i}) = 0$, i.e. $(X, \tau_{0,1})$ is fuzzy compact.

Now Proposition 5.4 implies (Notice that we have $\tau_0 = \{\mu_G : G \in \tau\}$.): (X, τ_0) is fuzzy compact $\Rightarrow (X, \tau_{0,1})$ is fuzzy compact $\Rightarrow (X, \tau)$ is fuzzy compact. In view of this proposition we can obtain the following results easily:

Corollary 5.6 *A GIFTS (X, τ) is fuzzy compact iff every family $\{\langle x, \mu_{K_i}, \gamma_{K_i} \rangle : i \in J\}$ of GIFCSs in X having the FIP has a nonempty intersection.*

Corollary 5.7 *Let (X, τ), (Y, Φ) be GIFTSs and $f : X \to Y$ a fuzzy continuous surjection. If (X, τ) is fuzzy compact, then so is (Y, Φ).*

Since fuzzy compactness of a GIFTS (X, τ) is identical to the fuzzy compactness of the fuzzy topological space (X, τ_0), we must define the fuzzy compactness of a GIFS in (X, τ) as follows:

Definition 5.8 Let (X, τ) be a GIFTS and A a GIFS in X.

(a) If a family $\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\}$ of GIFOSs in X satisfies the condition $A \subseteq \bigcup\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\}$, then it is called a fuzzy open cover of A. A finite subfamily of the fuzzy open cover $\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\}$ of A, which is also a fuzzy open cover of A, is called a finite subcover of $\{\langle x, \mu_{G_i}, \gamma_{G_i}\rangle : i \in J\}$.

(b) A GIFS $A = \langle x, \mu_A, \gamma_A\rangle$ in a GIFTS (X, τ) is called fuzzy compact iff every fuzzy open cover of A has a finite subcover.

Corollary 5.9 *A GIFS $A = \langle x, \mu_A, \gamma_A\rangle$ in a GIFTS (X, τ) is fuzzy compact iff for each family $\mathcal{G} = \{G_i : i \in J\}$, where $G_i = \langle x, \mu_{G_i}, \gamma_{G_i}\rangle$ $(i \in J)$, of GIFOSs in X with properties*

$$\mu_A \leq \bigvee_{i \in J} \mu_{G_i} \quad \textit{and} \quad 1 - \gamma_A \leq \bigvee_{i \in J} (1 - \gamma_{G_i}),$$

there exists a finite subfamily $\{G_i : i = 1, 2, \ldots, n\}$ of $\mathcal{G}$ such that

$$\mu_A \leq \bigvee_{i=1}^{n} \mu_{G_i} \quad \textit{and} \quad 1 - \gamma_A \leq \bigvee_{i=1}^{n} (1 - \gamma_{G_i}).$$

Example 5.10 Let (X, τ_0) be a fuzzy topological space in Chang's sense and $\mu_A \in I^X$ a fuzzy compact set in X. We can construct a GIFTS τ on X as in Example 3.2. Now the GIFS $A = \langle x, \mu_A, 1 - \mu_A\rangle$ is also fuzzy compact in (X, τ).

Example 5.11 Let $X = I$ and consider the GIFSs $(G_n)_{n \in \mathbf{N}}$ as follows:

First we define the GIFSs $G_n = \langle x, \mu_{G_n}, \gamma_{G_n}\rangle$, $n = 1, 2, 3, 4, \ldots$, and $G = \langle x, \mu_G, \gamma_G\rangle$ by

$$\mu_{G_n}(x) = \begin{cases} 0.6, & \text{if } x = 0, \\ nx, & \text{if } 0 < x \leq \frac{1}{n}, \\ 1, & \text{if } \frac{1}{n} < x \leq 1, \end{cases}$$

$$\gamma_{G_n}(x) = \begin{cases} 0.5, & \text{if } x = 0, \\ 1 - nx, & \text{if } 0 < x \leq \frac{1}{n}, \\ 0, & \text{if } \frac{1}{n} < x \leq 1, \end{cases}$$

$$\mu_G(x) = \begin{cases} 0.6, & \text{if } x = 0, \\ 1, & \text{otherwise,} \end{cases}$$

$$\gamma_G(x) = \begin{cases} 0.5, & \text{if } x = 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then $\tau = \{0_\sim, 1_\sim, G\} \cup \{G_n : n \in \mathbf{N}\}$ is a GIFT on X, and consider that GIFSs $C_{\alpha,\beta}$(see Definition 3.10) in (X, τ). Then the GIFSs $C_{0.65,0.45}, C_{0.65,0.55}, C_{0.55,0.45}$ are all fuzzy compact, but the GIFS $C_{0.55,0.55}$ is not fuzzy compact.

Corollary 5.12 *Let (X, τ), (Y, Φ) be GIFTSs and $f : X \to Y$ a fuzzy continuous function. If A is fuzzy compact in (X, τ), then so is $f(A)$ in (Y, Φ).*

Proof Let $\mathcal{B} = \{G_i : i \in J\}$, where $G_i = \langle y, \mu_{G_i}, \gamma_{G_i} \rangle$, $i \in J$, be a fuzzy open cover of $f(A)$. Then, by Definition 4.1 and Corollary 2.11, $\mathcal{A} = \{f^{-1}(G_i) : i \in J\}$ is a fuzzy open cover of A, too. Since A is fuzzy compact, there exists a finite subcover of $\mathcal{A}$, i.e, there is $G_i (i = 1, 2, \cdots, n)$ such that $A \subseteq \bigcup_{i=1}^{n} f^{-1}(G_i)$. Hence

$$f(A) \subseteq f\left(\bigcup_{i=1}^{n} f^{-1}(G_i)\right) = \bigcup_{i=1}^{n} f(f^{-1}(G_i)) \subseteq \bigcup_{i=1}^{n} G_i$$

follows. Therefore, $f(A)$ is also fuzzy compact. □

6 Generalized intuitionistic fuzzy C_5-connectedness

Here we define fuzzy C_5-disconnectedness and fuzzy C_5-connectedness whose origin is a paper of Chaudhuri and Das [12] :

Definition 6.1 Let (X, τ) be a GIFTS.

(a) X is said to be fuzzy C_5-disconnected if there exists a generalized intuitionistic fuzzy open and fuzzy closed set G such that $G \neq 1_\sim$ and $G \neq 0_\sim$.

(b) X is said to be fuzzy C_5-connected if it is not fuzzy C_5-disconnected.

Example 6.2 Let (X, τ_0) be a fuzzy topological space in Chang's sense. If X is fuzzy C_5-connected in this sense, then X is also fuzzy C_5-connected with respect to the GIFT as is constructed in Example 3.2 [Suppose, on the contrary, that $\exists$ a GIFOS $G = \langle x, \mu_G, 1 - \mu_G \rangle$ in X which is also a GIFCS such that $0_\sim \neq G \neq 1_\sim$; in this case X is fuzzy C_5-disconnected in (X, τ_0), an obvious contradiction.]

Example 6.3 Let $X = \{a, b\}$ and

$$A = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.3},\right), \left(\frac{a}{0.9}, \frac{b}{0.8}\right) \right\rangle,$$
$$B = \left\langle x, \left(\frac{a}{0.9}, \frac{b}{0.8}\right), \left(\frac{a}{0.4}, \frac{b}{0.3}\right) \right\rangle,$$
$$C = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.7}\right), \left(\frac{a}{0.6}, \frac{b}{0.8}\right) \right\rangle,$$
$$D = \left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.8}\right), \left(\frac{a}{0.5}, \frac{b}{0.7}\right) \right\rangle.$$

Then the family $\tau = \{0_\sim, 1_\sim, A, B, C, D\}$ is a GIFT on X, and (X, τ) is fuzzy C_5-disconnected, since A is a proper nonzero GIFOS and GIFCS in X.

Example 6.4 Let $X = 1, 2$ and define the GIFSs $G_{i,j}$ and H_i as follows ($i, j \in 2\mathbf{N}$):

$$G_{i,j} = \left\langle x, \left(\frac{1}{\frac{1}{i}}, \frac{2}{\frac{1}{i}}\right), \left(\frac{1}{\frac{1}{i}}, \frac{2}{\frac{1}{j}}\right) \right\rangle.$$

and

$$H_i = \left\langle x, \left(\frac{1}{\frac{1}{i}}, \frac{2}{\frac{1}{i}}\right), \left(\frac{1}{0}, \frac{2}{0}\right) \right\rangle.$$

In this case the family $\tau = \{0_\sim, 1_\sim\} \cup \{G_{i,j}, H_i : i, j \in 2\mathbf{N}\}$ is a GIFT on X, and it is fuzzy C_5-disconnected, too.

Now we shall obtain a characterization of fuzzy C_5-connectedness inspired by a paper of Ali and Srivastava [1]. For this purpose we shall construct a GIFTS on I as follows. Let I_D denote the unit interval I with the GIFT τ_D generated by the GIFSs $B = \langle i, id, 1-id\rangle$ and $B^c = \langle i, 1-id, id\rangle$, in other words, let

$$\tau_D = \{0_\sim, 1_\sim, B, B^c, B\cap B^c, B\cup B^c\},$$

where id denotes the identity map on I.

Proposition 6.5 *A GIFTS (X,τ) is fuzzy* C_5*-disconnected iff there exists a fuzzy continuous function $f : (X,\tau) \to (I_D, \tau_D)$ with $f \neq 0$ and $f \neq 1$.*

Proof This is obvious if we notice that f is fuzzy continuous iff the GIFSs

$$f^{-1}(B) = \langle x, f^{-1}(id), f^{-1}(1-id)\rangle = \langle x, f, 1-f\rangle,$$
$$f^{-1}(B^c) = \langle x, f^{-1}(1-id), f^{-1}(id)\rangle = \langle x, 1-f, f\rangle,$$

are GIFOSs in τ (notice that $f^{-1}(B^c) = f^{-1}(B)^c$) iff $f^{-1}(B)$ is both a GIFOS and GIFCS in X. □

Corollary 6.6 *A GIFTS (X,τ) is fuzzy* C_5*-connected iff there exists no fuzzy continuous function $f : (X,\tau) \to (I_D, \tau_D)$ with $f \neq 0$ and $f \neq 1$.*

Here we state that fuzzy C_5-connectedness is preserved under a fuzzy continuous surjection:

Proposition 6.7 *Let (X,τ),(Y,Φ) be GIFTSs and $f : X \to Y$ a fuzzy continuous surjection. If (X,τ) is fuzzy* C_5*-connected, then so is (Y,Φ).*

Proof On the contrary, suppose that (Y,Φ) is fuzzy C_5-disconnected. Then there exists a generalized intuitionistic fuzzy open and fuzzy closed set G such that $G \neq 1_\sim$ and $G \neq 0_\sim$. Since f is fuzzy continuous, $f^{-1}(G)$ is both a GIFOS and GIFCS by Proposition 4.5. The equalities $f^{-1}(G) = 1_\sim$ or $f^{-1}(G) = 0_\sim$ cannot hold. (Because, otherwise we have $G = f(f^{-1}(G)) = f(1_\sim) = 1_\sim$ and $G = f(f^{-1}(G)) = f(0_\sim) = 0_\sim$ by Corollary 2.10 (d), (k) and (l)). Hence (Y,Φ) is fuzzy C_5-connected,too. □

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