Almost Strongly θ-precontinuous Functions

거의 강한 θ-전연속함수

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요약

고전적 점집합 위상수학과 수학의 여러분야에서 중요한 역할을 해오는 함수의 연속성은 개집합보다 약한 α-개집합, 반개집합, 전개집합 및 β-개집합 등에 의해서 일반화되어 오고 있다. 1984년 Mashhour등은 전연속성을 소개하고 이 연속성에 대하여 조사하였는데, 이것은 Ptak에 의해서 near continuity라 소개되었고, 또한 Frolik과 Husain들에 의해서는 almost continuity라고 각각 소개되어졌다. Jankovic은 전연속성보다 약한 almost weak continuity을 소개하였다. Popa과 Noiri는 weak precontinuity을 소개하고 이것이 almost weak continuity과 동치라는 것을 보였다. 최근에, Noiri는 전연속함수보다 강하고 강한 θ-연속함수보다 약한 강한 θ-전연속함수를 소개하였다.

본논문에서는 약 전연속함수보다 강하고, 거의 강한 θ-연속함수와 강한 θ-전연속함수보다 약한 거의 강한 θ-전연속함수(almost strongly θ-precontinuous functions)족을 소개하고, Noiri에 의해서 만들어진 몇가지 결과들을 개선하였다. 또한, p-closed의 거의 강한 θ-전연속함수에 의한 상이 nearly compact임을 보였다.

1 Introduction

Continuity of functions have been played a significant role in the theory of classical point set topology and several branches of mathematics. This concept has been generalized by weaker forms of open sets such as α-open sets [24], semi-open sets [15], preopen sets [19], b-open sets [3] and β-open sets [1]. In 1984, Mashhour et al. [19] introduced and investigated the notion of precontinuous functions. Precontinuity was called near continuity by Pták [35] and also almost continuity by Frolík [12] and Husain [13]. Janković [14] introduced almost weak continuity as a weak form of precontinuity. Popa and Noiri [33] introduced weak precontinuity and showed that almost weak continuity is equivalent to weak precontinuity. Recently, Noiri [29] introduced and investigated the notion of strongly θ-precontinuous functions which is implied by that of strongly θ-continuous functions [17] and implies that of precontinuous functions.

In this paper, we introduce a new class of functions called almost strongly θ-precontinuous functions which is contained in the class of weakly precontinuous functions and contains both the class of almost strongly θ-continuous functions [27] and the class of strongly θ-precontinuous functions. We investigate almost strongly θ-precontinuous functions and obtain several improvements of results established by Noiri [29]. It is also shown that every almost strongly θ-precontinuous surjective image of p-closed (resp. countably p-closed) space is nearly compact (resp. nearly countably compact).

2 Preliminaries

Throughout this paper, spaces (X, τ) and (Y, σ) (simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. For a subset A of (X, τ), $\mathrm{cl}(A)$ and $\mathrm{int}(A)$ represent the closure of A and the interior of A with respect to τ, respectively. A subset A is said to be *regular open* (resp. *regular closed*) if $A = \mathrm{int}(\mathrm{cl}(A))$ (resp. $A = \mathrm{cl}(\mathrm{int}(A))$). A point x of X is called a θ-cluster [41] (resp. δ-cluster [41]) point of A if $\mathrm{cl}(U) \cap A \neq \emptyset$ (resp. $\mathrm{int}(\mathrm{cl}(U)) \cap A \neq \emptyset$) for every open set U of X containing x. The set of all θ-cluster (resp. δ-cluster) points of A is called the θ-closure [41] (resp. δ-closure [41]) of A and is denoted by $\theta\text{-cl}(A)$ (resp. $\delta\text{-cl}(A)$). A subset A is said to be θ-closed [41] (resp. δ-closed [41]) if $\theta\text{-cl}(A) = A$ (resp. $\delta\text{-cl}(A) = A$). The complement of a θ-closed (resp. δ-closed) set is said to be θ-open (resp. δ-open).

A subset A is said to be *preopen* [19] (resp. *α-open* [24], *semi-open* [15], *β-open* [1]) if $A \subset \mathrm{int}(\mathrm{cl}(A))$ (resp. $A \subset \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$, $A \subset \mathrm{cl}(\mathrm{int}(A))$, $A \subset \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)))$). The complement of a preopen (resp. α-open, semi-open, β-open) set is said to be *preclosed* (resp. α-closed, semi-closed, β-closed). The family of all preopen sets of X is denoted by $\mathbf{PO}(X)$ and the family $\{U \in \mathbf{PO}(X) : x \in U\}$ is denoted by $\mathbf{PO}(X, x)$, where x is a point of X. The intersection of all preclosed sets of X containing A is called the *preclosure* [10] of A and is denoted by $\mathrm{pcl}(A)$. The α-closure, semi-closure and β-closure are similarly defined and are denoted by $\alpha\text{-cl}(A)$, $\mathrm{scl}(A)$ and $\beta\text{-cl}(A)$. The union of all preopen sets of X contained in A is called *preinterior* and is denoted by $\mathrm{pint}(A)$. A point x of X is called a pre-θ-cluster point of A if $\mathrm{pcl}(U) \cap A \neq \emptyset$ for every preopen set U of X containing

x. The set of all pre-θ-cluster points of A is called the pre-θ-closure of A and is denoted by θ-pcl(A). A subset A is said to be pre-θ-closed [32] if $A = \theta$-pcl(A). The complement of a pre-θ-closed set is said to be pre-θ-open.

Definition 2.1 A function $f : X \to Y$ is said to be

(a) *almost continuous* [40] (briefly, *a.c.S.*) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open set U containing x such that $f(U) \subset \text{int}(\text{cl}(V))$;

(b) *δ-continuous* [26] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open set U containing x such that $f(\text{int}(\text{cl}(U)) \subset \text{int}(\text{cl}(V))$;

(c) *precontinuous* [19] or *almost continuous* [13] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \mathbf{PO}(X, x)$ such that $f(U) \subset V$;

(d) *weakly precontinuous* [33] or *almost weakly continuous* [14] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \mathbf{PO}(X, x)$ such that $f(U) \subset \text{cl}(V)$;

(e) *strongly θ-precontinuous* [29] (briefly, st.θ.p.c.) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \mathbf{PO}(X, x)$ such that $f(\text{pcl}(U)) \subset V$.

Definition 2.2 A function $f : X \to Y$ is said to be *almost strongly θ-precontinuous* (briefly, a.st.θ.p.c.) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \mathbf{PO}(X, x)$ such that $f(\text{pcl}(U)) \subset \text{int}(\text{cl}(V))$.

Definition 2.3 A function $f : X \to Y$ is said to be *strongly θ-continuous* [26] (resp. *almost strongly θ-continuous* [27] (briefly, a.st.θ.c.)) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open neighborhood U of x such that $f(\text{cl}(U)) \subset V$ (resp. $f(\text{cl}(U)) \subset \text{int}(\text{cl}(V))$).

Remark 2.4 Almost strongly θ-precontinuity is implied by both almost strongly θ-continuity and strongly θ-precontinuity and implies weak precontinuity. None of these implications is reversible as the following examples show. Moreover, the following Examples 2.5 and 2.7 show that almost strongly θ-precontinuity and continuity are independent of each other.

Example 2.5 Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$ and $\sigma = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Define a function $f : (X, \tau) \to (X, \sigma)$ as follows: $f(a) = f(b) = b$ and $f(c) = f(d) = a$. Then f is a.st.θ.p.c. (even a.st.θ.c.) but it is neither continuous nor st.θ.p.c.

Example 2.6 Let (X, τ) and (X, σ) be the spaces in Example 2.5. Define a function $f : (X, \tau) \to (X, \sigma)$ as follows: $f(a) = b$ and $f(b) = f(c) = f(d) = c$. Then f is a.st.θ.p.c. but it is not a.st.θ.c.

Example 2.7 Let (X, τ) be the space in Example 2.5. Then the identity function $f : (X, \tau) \to (X, \tau)$ is continuous (hence weakly precontinuous) but not a.st.θ.p.c.

3 Characterizations

Theorem 3.1 *For a function $f : X \to Y$, the following are equivalent:*

(a) *f is a.st.θ.p.c.;*

(b) *$f^{-1}(V)$ is pre-θ-open in X for each regular open set V of Y;*

(c) *$f^{-1}(F)$ is pre-θ-closed in X for each regular closed set F of Y;*

(d) *for each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists $U \in \mathbf{PO}(X, x)$ such that $f(\mathrm{pcl}(U)) \subset V$;*

(e) *$f^{-1}(V)$ is pre-θ-open in X for each δ-open set V of Y;*

(f) $f^{-1}(F)$ *is pre-*θ*-closed in* X *for each* δ*-closed set* F *of* Y*;*

(g) $f(\theta\text{-pcl}(A)) \subset \delta\text{-cl}(f(A))$ *for each subset* A *of* X*;*

(h) $\theta\text{-pcl}(f^{-1}(B)) \subset f^{-1}(\delta\text{-cl}(B))$ *for each subset* B *of* Y.

Proof (a)⇒(b): Let V be any regular open set of Y and $x \in f^{-1}(V)$. There exists $U \in \mathbf{PO}(X, x)$ such that $f(\text{pcl}(U)) \subset V$. Therefore, we have $x \in U \subset \text{pcl}(U) \subset f^{-1}(V)$. This shows that $f^{-1}(V)$ is pre-θ-open in X.

(b)⇒(c): Let F be any regular closed set of Y. By (b), $f^{-1}(F) = X - f^{-1}(Y - F)$ is pre-θ-closed in X.

(c)⇒(d): Let $x \in X$ and V be any regular open set of Y containing $f(x)$. By (c), $f^{-1}(Y - V) = X - f^{-1}(V)$ is pre-θ-closed in X. Since $f^{-1}(V)$ is a pre-θ-open set containing x, there exists $U \in \mathbf{PO}(X, x)$ such that $\text{pcl}(U) \subset f^{-1}(V)$. Therefore, we have $f(\text{pcl}(U)) \subset V$.

(d)⇒(e): Let V be any δ-open set of Y and $x \in f^{-1}(V)$. There exists a regular open set G of Y such that $f(x) \in G \subset V$. By (d), there exists $U \in \mathbf{PO}(X, x)$ such that $f(\text{pcl}(U)) \subset G$. Therefore, we obtain $x \in U \subset \text{pcl}(U) \subset f^{-1}(V)$. This shows that $f^{-1}(V)$ is pre-θ-open in X.

(e)⇒(f): Let F be any δ-closed set of Y. By (e), $f^{-1}(F) = X - f^{-1}(Y - F)$ is pre-θ-closed in X.

(f)⇒(g): Let A be any subset of Y. Since $\delta\text{-cl}(f(A))$ is δ-closed in Y, $f^{-1}(\delta\text{-cl}(f(A)))$ is pre-θ-closed in X. Let $x \notin f^{-1}(\delta\text{-cl}(f(A)))$. There exists $U \in \mathbf{PO}(X, x)$ such that $\text{pcl}(U) \cap f^{-1}(\delta\text{-cl}(f(A))) = \emptyset$ and thus $\text{pcl}(U) \cap A = \emptyset$. Hence $x \notin \theta\text{-pcl}(A)$. Therefore, we have $f(\theta\text{-pcl}(A)) \subset \delta\text{-cl}(f(A))$.

(g)⇒(h): Let B be any subset of Y. By (g), we have $f(\theta\text{-pcl}(f^{-1}(B))) \subset \delta\text{-cl}(B)$ and hence $\theta\text{-pcl}(f^{-1}(B)) \subset f^{-1}(\delta\text{-cl}(B))$.

(h)⇒(a): Let $x \in X$ and V be any open set of Y containing $f(x)$. Then $G = Y - \text{int}(\text{cl}(V))$ is regular closed and hence δ-closed in Y. By (h), $\theta\text{-pcl}(f^{-1}(G)) \subset f^{-1}(G)$ and hence $f^{-1}(G)$ is pre-θ-closed in X. Therefore, $f^{-1}(\text{int}(\text{cl}(V)))$ is pre-θ-open set containing x. There exists $U \in \mathbf{PO}(X, x)$ such that $\text{pcl}(U) \subset f^{-1}(\text{int}(\text{cl}(V)))$. Therefore, we obtain $f(\text{pcl}(U)) \subset \text{int}(\text{cl}(V))$. This shows that f is a.st.θ.p.c. □

It is known that the family of all δ-open sets in a space (X, τ) form a topology for X which is denoted by τ_δ. However, τ_δ is identical with the semiregularization τ_s of τ and hence we use τ_s in the place of τ_δ. For simplicity, we shall denote (X, τ_s) by X_s.

Lemma 3.2 (Andrijević [4]) $\text{scl}(V) = \text{int}(\text{cl}(V))$ *for each preopen set* V *of a space* X.

Theorem 3.3 *For a function* $f : X \to Y$ *the following are equivalent:*

(a) f *is a.st.θ.p.c.;*

(b) *for each* $x \in X$ *and each open set* V *of* Y *containing* $f(x)$, *there exists* $U \in \mathbf{PO}(X, x)$ *such that* $f(\text{pcl}(U)) \subset \text{scl}(V)$;

(c) $f^{-1}(V) \subset \theta\text{-pcl}(f^{-1}(\text{int}(\text{cl}(V))))$ *for each open set* V *of* Y;

(d) $f : X \to Y_s$ *is st.θ.p.c.*

Proof (a)⇔(b): It follows from Lemma 3.2.

(a)⇒(c): Let V be any open set of Y and $x \in f^{-1}(V)$. By (a), there exists $U \in \mathbf{PO}(X, x)$ such that $f(\text{pcl}(U)) \subset \text{int}(\text{cl}(V))$. Therefore, we have $x \in U \subset \text{pcl}(U) \subset f^{-1}(\text{int}(\text{cl}(V)))$ and hence $x \in \theta\text{-pcl}(f^{-1}(\text{int}(\text{cl}(V))))$. It follows that $f^{-1}(V) \subset \theta\text{-pcl}(f^{-1}(\text{int}(\text{cl}(V))))$.

(c)$\Rightarrow$(d): Let $x \in X$ and V be any open set of Y_s containing $f(x)$. There exists a regular open set G of Y such that $f(x) \in G \subset V$. By (c), we have $x \in f^{-1}(G) \subset \theta\text{-pcl}(f^{-1}(G))$ and hence there exists $U \in \mathbf{PO}(X)$ such that $x \in U \subset \text{pcl}(U) \subset f^{-1}(G)$. Therefore, we obtain $f(\text{pcl}(U)) \subset V$. This shows that $f : X \to Y_s$ is st.θ.p.c.

(d)$\Rightarrow$(a): Let V be regular open set of Y. For any $x \in f^{-1}(V)$, $f(x) \subset V$ and V is open in Y_s. There exists $U \in \mathbf{PO}(X, x)$ such that $f(\text{pcl}(U)) \subset V$ and hence $\text{pcl}(U) \subset f^{-1}(V)$. Therefore, we have $f^{-1}(V) \subset \theta\text{-pcl}(f^{-1}(V))$ and $f^{-1}(V)$ is pre-θ-open in X. By Theorem 3.1, f is a.st.θ.p.c. □

Lemma 3.4 [27] *For a subset V of a space X, then following hold:*

(a) $\alpha\text{-cl}(V) = \text{cl}(V)$ *for each β-open set V of X.*

(b) $\text{pcl}(V) = \text{cl}(V)$ *for each semi-open set V of X.*

Theorem 3.5 *For a function $f : X \to Y$, the following are equivalent:*

(a) *f is a.st.θ.a.c.;*

(b) $\theta\text{-pcl}(f^{-1}(V)) \subset f^{-1}(\text{cl}(V))$ *for each β-open set V of Y;*

(c) $\theta\text{-pcl}(f^{-1}(V)) \subset f^{-1}(\text{cl}(V))$ *for each semi-open set V of Y;*

(d) $\theta\text{-pcl}(f^{-1}(V)) \subset f^{-1}(\alpha\text{-cl}(V))$ *for each β-open set V of Y;*

(e) $\theta\text{-pcl}(f^{-1}(V)) \subset f^{-1}(\text{pcl}(V))$ *for each semi-open set V of Y.*

Proof (a)$\Rightarrow$(b): Let V be any β-open set of Y. Then by Theorem 2.4 in [4], $\text{cl}(V)$ is regular open in Y. Since f is a.st.θ.a.c., $f^{-1}(\text{cl}(V))$ is pre-θ-closed in X and hence $\theta\text{-pcl}(f^{-1}(V)) \subset f^{-1}(\text{cl}(V))$.

(b)$\Rightarrow$(c): This is obvious since every semi-open set is β-open.

(c)⇒(a): Let F be any regular closed set of Y. Then F is semi-open in Y and hence $\theta\text{-pcl}(f^{-1}(F)) \subset f^{-1}(\text{cl}(F)) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is pre-θ-closed in X. Therefore, f is a.st.δ.a.c.

(b)⇔(d): It follows from Lemma 3.4 (a).

(c)⇔(e): It follows from Lemma 3.4 (b).

□

Recall that a space X is said to be *almost regular* [38] (resp. *semi-regular*) if for any regular open (resp. open) set U of X and each point $x \in U$, there exits a regular open set V of X such that $x \in V \subset \text{cl}(V) \subset U$ (resp. $x \in V \subset U$).

Theorem 3.6 *Let $f : X \to Y$ be a function. Then, the following properties hold:*

(a) *If f is precontinuous and Y is almost regular, then f is a.st.θ.p.c.*

(b) *f is a.st.θ.p.c. and Y is semi-regular, then f is st.θ.p.c.*

Proof (a): Let $x \in X$ and V be any regular open set of Y containing $f(x)$. Since Y is almost regular, there exists an open set W such that $f(x) \in W \subset \text{cl}(W) \subset V$. Since f is precontinuous, there exists $U \in \mathbf{PO}(X, x)$ such that $f(U) \subset W$. We shall show that $f(\text{pcl}(U)) \subset \text{cl}(W)$. Suppose that $y \notin \text{cl}(W)$. There exists an open neighborhood of y such that $G \cap W = \emptyset$. Since f is precontinuous, $f^{-1}(G) \in \mathbf{PO}(X)$ and $f^{-1}(G) \cap U = \emptyset$ and hence $f^{-1}(G) \cap \text{pcl}(U) = \emptyset$. Therefore, we obtain $G \cap f(\text{pcl}(U)) = \emptyset$ and $y \notin f(\text{pcl}(U))$. Consequently, we have $f(\text{pcl}(U)) \subset \text{cl}(W) \subset V$.

(b): Let $x \in X$ and V be any open set of Y conntaining $f(x)$. Since Y is semi-regular, there exists a regular open set W such that $f(x) \subset W \subset V$. Since f is a.st.θ.p.c., there exists $U \in \mathbf{PO}(X, x)$ such that $f(\text{pcl}(U)) \subset W$. Therefore, we have $f(\text{pcl}(U)) \subset V$. □

Corollary 3.7 *Let Y be a regular space. Then, the following preperties are equivalent for a function $f : X \to Y$:*

(a) *f is weakly precontinuous;*

(b) *f is precontinuous;*

(c) *f is a.st.θ.p.c.;*

(d) *f is st.θ.p.c.*

Proof It follows from Theorem 3.2 of [29] and Theorem 3.6. □

Definition 3.8 A space X is said to be p-regular [10] (resp. almost p-regular [18]) if for each closed (resp. regular closed) set F and each point $x \in X - F$, there exist disjoint preopen sets U and V such that $x \in U$ and $F \subset V$.

Theorem 3.9 (a) *If continuous function $f : X \to Y$ is a.st.θ.p.c., then X is almost p-regular.*

(b) *If $f : X \to Y$ is a.c.S. (resp. δ-continuous) and X is p-regular (resp. almost p-regular), then f is a.st.θ.p.c.*

Proof (a): Let $f : X \to X$ be the identity. Then f is continuous and hence a.st.θ.p.c. For any regular open set U of X and any point x of U, we have $f(x) = x \in U$ and there exists $G \in \mathbf{PO}(X, x)$ such that $f(\mathrm{pcl}(G)) \subset U$. Therefore, we have $x \in G \subset \mathrm{pcl}(G) \subset U$ and hence X is almost p-regular.

(b): Suppose that $f : X \to Y$ is almost continuous (resp. δ-continuous) and X is p-regular (resp. almost p-regular). For each $x \in X$ and any regular open set V containing $f(x)$, $f^{-1}(V)$ is an open (resp. regular open) set of X containing x. Since X is p-regular (resp. almost p-regular), there exists $U \in \mathbf{PO}(X, x)$ such that $x \in U \subset \mathrm{pcl}(U) \subset f^{-1}(V)$. Therefore, we have $f(\mathrm{pcl}(U)) \subset V$. This shows that f is a.st.θ.p.c. □

A space X is said to be submaximal [37] if each dense subset of X is open in X. It is shown in [37] that a space X is submaximal if and only if every preopen set of X is open.

Theorem 3.10 *Let X be a submaximal space. Then $f : X \to Y$ is a.st.θ.p.c. if and only if f is a.st.θ.c.*

Proof Suppose that f is a.st.θ.p.c. Let $x \in X$ and V be any regular open set of Y containing $f(x)$. Since f is a.st.θ.p.c., there exists $U \in \mathbf{PO}(X, x)$ such that $f(\mathrm{pcl}(U)) \subset V$. Since X is submaximal, U is open and $\mathrm{pcl}(U) = \mathrm{cl}(U)$. Therefore, we obtain $f(\mathrm{cl}(U)) \subset V$. This shows that f is a.st.θ.c. □

Corollary 3.11 *Let X is a submaximal space and Y be a semi-regular space. Then the following properties are equivalent for $f : X \to Y$:*

(a) *f is a.st.θ.p.c.;*

(b) *f is st.θ.p.c.;*

(c) *f is a.st.θ.c.;*

(d) *f is strongly θ-continuous.*

Proof It follows from Theorems 3.6 and 3.10 and Theorem 4.1 of [27].

4 Some properties

A space X is said to be *pre-regular* [32] if for each preclosed set F and each point $x \in X - F$, there exist disjoint preopen sets U and V such that $x \in U$ and $F \subset V$.

Theorem 4.1 *Let $f : X \to Y$ be a function and $g : X \to X \times Y$ be the graph function of f. Then, the following properties hold:*

(a) *If g is a.st.θ.p.c., then f is a.st.θ.p.c. and X is almost p-regular.*

(b) *If f is a.st.θ.p.c. and X is pre-regular, then g is a.st.θ.p.c.*

Proof (a): Suppose that g is a.st.θ.p.c. First, we show that f is a.st.θ.p.c. Let $x \in X$ and V be a regular open set of Y containing $f(x)$. Then $X \times V$ is a regular open set of $X \times Y$ containing $g(x)$. Since g is a.st.θ.p.c., there exists $U \in \mathbf{PO}(X, x)$ such that $g(\mathrm{pcl}(U)) \subset X \times V$. Therefore, we obtain $f(\mathrm{pcl}(U)) \subset V$. Next, we show that X is almost p-regular. Let U be any regular open set of X and $x \in U$. Since $g(x) \in U \times Y$ and $U \times Y$ is regular open in $X \times Y$, there exists $G \in \mathbf{PO}(X, x)$ such that $g(\mathrm{pcl}(G)) \subset U \times Y$. Therefore, we obtain $x \in G \subset \mathrm{pcl}(G) \subset U$ and hence X is almost p-regular.

(b): Let $x \in X$ and W be any regular open set of $X \times Y$ containing $g(x)$. There exist regular open sets $U_1 \subset X$ and $V \subset Y$ such that $g(x) = (x, f(x)) \in U_1 \times V \subset W$. Since f is a.st.θ.p.c., there exists $U_2 \in \mathbf{PO}(X, x)$ such that $f(\mathrm{pcl}(U_2)) \subset V$. Since X is pre-regular and $U_1 \cap U_2 \in \mathbf{PO}(X, x)$, there exists $U \in \mathbf{PO}(X, x)$ such that $x \in U \subset \mathrm{pcl}(U) \subset U_1 \cap U_2$ [23, Lemma 4.2]. Therefore, we obtain $g(\mathrm{pcl}(U)) \subset U_1 \times f(\mathrm{pcl}(U_2)) \subset U_1 \times V \subset W$. This shows that g is a.st.θ.p.c. □

Corollary 4.2 *Let X be a pre-regular space. Then, a functionf $: X \to Y$ is a.st.θ.p.c. if and only if the graph functiion $g : X \to X \times Y$ is a.st.θ.p.c.*

Lemma 4.3 (Mashhour et al. [22]) *Let A and X_0 be subsets of a space X.*

(a) *If $A \in \mathbf{PO}(X)$ and X_0 is semi-open in X, then $A \cap X_0 \in \mathbf{PO}(X)$.*

(b) *If $A \in \mathbf{PO}(X_0)$ and $X_0 \in \mathbf{PO}(X)$, then $A \in \mathbf{PO}(X)$.*

Lemma 4.4 (Dontchev et al. [8]) *Let A and X_0 be subsets of a space X such that $A \subset X_0 \subset X$. Let $\mathrm{pcl}_{X_0}(A)$ denote the preclosure of A in the subspace X_0.*

(a) *If* X_0 *is semi-open in* X, *then* $\mathrm{pcl}_{X_0}(A) \subset \mathrm{pcl}(A)$.

(b) *If* $A \in \mathbf{PO}(X_0)$ *and* $X_0 \in \mathbf{PO}(X)$, *then* $\mathrm{pcl}(A) \subset \mathrm{pcl}_{X_0}(A)$.

Theorem 4.5 *If* $f : X \to Y$ *is a.st.*θ*.p.c. and* X_0 *is a semi-open subset of* X, *then the restriction* $f|X_0 : X_0 \to Y$ *is a.st.*θ*.p.c.*

Proof For any $x \in X_0$ and any regular open set V of Y containing $f(x)$, there exists $U \in \mathbf{PO}(X, x)$ such that $f(\mathrm{pcl}(U)) \subset V$ since f is a.st.θ.p.c. Put $U_0 = U \cap X_0$, then by Lemmas 4.3 and 4.4, $U_0 \in \mathbf{PO}(X_0. x)$ and $\mathrm{pcl}_{X_0}(U_0) \subset \mathrm{pcl}(U_0)$. Therefore, we obtain

$$(f|X_0)(\mathrm{pcl}_{X_0}(U_0)) = f(\mathrm{pcl}_{X_0}(U_0) \subset f(\mathrm{pcl}(U_0)) \subset f(\mathrm{pcl}(U)) \subset V.$$

This shows that $f|X_0$ is a.st.θ.p.c. □

Theorem 4.6 *A function* $f : X \to Y$ *is a.st.*θ*.p.c. if for each* $x \in X$ *there exists* $X_0 \in \mathbf{PO}(X, x)$ *such that the restriction* $f|X_0 : X_0 \to Y$ *is a.st.*θ*.p.c.*

Proof Let $x \in X$ and V be any regular open set of Y containing $f(x)$. There exists $X_0 \in \mathbf{PO}(X, x)$ such that $f|X_0 : X_0 \to Y$ is a.st.θ.p.c. Therefore, there exists $U \in \mathbf{PO}(X_0, x)$ such that $(f|X_0)(\mathrm{pcl}_{X_0}(U)) \subset V$. By Lemmas 4.3 and 4.4, $U \in \mathbf{PO}(X, x)$ and $\mathrm{pcl}(U) \subset \mathrm{pcl}_{X_0}(U)$. Hence, we have, $f(\mathrm{pcl}(U)) = (f|X_0)(\mathrm{pcl}(U)) \subset (f|X_0)(\mathrm{pcl}_{X_0}(U)) \subset V$. This shows that f is a.st.θ.p.c. □

In order to obtain some properties of the compositions of a.st.θ.p.c. functions, we shall recall some definitions.

Definition 4.7 A function $f : X \to Y$ is said to be

(a) *pre-irresolute* [36] if for each $x \in X$ and each $V \in \mathbf{PO}(Y, f(x))$, there exists $U \in \mathbf{PO}(X, x)$ such that $f(U) \subset V$.

(b) M-preopen [20] if $f(U) \in \mathbf{PO}(Y)$ for each $U \in \mathbf{PO}(X)$.

Lemma 4.8 (Noiri [29]) *If $f : X \to Y$ is pre-irresolute and V is a pre-θ-open set of Y, then $f^{-1}(V)$ is pre-θ-open in X.*

Theorem 4.9 *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then, the following properties hold:*

(a) *If f is a.st.θ.p.c. and g is δ-continuous, then the composition $g \circ f : X \to Z$ is a.st.θ.p.c.*

(b) *If f is pre-irresolute and g is a.st.θ.p.c., then $g \circ f$ is a.st.θ.p.c.*

(c) *If $f : X \to Y$ is an M-preopen bijection and $g \circ f : X \to Z$ is a.st.θ.p.c., then g is a.st.θ.p.c.*

Proof (a): It is obvious from Theorem 3.1.

(b): It follows immmediately from Theorem 3.1 and Lemma 4.8.

(c): Let W be any regular open set of Z. Since $g \circ f$ is a.st.θ.p.c., $(g \circ f)^{-1}(W)$ is pre-θ-open in X. Since f is M−preopen and bijective, f^{-1} is pre-irresolute and by Lemma 4.8, we have $g^{-1}(W) = f((g \circ f)^{-1}(W))$ is pre-θ-open in Y. Hence, by Theorem 3.1 g is a.st.θ.p.c. □

Let $\{X_\alpha : \alpha \in \Lambda\}$ be a family of spaces, A_α be a nonempty subset of X_α for each $\alpha \in \Lambda$ and the product space $\prod\{X_\alpha : \alpha \in \Lambda\}$ will be denoted by $\prod X_\alpha$.

Lemma 4.10 (El-Deeb et al. [10]) *Let n be a positive integer and $A = \prod_{j=1}^{n} A_{\alpha_j} \times \prod_{\alpha \neq \alpha_j} X_\alpha$.*

(a) $A \in \mathbf{PO}(X)$ *if and only if* $A_{\alpha_j} \in \mathbf{PO}(X_{\alpha_j})$ *for each* $j = 1, \cdots, n$.

(b) $\mathrm{pcl}(\prod_{\alpha\in\Lambda} A_\alpha) \subset \prod_{\alpha\in\Lambda} \mathrm{pcl}(A_\alpha)$.

Theorem 4.11 *If a function* $f_\alpha : X_\alpha \to Y_\alpha$ *is a.st.θ.p.c. for each* $\alpha \in \Lambda$, *the product function* $f : \prod X_\alpha \to \prod Y_\alpha$, *defined by* $f(\{x_\alpha\}) = \{f_\alpha(x_\alpha)\}$ *for each* $x = \{x_\alpha\}$, *is a.st.θ.p.c.*

Proof Let $x = \{x_\alpha\} \in \prod X_\alpha$ and W be any regular open set of $\prod Y_\alpha$ containing $f(x)$. Then, there exists a regular open set V_{α_j} of Y_{α_j} such that

$$f(x) = \{f_\alpha(x_\alpha)\} \in \prod_{j=1}^{n} V_{\alpha_j} \times \prod_{\alpha\neq\alpha_j} Y_\alpha \subset W.$$

Since f_α is a.st.θ.p.c. for each α, there exists $U_{\alpha_j} \in \mathbf{PO}(X_{\alpha_j}, x_{\alpha_j})$ such that $f_{\alpha_j}(\mathrm{pcl}(U_{\alpha_j})) \subset V_{\alpha_j}$ for $j = 1, \cdots, n$. Now, put $U = \prod_{j=1}^{n} U_{\alpha_j} \times \prod_{\alpha\neq\alpha_j} X_\alpha$. Then, by Lemma 4.10 we have $U \in \mathbf{PO}(\prod X_\alpha, x)$ and

$$f(\mathrm{pcl}(U)) \subset f(\textstyle\prod_{j=1}^{n} \mathrm{pcl}(U_{\alpha_j}) \times \prod_{\alpha\neq\alpha_j} X_\alpha) \subset \prod_{j=1}^{n} f_{\alpha_j}(\mathrm{pcl}(U_{\alpha_j})) \times \prod_{\alpha\neq\alpha_j} Y_\alpha$$

$$\subset \textstyle\prod_{j=1}^{n} V_{\alpha_j} \times \prod_{\alpha\neq\alpha_j} Y_\alpha \subset W.$$

This shows that f is a.st.θ.p.c. □

5 Separation axioms and a.st.θ.p.c. functions

Definition 5.1 A space X is said to be

(a) *pre-*T_2 (resp. *pre-Urysohn*) [30] if for each pair of distinct points x and y in X, there exist $U \in \mathbf{PO}(X, x)$ and $V \in \mathbf{PO}(X, y)$ such that $U \cap V = \emptyset$ (resp. $\mathrm{pcl}(U) \cap \mathrm{pcl}(V) = \emptyset$);

(b) rT_0 [4] if for any two distinct points of X, there exists a regular open set containing one of the points but not the other.

Theorem 5.2 (a) *If $f : X \to Y$ is an a.st.θ.p.c. injection and Y is rT_0, then X is pre-T_2.*

(b) *If $f : X \to Y$ is an a.st.θ.p.c. injection and Y is Hausdorff, then X is pre-Urysohn.*

Proof (a): Let x and y be any distinct points of X. Since f is injective, $f(x) \neq f(y)$ and there exists either a regular open set V containing $f(x)$ not containing $f(y)$ or a regular open set W containing $f(y)$ not containing $f(x)$. If the first case holds, then there exists $U \in \mathbf{PO}(X, x)$ such that $f(\mathrm{pcl}(U)) \subset V$. Therefore, we obtain $f(y) \notin f(\mathrm{pcl}(U))$ and hence $X - \mathrm{pcl}(U) \in \mathbf{PO}(X, y)$. If the second case holds, then we obtain a similar result. Therefore, X is pre-T_2.

(b): Let x and y be any distinct points of X. Then $f(x) \neq f(y)$. Since Y is Hausdorff, there exist open sets V and W containing $f(x)$ and $f(y)$, respectively, such that $\mathrm{int}(\mathrm{cl}(V)) \cap \mathrm{int}(\mathrm{cl}(W)) = \emptyset$. Since f is a.st.θ.p.c., there exist $G \in \mathbf{PO}(X, x)$ and $H \in \mathrm{PO}(X, y)$ such that $f(\mathrm{pcl}(G)) \subset \mathrm{int}(\mathrm{cl}(V))$ and $f(\mathrm{pcl}(H)) \subset \mathrm{int}(\mathrm{cl}(W))$. It follows that $\mathrm{pcl}(G) \cap \mathrm{pcl}(H) = \emptyset$. This shows that X is pre-Urysohn. □

Corollary 5.3 (Noiri [29]) *If $f : X \to Y$ is a st.θ.p.c. injection and Y is Hausdorff, then X is pre-Urysohn.*

Theorem 5.4 *If $f : X \to Y$ is an a.st.θ.p.c. function and Y is Hausdorff, then the subset $E = \{(x, y) : f(x) = f(y)\}$ is pre-θ-closed in $X \times X$.*

Proof Suppose that $(x, y) \notin E$. Then $f(x) \neq f(y)$. Since Y is Hausdorff, there exist open sets V and W of Y containing $f(x)$ and $f(y)$, respectively, such that $\mathrm{int}(\mathrm{cl}(V)) \cap \mathrm{int}(\mathrm{cl}(W)) = \emptyset$. Since f is a.st.θ.p.c., there exist $U \in \mathbf{PO}(X, x)$ and

$G \in \mathbf{PO}(X, y)$ such that $f(\mathrm{pcl}(U)) \subset \mathrm{int}(\mathrm{cl}(V))$ and $f(\mathrm{pcl}(G)) \subset \mathrm{int}(\mathrm{cl}(W))$. Set $D = U \times G$. It follows that $(x, y) \in D \in \mathbf{PO}(X \times X)$ and $\mathrm{pcl}(U \times G) \cap E \subset [\mathrm{pcl}(U) \times \mathrm{pcl}(G)] \cap E = \emptyset$. Therefore, E is pre-θ-closed in $X \times X$. □

Corollary 5.5 (Noiri [29]) *If $f : X \to Y$ is a st.θ.p.c. function and Y is Hausdorff, then the subset $E = \{(x, y) : f(x) = f(y)\}$ is pre-θ-closed in $X \times X$.*

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\}$ of $X \times Y$ is called the graph of f is denoted by $G(f)$.

Definition 5.6 The graph $G(f)$ of a function $f : X \to Y$ is said to be *strongly pre-closed* [29] (resp. *pre-θ-closed*) if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in \mathbf{PO}(X, x)$ and an open set V in Y containing y such that $(\mathrm{pcl}(U) \times V) \cap G(f) = \emptyset$ (resp. $(\mathrm{pcl}(U) \times \mathrm{cl}(V)) \cap G(f) = \emptyset$).

Lemma 5.7 *The graph $G(f)$ of a function $f : X \to Y$ is pre-θ-closed if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in \mathbf{PO}(X, x)$ and an open set V in Y containing y such that $f(\mathrm{pcl}(U)) \cap \mathrm{cl}(V) = \emptyset$.*

Theorem 5.8 *If $f : X \to Y$ is a.st.θ.p.c. and Y is Hausdorff, then $G(f)$ is pre-θ-closed in $X \times Y$.*

Proof Let $(x, y) \in (X \times Y) - G(f)$. Then $f(x) \neq y$. Since Y is hausdorff, there exist open sets V and W in Y containing $f(x)$ and y, respectively, such that $\mathrm{int}(\mathrm{cl}(V)) \cap \mathrm{cl}(W) = \emptyset$. Since f is a.st.θ.p.c., there exists $U \in \mathbf{PO}(X, x)$ such that $f(\mathrm{pcl}(U)) \subset \mathrm{int}(\mathrm{cl}(V))$. Therefore, $f(\mathrm{pcl}(U)) \cap \mathrm{cl}(W) = \emptyset$ and then by Lemma 5.7 $G(f)$ is pre-θ-closed in $X \times Y$. □

Corollary 5.9 (Noiri [29]) *If $f : X \to Y$ is st.θ.p.c. and Y is Hausdorff, then $G(f)$ is strongly pre-closed in $X \times Y$.*

6 Covering properties

Definition 6.1 A space X is said to be

(a) *quasi H-closed* [34] if every cover of X by open sets has finite subcover whose closures cover of X;

(b) *nearly compact* [39] if every cover of X by regular open sets has a finite subcover;

(c) *nearly countably compact* [11] if every countable cover of X by regular open sets has a finite subcover;

(d) *p-closed* [8] if every cover of X by preopen sets has a finite subcover whose preclosures cover X;

(e) *countably p-closed* [29] if every countable cover of X by preopen sets has a finite subcover whose preclosures cover X.

A subset K of a space X is said to be *quasi H-closed relative to* X [34] (resp. *N-closed relative to* X [39], p-closed relative to X [8]) if for every cover $\{V_\alpha : \alpha \in \Lambda\}$ of K by open (resp. regular open, preopen) sets of X, there exists a finite subset Λ_0 of Λ such that $K \subset \cup\{\mathrm{cl}(V_\alpha) : \alpha \in \Lambda_0\}$ (resp. $K \subset \cup\{V_\alpha : \alpha \in \Lambda_0\}$, $K \subset \cup\{\mathrm{pcl}(V_\alpha) : \alpha \in \Lambda_0\}$).

Theorem 6.2 *If $f : X \to Y$ is an a.st.θ.p.c. function and K is p-closed relative to X, then $f(K)$ is N-closed relative to Y.*

Proof Let $\{V_\alpha : \alpha \in \Lambda\}$ be a cover of $f(K)$ by regular open sets of Y. For each point $x \in K$, there exists $\alpha(x) \in \Lambda$ such that $f(x) \in V_{\alpha(x)}$. Since f is a.st.θ.p.c., there exists $U_x \in \mathbf{PO}(X, x)$ such that $f(\mathrm{pcl}(U_x)) \subset V_{\alpha(x)}$. The family $\{U_x : x \in K\}$ is a cover of K by preopen sets of X and hence there exists a finite subset K_0 of K such that $K \subset \cup_{x \in K_0} \text{pcl}(U_x)$. Therefore, we obtain $f(K) \subset \cup_{x \in K_0} V_{\alpha(x)}$. This shows that $f(K)$ is N-closed relative to Y. □

Corollary 6.3 *Let $f : X \to Y$ be an a.st.θ.p.c. surjection. Then, the following properties hold:*

(a) *If X is p-closed, then Y is nearly compact.*

(b) *If X is countably p-closed, then Y is nearly countably compact.*

Theorem 6.4 *If a function $f : X \to Y$ has a pre-θ-closed graph, then $f(K)$ is θ-closed in Y for each subset K which is p-closed relative to X.*

Proof Let K be a p-closed relative to X and $y \in Y - f(K)$. Then for each $x \in K$ we have $(x, y) \notin G(f)$ and by Lemma 5.7 there exist $U_x \in \mathbf{PO}(X, x)$ and an open set V_x of Y containing y such that $f(\text{pcl}(U_x)) \cap \text{cl}(V_x) = \emptyset$. The family $\{U_x : x \in K\}$ is a cover of K by preopen sets of X. Since K is p-closed relative to X, there exists a finite subset K_0 of K such that $K \subset \cup\{\text{pcl}(U_x) : x \in K_0\}$. Put $V = \cap\{V_x : x \in K_0\}$. Then V is an open set containing y and

$$f(K) \cap \text{cl}(V) \subset [\cup_{x \in K_0} f(\text{pcl}(U_x))] \cap \text{cl}(V) \subset \cup_{x \in K_0} [f(\text{pcl}(U_x)) \cap \text{cl}(V_x)] = \emptyset.$$

Therefore, we have $y \in \theta\text{-cl}(f(K))$ and hence $f(K)$ is θ-closed in Y. □

Theorem 6.5 *Let X is a submaximal space. If a function $f : X \to Y$ has a pre-θ-closed graph, then $f^{-1}(K)$ is θ-closed in X for each subset K which is quasi H-closed relative to Y.*

Proof Let K be a quasi H-closed set of Y and $x \notin f^{-1}(K)$. Then for each $y \in K$ we have $(x, y) \notin G(f)$ and by Lemma 5.7 there exists $U_y \in \mathbf{PO}(X, x)$ and an open set V_y of Y containing y such that $f(\mathrm{pcl}(U_y)) \cap \mathrm{cl}(V_y) = \emptyset$. The family $\{V_y : y \in K\}$ is an open cover of K and there exists a finite subset K_0 of K such that $K \subset \cup_{y \in K_0} \mathrm{cl}(V_y)$. Since X is submaximal, each U_y is open in X and $\mathrm{pcl}(U_y) = \mathrm{cl}(U_y)$. Set $U = \cap_{y \in K_0} U_y$, then U is an open set containing x and

$$f(\mathrm{cl}(U)) \cap \mathrm{cl}(K) \subset \cup_{y \in K_0} [f(\mathrm{cl}(U)) \cap \mathrm{cl}(V_y)] \subset \cup_{x \in K_0} [f(\mathrm{pcl}(U_y)) \cap \mathrm{cl}(V_y)] = \emptyset.$$

Therefore, we have $\mathrm{cl}(U) \cap f^{-1}(K) = \emptyset$ and hence $x \notin \theta\text{-}\mathrm{cl}(f^{-1}(K))$. This shows that $f^{-1}(K)$ is θ-closed in X. □

Corollary 6.6 *Let X be a submaximal space and Y be a quasi H-closed Hausdorff space. The following properties are equivalent for a function $f : X \to Y$:*

(a) *f is a.st.θ.p.c.;*

(b) *$G(f)$ is pre-θ-closed in $X \times Y$;*

(c) *f is a.st.θ.c.*

Proof (a)⇒(b): This follows from Theorem 5.8.

(b)⇒(c): Let F be any regular closed set Y. By (2.2) of [34] F is quasi H-closed and hence quasi H-closed relative to Y. It follows from Theorem 6.5 that $f^{-1}(F)$ is θ-closed in X. Therefore, f is a.st.θ.c.

(c)⇒(a): Obvious. □

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