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Confidence Intervals in Two-factor Mixed Model with One Covariate for Gauge R & R Study

by

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Confidence Intervals in Two-factor Mixed Model with One Covariate for Gauge R & R Study
계측기(R & R) 연구를 위한 공변량을 갖는 2요인 혼합 모형의 신뢰구간

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요 약

이 논문은 계측기 (R & R) 연구를 위한 공변량을 갖는 2요인 교차 혼합모형에 나타난 변동에 관한 신뢰구간을 제안하였다. 모형의 변동은 부품, 작업자, 측정의 분산을 의미하는데 제안되는 신뢰구간들은 수정대표본 방법, 일반화 P값에 의한 방법, SAS의 PROC MIXED에서 계산되는 잔차 최대우도방법에 기초하였다. 제안되는 세 가지 방법들을 비교하기 위하여 부품과 작업자와 측정값들의 여러 가지 값들에 대하여 SAS/IML로 시뮬레이션을 실행하였다. 시뮬레이션 결과 특히 총 분산에 대한 부품이나 작업자의 분산 비율이 작을 때는 수정대표본방법이나 일반화 P값에 의한 신뢰구간이 잔차 최대우도 방법보다 더 신뢰계수를 잘 지킨다는 것을 볼 수 있었고 수치 값이 들어있는 예제를 제시하였다.

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1. INTRODUCTION

Measurements obtained in manufacturing process are used to control the quality of final product. The sources of variability in the measurement process are due to the measurement system. In order to determine whether a measurement procedure is adequate for monitoring a manufacturing process, gauge capability analysis is needed. A gauge is used to obtain replicate measurements on units by several different operators, setups, or time periods. One of the common experiments employed in industry is to properly monitor manufacturing process, e.g., repeatability and reproducibility. The variability is inherent in the measurement system, which we generally think of as the precision of the gauge. Gauge repeatability and reproducibility(R & R) study include comparison of measurement system with manufacturing process. The sources of variability in gauge R & R study are expressed as variance components and confidence intervals on variance components are utilized for quality control. Burdick et al.(2005) define that repeatability represents the gauge variability when it is used to measure the same unit(with the same operator or setup or in the same time period). Reproducibility is referred to as the variability arising from different operators, setups, or time periods.

Montgomery and Runger(1993 a, 1993 b) provided complete background on gauge capability and designed experiments. They compared classical R & R analysis using $\bar{X}$ and R charts and ANOVA analysis by illustrating numerical examples. Burdick and Larsen(1997) proposed alternative confidence intervals on measures of variability in a balanced two-factor crossed random models. They compared a modified large sample(MLS) method and Restricted Maximum Likelihood(REML) method. Borror et al.(1997) compared MLS method with REML method for confidence intervals on gauge variability in balanced two-factor crossed random models with interaction. They concluded that REML method resulted in shorter intervals than MLS method but REML method did not maintain the stated level of confidence. Dolezal et al.(1998) considered a balanced two-factor crossed random model with fixed operators and compared two confidence intervals; operator as a random factor as usual and operator as a fixed factor. Burdick et al.(2003) provided more recent review of the methods for conducting and analyzing measurement systems capabilities, focusing on the analysis of variance approach. Adamec and Burdick(2003) proposed two confidence intervals on a discrimination ratio used to determine the adequacy of the measurement system considering a traditional two-factor model. Gong et al.(2005) considered methods for constructing confidence intervals in a two-factor gauge repeatability and reproducibility when there are unequal replicates. They provided easy and simple EXCEL and SAS codes for calculating confidence intervals. The classical R & R study gives a downward biased estimator of the variance component representing gauge reproducibility. Confidence intervals on variance components are therefore analyzed using ANOVA approach that leads to a direct and convenient method in an experimental design(Montgomery and Runger, 1993 b).

This thesis proposes confidence intervals on variance components in two factor crossed mixed model with one covarite for gauge R & R study. Computer simulation is used to compare the performance of the confidence intervals. A numerical example is provided and recommendations are presented.

2. CONFIDENCE INTERVALS IN TWO-FACTOR CROSSED MIXED MODEL WITH ONE COVARIATE

2.1 Two-factor Crossed Mixed Model with One Covaiate

Two-factor crossed mixed model with one covariate for gauge R & R study employs J operators randomly chosen to conduct measurements on I randomly selected parts from a manufacturing process. In this R&R study each operator measures each part k times. The model that explains this study is written as

$$Y_{ijk} = \mu + \beta X_{ijk} + P_i + O_j + E_{ijk} \qquad (2.1.1)$$

$$i = 1, ..., I; \;\; j = 1, ..., J; \;\; k = 1, ..., K$$

where Y_{ijk} is the kth measurement of the ith part measured by the jth operator, μ and β are unknown constants, X_{ijk} is a covariate, P_i is the ith randomly selected part, O_j is the jth randomly chosen operator, and E_{ijk} is the kth measurement error of the ith part measured by the jth operator. P_i, O_j, and E_{ijk} are jointly independent normal random variables with zero means and variances σ_P^2, σ_O^2, and σ_E^2, respectively. Since the X_{ijk} and β are fixed and P_i and O_j are random, model (2.1.1) is a two-factor crossed mixed model with one covariate.

2.2 Distributional Results

Model (2.1.1) is written in matrix notation,

$$\mathbf{y} \;=\; \mathbf{X}\,\underline{\alpha} \;+\; \mathbf{Z}_1\,\mathbf{p} \;+\; \mathbf{Z}_2\,\mathbf{o} \;+\; \mathbf{Z}_3\,\mathbf{e} \qquad (2.2.1)$$

where $\mathbf{y}$ is an $IJK \times 1$ vector of observations, $\mathbf{X}$ is an IJK matrix of known values with a column of 1's in the first column and a column of X_{ijk}'s in the second column, $\underline{\alpha}$ is a 2×1 vector of parameters with μ and β as elements, $\mathbf{Z}_1$ is an $IJK \times I$ design matrix with 0's and 1's, $\mathbf{p}$ is an $I \times 1$ vector of random part effects, $\mathbf{Z}_2$ is an $IJK \times J$ design matrix with 0's and 1's, $\mathbf{o}$ is a $J \times 1$ vector of random operator effects, $\mathbf{Z}_3$ is an $IJK \times IJK$ design matrix which is an identity matrix, and $\mathbf{e}$ is an $IJK \times 1$ vector of random measurement errors. In particular,

$$\mathbf{y} = \begin{bmatrix} Y_{111} \\ \vdots \\ \vdots \\ Y_{IJK} \end{bmatrix}, \qquad \mathbf{X} = \begin{bmatrix} 1 & X_{111} \\ \vdots & \vdots \\ \vdots & \vdots \\ 1 & X_{IJK} \end{bmatrix}, \qquad \underline{\alpha} = \begin{bmatrix} \mu \\ \beta \end{bmatrix},$$

$$\mathbf{p} = \begin{bmatrix} P_1 \\ \vdots \\ \vdots \\ P_I \end{bmatrix}, \qquad \mathbf{o} = \begin{bmatrix} O_1 \\ \vdots \\ \vdots \\ O_J \end{bmatrix}, \qquad \mathbf{e} = \begin{bmatrix} E_{111} \\ \vdots \\ \vdots \\ E_{IJK} \end{bmatrix},$$

$$
\mathbf{Z}_1 = \begin{bmatrix}
1 & 0 & \dots & 0 \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
1 & 0 & \dots & 0 \\
0 & 1 & \dots & 0 \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
0 & 1 & \dots & 0 \\
\vdots & \vdots & \dots & \vdots \\
0 & 0 & \dots & 1 \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
0 & 0 & \dots & 1
\end{bmatrix} = \bigoplus_{i=1}^{I} \underline{1}_{JK}, \quad
\mathbf{Z}_2 = \begin{bmatrix}
1 & 0 & \dots & 0 \\
\vdots & \vdots & \dots & \vdots \\
1 & 0 & \dots & 0 \\
0 & 1 & \dots & 0 \\
\vdots & \vdots & \dots & \vdots \\
0 & 1 & \dots & 0 \\
\vdots & \vdots & \dots & \vdots \\
0 & 0 & \dots & 1 \\
\vdots & \vdots & \dots & \vdots \\
0 & 0 & \dots & 1 \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
\vdots & \vdots & \dots & \vdots \\
1 & 0 & \dots & 0 \\
\vdots & \vdots & \dots & \vdots \\
1 & 0 & \dots & 0 \\
0 & 1 & \dots & 0 \\
\vdots & \vdots & \dots & \vdots \\
0 & 1 & \dots & 0 \\
\vdots & \vdots & \dots & \vdots \\
0 & 0 & \dots & 1 \\
\vdots & \vdots & \dots & \vdots \\
0 & 0 & \dots & 1
\end{bmatrix} = \underline{1}_I \otimes \left(\bigoplus_{j=1}^{J} \underline{1}_K \right),
$$

$$\mathbf{Z}_3 = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} = \mathbf{D}_{IJK},$$

$\underline{1}_{JK}$, $\underline{1}_I$, and $\underline{1}_K$ are, respectively, a $JK \times 1$ column vector of 1's, an $I \times 1$ column vector of 1's, and a $K \times 1$ column vector of 1's, $\oplus$ is the direct sum operator, $\otimes$ is the direct product operator, and $\mathbf{D}_{IJK}$ is an $IJK \times IJK$ identity matrix. The direct sum of two matrices A and B is defined as

$$\mathbf{A} \oplus \mathbf{B} = \begin{bmatrix} \mathbf{A} & 0 \\ 0 & \mathbf{B} \end{bmatrix}$$

and the direct product of two matrices A and B is defined as

$$\mathbf{A}_{p\times q} \otimes \mathbf{B}_{m\times n} = \begin{bmatrix} a_{11}\mathbf{B}_{m\times n} & \dots & a_{1q}\mathbf{B}_{m\times n} \\ \vdots & \ddots & \vdots \\ a_{p1}\mathbf{B}_{m\times n} & \dots & a_{pq}\mathbf{B}_{m\times n} \end{bmatrix}.$$

The variance-covariance matrix of $\mathbf{y}$ is

$$\begin{aligned} \mathbf{V} = V(\mathbf{y}) &= V(\mathbf{X}\underline{\alpha} + \mathbf{Z}_1\mathbf{p} + \mathbf{Z}_2\mathbf{o} + \mathbf{Z}_3\mathbf{e}) \\ &= \sigma_P^2\mathbf{Z}_1\mathbf{Z}_1' + \sigma_O^2\mathbf{Z}_2\mathbf{Z}_2' + \sigma_E^2\mathbf{Z}_3\mathbf{Z}_3'. \end{aligned}$$

In particular, the covariance of Y_{ijk}, and $Y_{i'j'k'}$ is obtained as

$$Cov(Y_{ijk}, Y_{i'j'k'}) = \begin{cases} 0 & \text{if } i \neq i' \\ \sigma_P^2 & \text{if } i = i', j \neq j' \\ \sigma_P^2 + \sigma_O^2 & \text{if } i = i', j = j', k \neq k' \\ \sigma_P^2 + \sigma_O^2 + \sigma_E^2 & \text{if } i = i', j = j', k = k' \end{cases}.$$

The variance-covariance matrix structure of $\mathbf{V}$ is

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_1 & 0 & \dots & 0 \\ 0 & \mathbf{V}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{V}_I \end{bmatrix}$$

where matrix $\mathbf{V}_i$ is

$$\mathbf{V}_i = \begin{array}{c} \\ Y_{i11} \\ \vdots \\ Y_{i1K} \\ \vdots \\ Y_{iJ1} \\ \vdots \\ Y_{iJK} \end{array} \begin{array}{c} \begin{array}{ccccccc} Y_{i11} & \dots & Y_{i1K} & \cdots & Y_{iJ1} & \dots & Y_{iJK} \end{array} \\ \left(\begin{array}{ccccccc} V_{POE} & \dots & V_{PO} & \cdots & V_P & \dots & V_P \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ V_{PO} & \dots & V_{POE} & \cdots & V_P & \dots & V_P \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ V_P & \dots & V_P & \cdots & V_{POE} & \dots & V_{PO} \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ V_P & \dots & V_P & \cdots & V_{PO} & \dots & V_{POE} \end{array} \right) \end{array},$$

$V_{POE} = \sigma_P^2 + \sigma_O^2 + \sigma_E^2$, $V_{PO} = \sigma_P^2 + \sigma_O^2$, and $V_P = \sigma_P^2$.

In order to form confidence intervals on the variance components, an appropriate set of sums of squares is needed. The analysis of variance for Y_{ijk}, X_{ijk}, and $X_{ijk}Y_{ijk}$ is shown in Table 2.2.1.

TABLE 2.2.1 ANOVA for Model (2.1.1)

SV	Sums of Squares and Products Y	X	XY	DF
Parts	SS_{PY}	SS_{PX}	SP_{PXY}	$I-1$
Operators	SS_{OY}	SS_{OX}	SP_{OXY}	$J-1$
Error	SS_{EY}	SS_{EX}	SP_{EXY}	$IJK-I-J+1$
Total	SST_Y	SST_X	SPT_{XY}	$IJK-1$

The notation in Table 2.2.1. is defined as

$$SS_{PY} = JK\sum_i(\bar{Y}_{i..} - \bar{Y}_{...})^2,$$

$$SS_{PX} = JK\sum_i(\bar{X}_{i..} - \bar{X}_{...})^2,$$

$$SP_{PXY} = JK\sum_i(\bar{X}_{i..} - \bar{X}_{...})(\bar{Y}_{i..} - \bar{Y}_{...}),$$

$$SS_{OY} = IK\sum_j(\bar{Y}_{.j.} - \bar{Y}_{...})^2,$$

$$SS_{OX} = IK\sum_j(\bar{X}_{.j.} - \bar{X}_{...})^2,$$

$$SP_{OXY} = IK\sum_j(\bar{X}_{.j.} - \bar{X}_{...})(\bar{Y}_{.j.} - \bar{Y}_{...}),$$

$$SS_{EY} = \sum_i \sum_j \sum_k (Y_{ijk} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})^2,$$

$$SS_{EX} = \sum_i \sum_j \sum_k (X_{ijk} - \bar{X}_{i..} - \bar{X}_{.j.} + \bar{X}_{...})^2,$$

$$SP_{EXY} = \sum_i \sum_j \sum_k (X_{ijk} - \bar{X}_{i..} - \bar{X}_{.j.} + \bar{X}_{...})(Y_{ijk} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...}),$$

$$SST_Y = \sum_i \sum_j \sum_k (Y_{ijk} - \bar{Y}_{...})^2,$$

$$SST_X = \sum_i \sum_j \sum_k (X_{ijk} - \bar{X}_{...})^2, \quad \text{and}$$

$$SPT_{XY} = \sum_i \sum_j \sum_k (X_{ijk} - \bar{X}_{...})(Y_{ijk} - \bar{Y}_{...}).$$

In order to focus on constructing confidence intervals on the variance components, Table 2.2.1 is modified in Table 2.2.2 that presents expected mean squares.

TABLE 2.2.2 Adjusted ANOVA for Model(2.1.1)

SV	SS	DF	MS	EMS
Parts	R_1	$I-2$	S_P^2	$\sigma_E^2 + JK\sigma_P^2$
Operators	R_2	$J-2$	S_O^2	$\sigma_E^2 + IK\sigma_O^2$
Replicates	R_3	$IJK-I-J$	S_E^2	σ_E^2

where sums of squares R_1, R_2, and R_3 and mean squares S_P^2, S_O^2, and S_E^2 are defined as

$$S_P^2 = \frac{R_1}{I-2},$$

$$S_O^2 = \frac{R_2}{J-2},$$

$$S_E^2 = \frac{R_3}{IJK-I-J},$$

$$R_1 = JK\ \mathbf{y}'\mathbf{W}_1'(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{y},$$

$$R_2 = IK\ \mathbf{y}'\mathbf{W}_2'(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{y},$$

$$R_3 = \mathbf{y}'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{y},$$

$$\mathbf{W}_1 = \frac{1}{JK}\mathbf{Z}_1',$$

$$\mathbf{W}_2 = \frac{1}{IK}\mathbf{Z}_2',$$

$$\mathbf{W}_3 = \mathbf{Z}_3' = \mathbf{D}_{IJK},$$

$$\mathbf{H}_1 = \mathbf{X}_1(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1',$$

$$\mathbf{H}_2 = \mathbf{X}_2(\mathbf{X}_2'\mathbf{X}_2)^{-1}\mathbf{X}_2',$$

$$\mathbf{H}_3 = \mathbf{X}_3(\mathbf{X}_3'\mathbf{X}_3)^{-}\mathbf{X}_3',$$

$$\mathbf{X}_1 = \mathbf{W}_1\mathbf{X},$$

$$\mathbf{X}_2 = \mathbf{W}_2\mathbf{X}, \text{ and}$$

$$\mathbf{X}_3 = [\mathbf{X}\ \ \mathbf{Z}_1\ \ \mathbf{Z}_2].$$

A set of jointly independent chi-squared variables is necessary for deriving confidence intervals on the variance components. The distributional properties of the sums of squares in Table 2.2.2 are now examined.

Theorem 2.2.1 : Under the distributional assumptions in (2.1.1), $R_1/(\sigma_E^2 + JK\sigma_P^2)$ is a chi-squared random variable with $I-2$ degrees of freedom.

Proof : Consider (2.2.1) and multiply both sides of the equation on the left by $(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1$

where

$$\mathbf{D}_I = \text{ an identity matrix of order } I,$$

$$\mathbf{H}_1 = \mathbf{X}_1(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1',$$

$$\mathbf{X}_1 = \mathbf{W}_1\mathbf{X}, \text{ and}$$

$$\mathbf{W}_1 = \frac{1}{JK}\mathbf{Z}_1'.$$

It follows that

$$(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{y} = (\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{X}\underline{\alpha} + (\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{Z}_1\mathbf{p} + (\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{Z}_2\mathbf{o} + (\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{Z}_3\mathbf{e}. \qquad (2.2.2)$$

Let $(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{y} = \mathbf{y}_1$. Since

$$(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{X} = (\mathbf{D}_I - \mathbf{H}_1)\mathbf{X}_1 = \mathbf{X}_1 - \mathbf{H}_1\mathbf{X}_1 = \mathbf{0},$$

$$\mathbf{W}_1\mathbf{Z}_1 = \frac{1}{JK}\mathbf{Z}_1'\mathbf{Z}_1 = \frac{1}{JK}JK\mathbf{D}_I = \mathbf{D}_I, \text{ and}$$

$$\mathbf{W}_1\mathbf{Z}_3 = \frac{1}{JK}\mathbf{Z}_1'\mathbf{Z}_3 = \frac{1}{JK}\mathbf{Z}_1',$$

(2.2.2) is written as

$$\mathbf{y}_1 = (\mathbf{D}_I - \mathbf{H}_1)\mathbf{p} + (\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{Z}_2\mathbf{o} + \frac{1}{JK}(\mathbf{D}_I - \mathbf{H}_1)\mathbf{Z}_1'\mathbf{e}.$$

Note that

$$\mathbf{W}_1\mathbf{Z}_2 = \frac{1}{JK}\mathbf{Z}_1'\mathbf{Z}_2 = \frac{1}{JK}K\underline{1}_I\underline{1}_J' = \frac{1}{J}\underline{1}_I\underline{1}_J' \text{ and}$$

$$\begin{aligned}(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{Z}_2\mathbf{Z}_2'\mathbf{W}_1'(\mathbf{D}_I - \mathbf{H}_1) &= (\mathbf{D}_I - \mathbf{H}_1)\frac{1}{J}\underline{1}_I\underline{1}_J'\frac{1}{J}\underline{1}_J\underline{1}_I'(\mathbf{D}_I - \mathbf{H}_1) \\ &= (\mathbf{D}_I - \mathbf{H}_1)\frac{1}{J^2}J\underline{1}_I\underline{1}_I'(\mathbf{D}_I - \mathbf{H}_1) \\ &= (\mathbf{D}_I - \mathbf{H}_1)\frac{1}{J}\underline{1}_I\underline{1}_I'(\mathbf{D}_I - \mathbf{H}_1) \\ &= \frac{1}{J}(\mathbf{D}_I - \mathbf{H}_1)\underline{1}_I[(\mathbf{D}_I - \mathbf{H}_1)\underline{1}_I]' \\ &= \frac{1}{J}\underline{0}\ \underline{0}' \\ &= \mathbf{0}\end{aligned}$$

since $\mathbf{H}_1$ is an idempotent matrix and

$$\begin{aligned}(\mathbf{D}_I - \mathbf{H}_1)\underline{1}_I &= \underline{1}_I - \mathbf{X}_1(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\underline{1}_I \\ &= \underline{1}_I - \underline{1}_I \\ &= \underline{0}.\end{aligned}$$

Considering that

$$\mathbf{Z}_1'\mathbf{Z}_1 = JK\mathbf{D}_I,$$

the variance of $\mathbf{y}_1$ is

$$\begin{aligned}V(\mathbf{y}_1) &= V((\mathbf{D}_I - \mathbf{H}_1)\mathbf{p}) + V((\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{Z}_2\mathbf{o}) + V(\frac{1}{JK}(\mathbf{D}_I - \mathbf{H}_1)\mathbf{Z}_1'\mathbf{e}) \\ &= \sigma_P^2(\mathbf{D}_I - \mathbf{H}_1) + \frac{1}{JK}\sigma_E^2(\mathbf{D}_I - \mathbf{H}_1) \\ &= \frac{1}{JK}(\sigma_E^2 + JK\sigma_P^2)(\mathbf{D}_I - \mathbf{H}_1).\end{aligned}$$

The distribution of R_1 is determined by writing

$$\begin{aligned}
&\frac{R_1}{\sigma_E^2 + JK\sigma_P^2} \\
&= \mathbf{y}'\mathbf{W}_1'(\mathbf{D}_I - \mathbf{H}_1)\Big[\frac{JK(\mathbf{D}_I - \mathbf{H}_1)}{\sigma_E^2 + JK\sigma_P^2}\Big](\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{y} \qquad (2.2.3) \\
&= \mathbf{y}_1'\Big[\frac{JK(\mathbf{D}_I - \mathbf{H}_1)}{\sigma_E^2 + JK\sigma_P^2}\Big]\mathbf{y}_1
\end{aligned}$$

and noting

$$\begin{aligned}
&\Big[\frac{JK(\mathbf{D}_I - \mathbf{H}_1)}{\sigma_E^2 + JK\sigma_P^2}\Big]V(\mathbf{y}_1) \\
&= \Big[\frac{JK(\mathbf{D}_I - \mathbf{H}_1)}{\sigma_E^2 + JK\sigma_P^2}\Big]\frac{1}{JK}(\sigma_E^2 + JK\sigma_P^2)(\mathbf{D}_I - \mathbf{H}_1) \\
&= (\mathbf{D}_I - \mathbf{H}_1),
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{2}E(\mathbf{y}_1)'\Big[\frac{JK(\mathbf{D}_I - \mathbf{H}_1)}{\sigma_E^2 + JK\sigma_P^2}\Big](\mathbf{D}_I - \mathbf{H}_1)E(\mathbf{y}_1) \\
&= \frac{1}{2}\underline{\alpha}'\mathbf{X}'\mathbf{W}_1'(\mathbf{D}_I - \mathbf{H}_1)\Big[\frac{JK(\mathbf{D}_I - \mathbf{H}_1)}{\sigma_E^2 + JK\sigma_P^2}\Big](\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{X}\underline{\alpha} \\
&= 0, \quad \text{and}
\end{aligned}$$

$$r(\mathbf{D}_I - \mathbf{H}_1) = I - 2.$$

By Theorem 7.3 in Searle(1987, p. 232), $R_1/(\sigma_E^2 + JK\sigma_P^2)$ is a chi-squared random variable with $I-2$ degrees of freedom. ■

Theorem 2.2.2 : Under the distributional assumptions in (2.1.1), $R_2/(\sigma_E^2 + IK\sigma_O^2)$ is a chi-squared random variable with $J-2$ degrees of freedom.

Proof : Multiplying (2.2.1) both sides of the equation on the left by $(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2$ yields that

$$\begin{aligned}(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{y} = & (\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{X}\underline{\alpha} + (\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{Z}_1\mathbf{p} \\ & + (\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{Z}_2\mathbf{o} + (\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{Z}_3\mathbf{e}\end{aligned} \tag{2.2.4}$$

where $\mathbf{D}_J$ is an identity matrix of order J. Let $(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{y} = \mathbf{y}_2$. Since

$$(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{X} = (\mathbf{D}_J - \mathbf{H}_2)\mathbf{X}_2 = \mathbf{X}_2 - \mathbf{H}_2\mathbf{X}_2 = \mathbf{0},$$

$$\mathbf{W}_2\mathbf{Z}_1 = \frac{1}{IK}\mathbf{Z}_2'\mathbf{Z}_1 = \frac{1}{IK}K\underline{1}_J\underline{1}_I' = \frac{1}{I}\underline{1}_J\underline{1}_I',$$

$$\mathbf{W}_2\mathbf{Z}_2 = \frac{1}{IK}\mathbf{Z}_2'\mathbf{Z}_2 = \frac{1}{IK}IK\mathbf{D}_J = \mathbf{D}_J, \text{ and}$$

$$\mathbf{W}_2\mathbf{Z}_3 = \frac{1}{IK}\mathbf{Z}_2'\mathbf{Z}_3 = \frac{1}{IK}\mathbf{Z}_2',$$

(2.2.4) is written as

$$\mathbf{y}_2 = \frac{1}{I}(\mathbf{D}_J - \mathbf{H}_2)\underline{1}_J\underline{1}_I'\mathbf{p} + (\mathbf{D}_J - \mathbf{H}_2)\mathbf{o} + \frac{1}{IK}(\mathbf{D}_J - \mathbf{H}_2)\mathbf{Z}_2'\mathbf{e}.$$

Note that

$$\mathbf{Z}_2'\mathbf{Z}_2 = IK\mathbf{D}_J \text{ and}$$

$$\begin{aligned}(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{Z}_1\mathbf{Z}_1'\mathbf{W}_2'(\mathbf{D}_J - \mathbf{H}_2) &= (\mathbf{D}_J - \mathbf{H}_2)\frac{1}{I}\underline{1}_J\underline{1}_I'\frac{1}{I}\underline{1}_I\underline{1}_J'(\mathbf{D}_J - \mathbf{H}_2)\\ &= (\mathbf{D}_J - \mathbf{H}_2)\frac{1}{I^2}I\underline{1}_J\underline{1}_J'(\mathbf{D}_J - \mathbf{H}_2)\\ &= (\mathbf{D}_J - \mathbf{H}_2)\frac{1}{I}\underline{1}_J\underline{1}_J'(\mathbf{D}_J - \mathbf{H}_2)\\ &= \frac{1}{I}(\mathbf{D}_J - \mathbf{H}_2)\underline{1}_J[(\mathbf{D}_J - \mathbf{H}_2)\underline{1}_J]'\\ &= \frac{1}{I}\underline{0}\ \underline{0}'\\ &= \underline{0}\end{aligned}$$

since $\mathbf{H}_2$ is an idempotent matrix and

$$\begin{aligned}(\mathbf{D}_J - \mathbf{H}_2)\underline{1}_J &= \underline{1}_J - \mathbf{X}_2(\mathbf{X}_2'\mathbf{X}_2)^{-1}\mathbf{X}_2'\underline{1}_J\\ &= \underline{1}_J - \underline{1}_J\\ &= \underline{0}.\end{aligned}$$

The variance of $\mathbf{y}_2$ is

$$\begin{aligned}V(\mathbf{y}_2) &= V(\frac{1}{I}(\mathbf{D}_J - \mathbf{H}_2)\underline{1}_J\underline{1}_I'\mathbf{p}) + V((\mathbf{D}_J - \mathbf{H}_2)\mathbf{o}) + V(\frac{1}{IK}(\mathbf{D}_J - \mathbf{H}_2)\mathbf{Z}_2'\mathbf{e})\\ &= \sigma_O^2(\mathbf{D}_J - \mathbf{H}_2) + \frac{1}{IK}\sigma_E^2(\mathbf{D}_J - \mathbf{H}_2)\\ &= \frac{1}{IK}(\sigma_E^2 + IK\sigma_O^2)(\mathbf{D}_J - \mathbf{H}_2).\end{aligned}$$

The distribution of R_2 is determined by writing

$$\frac{R_2}{\sigma_E^2 + IK\sigma_O^2}$$

$$= \mathbf{y}'\mathbf{W}_2'(\mathbf{D}_J - \mathbf{H}_2)\Big[\frac{IK(\mathbf{D}_J - \mathbf{H}_2)}{\sigma_E^2 + IK\sigma_O^2}\Big](\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{y} \qquad (2.2.5)$$

$$= \mathbf{y}_2'\Big[\frac{IK(\mathbf{D}_J - \mathbf{H}_2)}{\sigma_E^2 + IK\sigma_O^2}\Big]\mathbf{y}_2$$

and noting

$$\Big[\frac{IK(\mathbf{D}_J - \mathbf{H}_2)}{\sigma_E^2 + IK\sigma_O^2}\Big]V(\mathbf{y}_2)$$

$$= \Big[\frac{IK(\mathbf{D}_J - \mathbf{H}_2)}{\sigma_E^2 + IK\sigma_O^2}\Big]\frac{1}{IK}(\sigma_E^2 + IK\sigma_O^2)(\mathbf{D}_J - \mathbf{H}_2)$$

$$= (\mathbf{D}_J - \mathbf{H}_2),$$

$$\frac{1}{2}E(\mathbf{y}_2)'\Big[\frac{IK(\mathbf{D}_J - \mathbf{H}_2)}{\sigma_E^2 + IK\sigma_O^2}\Big]E(\mathbf{y}_2)$$

$$= \frac{1}{2}\underline{\alpha}'\mathbf{X}'\mathbf{W}_2'(\mathbf{D}_J - \mathbf{H}_2)\Big[\frac{IK(\mathbf{D}_J - \mathbf{H}_2)}{\sigma_E^2 + IK\sigma_O^2}\Big](\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{X}\underline{\alpha}$$

$$= 0, \quad \text{and}$$

$$r(\mathbf{D}_J - \mathbf{H}_2) = J - 2.$$

By Theorem 7.3 in Searle(1987, p. 232), $R_2/(\sigma_E^2 + IK\sigma_O^2)$ is a chi-squared random variable with $J - 2$ degrees of freedom. ■

Theorem 2.2.3 : Under the distributional assumptions in (2.1.1), R_3/σ_E^2 is a chi-squared random variable with $IJK - I - J$ degrees of freedom.

Proof : Multiplying (2.2.1) both sides of the equation on the left by $(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3$ yields that

$$\begin{aligned}(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{y} =& \ (\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{X}\underline{\alpha} + (\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{Z}_1\mathbf{p} \\ &+ \ (\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{Z}_2\mathbf{o} + (\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{Z}_3\mathbf{e}.\end{aligned}$$

It can be shown that

$$\begin{aligned}(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{X} =& \ (\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{X} \\ =& \ \mathbf{X} - \mathbf{H}_3\mathbf{X} \\ =& \ \mathbf{0}, \\ \mathbf{H}_3\mathbf{W}_3\mathbf{Z}_1 =& \ \mathbf{Z}_1, \text{ and} \\ \mathbf{H}_3\mathbf{W}_3\mathbf{Z}_2 =& \ \mathbf{Z}_2\end{aligned} \tag{2.2.6}$$

since

$$\begin{aligned}\mathbf{W}_3 =& \ \mathbf{Z}_3 = \ \mathbf{D}_{IJK}, \\ \mathbf{H}_3\mathbf{X} =& \ \mathbf{X}_3(\mathbf{X}_3'\mathbf{X}_3)^-\mathbf{X}_3'\mathbf{X} = \ \mathbf{X}, \\ \mathbf{H}_3\mathbf{Z}_1 =& \ \mathbf{X}_3(\mathbf{X}_3'\mathbf{X}_3)^-\mathbf{X}_3'\mathbf{Z}_1 = \ \mathbf{Z}_1, \text{ and} \\ \mathbf{H}_3\mathbf{Z}_2 =& \ \mathbf{X}_3(\mathbf{X}_3'\mathbf{X}_3)^-\mathbf{X}_3'\mathbf{Z}_2 = \ \mathbf{Z}_2.\end{aligned}$$

Let $(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{y} = \mathbf{y}_3$. The variance of $\mathbf{y}_3$ is

$$\begin{aligned}V(\mathbf{y}_3) =& \ V((\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{X}\underline{\alpha}) + V((\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{Z}_1\mathbf{p}) \\ &+ V((\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{Z}_2\mathbf{o}) + V((\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{Z}_3\mathbf{e}) \\ =& \ V((\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{Z}_3\mathbf{e}) \\ =& \ \sigma_E^2(\mathbf{D}_{IJK} - \mathbf{H}_3).\end{aligned}$$

The distribution of R_3 is determined by writing

$$\begin{aligned} \frac{R_3}{\sigma_E^2} &= \mathbf{y}'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\Big[\frac{(\mathbf{D}_{IJK} - \mathbf{H}_3)}{\sigma_E^2}\Big](\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{y} \quad (2.2.7) \\ &= \mathbf{y}_3'\Big[\frac{(\mathbf{D}_{IJK} - \mathbf{H}_3)}{\sigma_E^2}\Big]\mathbf{y}_3 \end{aligned}$$

and noting

$$\begin{aligned} &\Big[\frac{(\mathbf{D}_{IJK} - \mathbf{H}_3)}{\sigma_E^2}\Big]V(\mathbf{y}_3) \\ &= \Big[\frac{(\mathbf{D}_{IJK} - \mathbf{H}_3)}{\sigma_E^2}\Big]\sigma_E^2(\mathbf{D}_{IJK} - \mathbf{H}_3) \\ &= \mathbf{D}_{IJK} - \mathbf{H}_3, \end{aligned}$$

$$\begin{aligned} &\frac{1}{2}E(\mathbf{y}_3)'\Big[\frac{(\mathbf{D}_{IJK} - \mathbf{H}_3)}{\sigma_E^2}\Big]E(\mathbf{y}_3) \\ &= \frac{1}{2}\underline{\alpha}'\mathbf{X}'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\Big[\frac{(\mathbf{D}_{IJK} - \mathbf{H}_3)}{\sigma_E^2}\Big](\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3\mathbf{X}\underline{\alpha} \\ &= 0, \quad \text{and} \end{aligned}$$

$$r(\mathbf{D}_{IJK} - \mathbf{H}_3) = IJK - I - J.$$

By Theorem 7.3 in Searle(1987, p. 232), R_3/σ_E^2 is a chi-squared random variable with $IJK - I - J$ degrees of freedom. ■

<u>*Theorem 2.2.4*</u> : Under the distributional assumptions in (2.1.1), $R_1/(\sigma_E^2 + JK\sigma_P^2)$ and R_3/σ_E^2 are independent and $R_2/(\sigma_E^2 + IK\sigma_O^2)$ and R_3/σ_E^2 are independent.

Proof : It can be shown from (2.2.3) and (2.2.7) that

$$
\begin{aligned}
& \mathbf{W}_1'(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1 V(\mathbf{y})\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3 \\
= \ & \sigma_P^2 \mathbf{W}_1'(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{Z}_1\mathbf{Z}_1'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3 \\
& + \sigma_O^2 \mathbf{W}_1'(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{Z}_2\mathbf{Z}_2'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3 \\
& + \sigma_E^2 \mathbf{W}_1'(\mathbf{D}_I - \mathbf{H}_1)\mathbf{W}_1\mathbf{Z}_3\mathbf{Z}_3'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3 \\
= \ & \mathbf{0}
\end{aligned}
$$

since

$$
\begin{aligned}
\mathbf{Z}_1'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3) &= \mathbf{Z}_1' - \mathbf{Z}_1'\mathbf{H}_3 \\
&= \mathbf{Z}_1' - \mathbf{Z}_1' \\
&= \mathbf{0}, \\
\mathbf{Z}_2'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3) &= \mathbf{Z}_2' - \mathbf{Z}_2'\mathbf{H}_3 \\
&= \mathbf{Z}_2' - \mathbf{Z}_2' \\
&= \mathbf{0}, \quad \text{and} \\
\mathbf{W}_1\mathbf{Z}_3\mathbf{Z}_3'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3 &= \mathbf{W}_1(\mathbf{D}_{IJK} - \mathbf{H}_3) \\
&= \frac{1}{JK}\mathbf{Z}_1'(\mathbf{D}_{IJK} - \mathbf{H}_3) \\
&= \frac{1}{JK}(\mathbf{Z}_1' - \mathbf{Z}_1'\mathbf{H}_3) \\
&= \frac{1}{JK}(\mathbf{Z}_1' - \mathbf{Z}_1') \\
&= \mathbf{0}.
\end{aligned}
$$

By Theorem 7.4 in Searle(1987, p. 232), $R_1/(\sigma_E^2 + JK\sigma_P^2)$ and R_3/σ_E^2 are independent.

Similarly, it can be shown from (2.2.5) and (2.2.7) that

$$\begin{aligned}
&\mathbf{W}_2'(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2 V(\mathbf{y})\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3 \\
=\ & \sigma_P^2 \mathbf{W}_2'(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{Z}_1\mathbf{Z}_1'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3 \\
& + \sigma_O^2 \mathbf{W}_2'(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{Z}_2\mathbf{Z}_2'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3 \\
& + \sigma_E^2 \mathbf{W}_2'(\mathbf{D}_J - \mathbf{H}_2)\mathbf{W}_2\mathbf{Z}_3\mathbf{Z}_3'\mathbf{W}_3'(\mathbf{D}_{IJK} - \mathbf{H}_3)\mathbf{W}_3 \\
=\ & \mathbf{0}.
\end{aligned}$$

By Theorem 7.4 in Searle(1987, p. 232), $R_2/(\sigma_E^2 + IK\sigma_O^2)$ and R_3/σ_E^2 are independent. ■

Using the results of Theorems 2.2, the expected mean squares are

$$E(S_P^2) = \sigma_E^2 + JK\sigma_P^2 = \theta_P, \tag{2.2.8a}$$

$$E(S_O^2) = \sigma_E^2 + IK\sigma_O^2 = \theta_O, \text{ and} \tag{2.2.8b}$$

$$E(S_E^2) = \sigma_E^2 = \theta_E. \tag{2.2.8c}$$

2.3 Confidence Intervals on Variance Components Using Ting et al. Method

We use the distributional results of Section 2.2 to construct confidence intervals on σ_P^2, σ_O^2, and σ_E^2. Since $R_3/\sigma_E^2 \sim \chi^2_{IJK-I-J}$, an exact confidence interval on σ_E^2 is obtained. An exact $100(1-\alpha)\%$ two-sided confidence interval on σ_E^2 is

$$\left[\frac{S_E^2}{F_{(\alpha/2:IJK-I-J,\infty)}} \quad ; \quad \frac{S_E^2}{F_{(1-\alpha/2:IJK-I-J,\infty)}}\right] \tag{2.3.1}$$

where $F_{(\delta:\nu_1,\nu 2)}$ is the F-value for ν_1 and ν_2 degrees of freedom with δ area to the right.

The variance component of σ_P^2 can be represented by (2.2.8)

$$\sigma_P^2 = \frac{\theta_P - \theta_E}{JK}. \tag{2.3.2}$$

Ting, Burdick, Graybill, Jeyaratnam, and Lu (1990) proposed a general method for constructing confidence intervals on $\gamma = \Sigma_{q=1}^{P} c_q\theta_q - \Sigma_{r=1}^{Q} c_r\theta_r$ where $c_q, c_r \geq 0$, and the sign of γ is unknown. The intervals are extended to the general cases where $Q, P > 2$. Their two-sided intervals are very close to the stated confidence coefficients. Thus a confidence interval on σ_P^2 can be constructed using the method of Ting et al.(1990). An approximate $100(1-\alpha)\%$ two-sided confidence interval on σ_P^2 is

$$\frac{1}{JK}\left[\ S_P^2 - S_E^2 - (G_1^2 S_P^4 + H_2^2 S_E^4 + G_{12} S_P^2 S_E^2)^{\frac{1}{2}}\ ;\right.$$

$$\left. S_P^2 - S_E^2 + (H_1^2 S_P^4 + G_2^2 S_E^4 + H_{12} S_P^2 S_E^2)^{\frac{1}{2}}\ \right] \tag{2.3.3}$$

where

$$G_1 = 1 - \frac{1}{F_{(\frac{\alpha}{2}:I-2,\infty)}},$$

$$H_2 = \frac{1}{F_{(1-\frac{\alpha}{2}:IJK-I-J,\infty)}} - 1,$$

$$G_{12} = \frac{(F_1 - 1)^2 - G_1^2 F_1^2 - H_2^2}{F_1},$$

$$H_1 = \frac{1}{F_{(1-\frac{\alpha}{2}:I-2,\infty)}} - 1,$$

$$G_2 = 1 - \frac{1}{F_{(\frac{\alpha}{2}:IJK-I-J,\infty)}},$$

$$H_{12} = \frac{(1 - F_2)^2 - H_1^2 F_2^2 - G_2^2}{F_2},$$

$$F_1 = F_{(\frac{\alpha}{2}:I-2,IJK-I-J)}, \quad \text{and}$$

$$F_2 = F_{(1-\frac{\alpha}{2}:I-2,IJK-I-J)}.$$

Since $\sigma_P^2 > 0$, any negative bound is defined to be zero. The variance component of σ_O^2 can be represented by (2.2.8)

$$\sigma_O^2 = \frac{\theta_O - \theta_E}{IK}. \tag{2.3.4}$$

The same Ting et al. method can be applied to form a confidence interval on σ_O^2. In particular, an approximate $100(1-\alpha)\%$ two-sided confidence interval on σ_O^2 is

$$\frac{1}{IK}[\ S_O^2 - S_E^2 - (G_3^2 S_O^4 + H_4^2 S_E^4 + G_{34} S_O^2 S_E^2)^{\frac{1}{2}}\ ;$$

$$S_O^2 - S_E^2 + (H_3^2 S_O^4 + G_4^2 S_E^4 + H_{34} S_O^2 S_E^2)^{\frac{1}{2}}] \qquad (2.3.5)$$

where

$$G_3 = 1 - \frac{1}{F_{(\frac{\alpha}{2}:J-2,\infty)}},$$

$$H_4 = \frac{1}{F_{(1-\frac{\alpha}{2}:IJK-I-J,\infty)}} - 1 = H_2,$$

$$G_{34} = \frac{(F_3 - 1)^2 - G_3^2 F_3^2 - H_4^2}{F_3},$$

$$H_3 = \frac{1}{F_{(1-\frac{\alpha}{2}:J-2,\infty)}} - 1,$$

$$G_4 = 1 - \frac{1}{F_{(\frac{\alpha}{2}:IJK-I-J,\infty)}} = G_2,$$

$$H_{34} = \frac{(1 - F_4)^2 - H_3^2 F_4^2 - G_4^2}{F_4},$$

$$F_3 = F_{(\frac{\alpha}{2}:J-2,IJK-I-J)}, \quad \text{and}$$

$$F_4 = F_{(1-\frac{\alpha}{2}:J-2,IJK-I-J)}.$$

TING method refers to confidence intervals using (2.3.3) and (2.3.5).

2.4 Generalized Confidence Intervals on Variance Components

Tsui and Weerahandi(1989) introduced the concept on generalized inference for testing hypotheses in situations where exact method do not exist. Their method can be applied to form approximate confidence intervals on variance components. Since an exact $100(1-\alpha)\%$ two-sided confidence interval on σ_E^2 exists, we focus on constructing confidence intervals on σ_P^2 and σ_O^2. Since $R_1/(\sigma_E^2+JK\sigma_P^2) \sim \chi_{I-2}^2$, σ_P^2 can be written as a generalized pivotal quantity as follows:

$$\sigma_P^2 = \frac{1}{JK}\Big[\ \frac{(I-2)}{P^*}s_P^2 - \frac{(IJK-I-J)}{E^*}s_E^2\ \Big] \tag{2.4.1}$$

where s_P^2 and s_E^2 are, respectively, observed values of S_P^2 and S_E^2, $P^* = (I-2)S_P^2/(\sigma_E^2+JK\sigma_P^2)$, and $E^* = (IJK-I-J)S_E^2/\sigma_E^2$. Define R_P as the solution for σ_P^2. The distribution of R_P is completely determined by P^* and E^*. An approximate $100(1-\alpha)\%$ two-sided confidence interval on σ_P^2 is

$$\Big[R_{P_{(\frac{\alpha}{2})}} \quad ; \quad R_{P_{(\frac{1-\alpha}{2})}}\Big] \tag{2.4.2}$$

where $R_{P_{(\frac{\alpha}{2})}}$ and $R_{P_{(\frac{1-\alpha}{2})}}$ are the percentile of $\alpha/2$ and $1-\alpha/2$ of the distribution R_P, respectively.

Similarly, we can form confidence intervals on σ_O^2 using generalized pivotal quantity. Since $R_2/(\sigma_E^2+IK\sigma_O^2) \sim \chi_{J-2}^2$, σ_O^2 can be written as a generalized pivotal quantity as follows:

$$\sigma_O^2 = \frac{1}{IK}\Big[\ \frac{(J-2)}{O^*}s_O^2 - \frac{(IJK-I-J)}{E^*}s_E^2\ \Big] \tag{2.4.3}$$

where s_O^2 is an observed value of S_O^2 and $O^* = (J-2)S_O^2/(\sigma_E^2 + IK\sigma_O^2)$. Define R_O as the solution for σ_O^2. The distribution of R_O is completely determined by O^* and E^*. An approximate $100(1-\alpha)\%$ two-sided confidence interval on σ_O^2 is

$$\left[R_{O_{(\frac{\alpha}{2})}} \quad ; \quad R_{O_{(\frac{1-\alpha}{2})}}\right] \tag{2.4.4}$$

where $R_{O_{(\frac{\alpha}{2})}}$ and $R_{O_{(\frac{1-\alpha}{2})}}$ are the percentile of $\alpha/2$ and $1-\alpha/2$ of the distribution R_O, respectively. The GEN method refers to confidence intervals using (2.4.2) and (2.4.4).

2.5 Confidence Intervals on Variance Components Using SAS PROC MIXED Procedure

SAS PROC MIXED procedure can be employed to fit a variety of mixed linear models to data. It enables one to use these fitted models to make statistical inferences about data. PROC MIXED is a generalization of the GLM procedure in the sense that PROC GLM fits standard linear models and PROC MIXED fits wider class of mixed linear models. PROC MIXED fits the data using the method of restricted maximum likelihood(REML), also known as residual maximum likelihood. The procedure permits one to exhibit asymptotic covariances among variance components by using options of MIXED procedure.

In order to form confidence intervals on σ_P^2 and σ_O^2 using REML estimator, a PROC MIXED statement such as TABLE 2.5.1. can be performed.

TABLE 2.5.1 An Example Statement of PROC MIXED

```
PROC MIXED   DATA=EXAMPLE   ASYCOV   CL   ALPHA=0.10 ;
CLASSES   SIGMAP   SIGMAO;
MODEL   Y = X;
RANDOM   SIGMAP   SIGMAO;
```

where EXAMPLE is a data set name, ASYCOV requests asymptotic covariances among variance components, and CL and ALPHA=0.10 produce 90 % confidence intervals using REML estimators of σ_P^2 and σ_O^2. CLASSES SIGMAP SIGMAO names classification variables and RANCOM SIGMAP SIGMAO defines random effects in this model.

As a result of executing PROC MIXED, an approximate confidence interval on σ_P^2 is obtained using Wald Statistics. The interval is computed as

$$\left[\frac{\hat{\sigma}_P^2}{F_{(1-\alpha/2:\nu,\infty)}}\ ;\ \frac{\hat{\sigma}_P^2}{F_{(\alpha/2:\nu,\infty)}}\right] \tag{2.5.1}$$

where $\hat{\sigma}_P^2$ is the REML estimate of σ_P^2, $\nu = 2Z^2$, Z is the Wald Z score,

$$Z = \frac{\hat{\sigma}_P^2}{S.E.(\hat{\sigma}_P^2)}, \tag{2.5.2}$$

and $S.E.$ stands for standard error.

Similarly, an approximate confidence interval on σ_O^2 is obtained using Wald Statistics. The interval is computed as

$$\left[\frac{\hat{\sigma}_O^2}{F_{(1-\alpha/2:\nu,\infty)}}\ ;\ \frac{\hat{\sigma}_O^2}{F_{(\alpha/2:\nu,\infty)}}\right] \tag{2.5.3}$$

where $\hat{\sigma}_O^2$ is the REML estimate of σ_O^2, $\nu = 2Z^2$, and Z is the Wald Z score. MIXED method refers to confidence intervals using (2.5.1) and (2.5.3).

3. SIMULATION AND CONCLUSION

3.1 Simulation Study

We compare the confidence intervals on variance components proposed from Sections 2.3 to 2.5. Computer simulation is used to compare the confidence coefficients and expected interval lengths. The criteria for analyzing the performance of the methods are: i) their ability to obtain stated confidence coefficients, and ii) the average length of two-sided confidence intervals. Although shorter average lengths are preferable, it is necessary that the methods maintain the stated confidence coefficients. Dolezal et al.(1998) used eight designs with several combinations of I, J, and K for two-factor R & R study with fixed operators. Four more designs are added to their designs and the designs shown in Table 3.1.1 are used for simulation study.

TABLE 3.1.1 Eight Designs Used in Simulation Study

I	J	K	I	J	K	I	J	K
6	3	2	12	3	2	24	3	2
6	3	4	12	3	4	24	3	4
6	6	2	12	6	2	24	6	2
6	6	4	12	6	4	24	6	4

Let $\rho = \frac{\sigma_P^2}{\sigma_P^2+\sigma_O^2+\sigma_E^2}$. Without loss of generality $\sigma_P^2 = 1-\sigma_O^2-\sigma_E^2$. The values of σ_P^2 and σ_O^2 are varied form 0.1 to 0.8 in increments of 0.1. In particular, the values used in this study are presented in Table 3.1.2

TABLE 3.1.2 Parameter Values Used in Simulation Study

σ_P^2	σ_O^2	σ_E^2	σ_P^2	σ_O^2	σ_E^2	σ_P^2	σ_O^2	σ_E^2
0.1	0.1	0.8	0.2	0.5	0.3	0.4	0.4	0.2
0.1	0.2	0.7	0.2	0.6	0.2	0.4	0.5	0.1
0.1	0.3	0.6	0.2	0.7	0.1	0.5	0.1	0.4
0.1	0.4	0.5	0.3	0.1	0.6	0.5	0.2	0.3
0.1	0.5	0.4	0.3	0.2	0.5	0.5	0.3	0.2
0.1	0.6	0.3	0.3	0.3	0.4	0.5	0.4	0.1
0.1	0.7	0.2	0.3	0.4	0.3	0.6	0.1	0.3
0.1	0.8	0.1	0.3	0.5	0.2	0.6	0.2	0.2
0.2	0.1	0.7	0.3	0.6	0.1	0.6	0.3	0.1
0.2	0.2	0.6	0.4	0.1	0.5	0.7	0.1	0.2
0.2	0.3	0.5	0.4	0.2	0.4	0.7	0.2	0.1
0.2	0.4	0.4	0.4	0.3	0.3	0.8	0.1	0.1

36 combinations of σ_P^2, σ_O^2, and σ_E^2 in Table 3.1.2 are simulated for 2000 times for each design in Table 3.1.1. Simulated values for S_P^2, S_O^2, and S_E^2 are substituted into TING, GEN, and MIXED methods in Section 2.3 to 2.5 and the intervals are computed. Confidence coefficients are determined by counting the number of the intervals that contain the parameters. Using the normal approximation to the binomial, if the true confidence coefficient is 0.90, there is less than a 2.5 % chance that an estimated confidence coefficient based on 2000 replications will be less than 0.8866. The average lengths of the two-sided confidence intervals are calculated.

3.2 Simulation Results

Tables 3.2.1 to 3.2.12 and Tables 3.2.13 to 3.2.24 present simulated confidence coefficients for stated two-sided 90% confidence intervals on σ_P^2 and σ_O^2, respectively. Tables 3.2.25 to 3.2.36 and Tables 3.2.37 to 3.2.48 report average confidence interval lengths for stated two-sided 90% confidence intervals on σ_P^2 and σ_O^2, respectively. Figures 3.2.1 and 3.2.2 show boxplots for simulated confidence coefficients for stated two-sided 90% confidence intervals on σ_P^2 and σ_O^2, respectively. Figures 3.2.3 and 3.2.4 show boxplots for average confidence interval lengths for stated two-sided 90% confidence intervals on σ_P^2 and σ_O^2, respectively. The boxplots for MIXED method were not presented because it generates too much wide average confidence interval lengths when PROC MIXED executes using the data generated by random numbers for 2000 times in simulation study.

It is known from Figures 3.2.1 that TING and GEN methods for confidence intervals on σ_P^2 generally maintain the stated confidence coefficients for all combinations of I, J, and K. However, MIXED method in general generates conservative confidence intervals, comparing with TING and GEN. The performance of MIXED slightly improves as J and K increase. MIXED dramatically performs better especially as I increases. MIXED produces slightly larger simulated confidence coefficients than TING and GEN especially for $I = 24$, $J = 6$, and $K = 4$. This is because MIXED uses Wald Z statistics in computing confidence intervals which is valid for large samples.

From Figures 3.2.2 simulated confidence coefficients for σ_O^2 show similar trend to ones for σ_P^2. It is known from Figures 3.2.2 that TING and GEN methods for confidence intervals on σ_O^2 generally maintain the stated confidence coefficients for all combinations of I, J, and K. However, MIXED method in general generates conservative confidence intervals, comparing with TING and GEN. The performance of MIXED slightly improves as J and K increase.

It can be noticed from Figures 3.2.3 and 3.2.4 that TING and GEN methods for average confidence interval lengths on σ_P^2 and σ_O^2 are very similar to each other for all combinations of I, J, and K. MIXED method was eliminated since it produces too much wide average confidence interval lengths.

TABLE 3.2.1 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I = 6,\ J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9070	0.9010	0.7872	0.3	0.5	0.2	0.8880	0.9015	0.9446
0.1	0.2	0.7	0.9060	0.8945	0.8170	0.3	0.6	0.1	0.8945	0.9040	0.9449
0.1	0.3	0.6	0.9135	0.9065	0.8653	0.4	0.1	0.5	0.8940	0.8915	0.9335
0.1	0.4	0.5	0.9035	0.9020	0.8577	0.4	0.2	0.4	0.9045	0.9100	0.9410
0.1	0.5	0.4	0.8990	0.9085	0.8663	0.4	0.3	0.3	0.8985	0.9055	0.9415
0.1	0.6	0.3	0.8935	0.8960	0.8926	0.4	0.4	0.2	0.9025	0.9065	0.9406
0.1	0.7	0.2	0.9065	0.9015	0.9269	0.4	0.5	0.1	0.8895	0.9030	0.9328
0.1	0.8	0.1	0.8910	0.9055	0.9303	0.5	0.1	0.4	0.9010	0.9080	0.9509
0.2	0.1	0.7	0.8995	0.9135	0.8924	0.5	0.2	0.3	0.8945	0.8960	0.9406
0.2	0.2	0.6	0.9125	0.9025	0.8894	0.5	0.3	0.2	0.8985	0.9025	0.9477
0.2	0.3	0.5	0.9025	0.9130	0.9149	0.5	0.4	0.1	0.8980	0.9070	0.9379
0.2	0.4	0.4	0.9075	0.9025	0.9150	0.6	0.1	0.3	0.9155	0.9055	0.9518
0.2	0.5	0.3	0.9085	0.8930	0.9298	0.6	0.2	0.2	0.8935	0.9055	0.9453
0.2	0.6	0.2	0.9000	0.9010	0.9301	0.6	0.3	0.1	0.8915	0.8990	0.9170
0.2	0.7	0.1	0.8945	0.8930	0.9437	0.7	0.1	0.2	0.8980	0.9005	0.9435
0.3	0.1	0.6	0.9035	0.9075	0.9178	0.7	0.2	0.1	0.8985	0.8945	0.9095
0.3	0.2	0.5	0.8920	0.8985	0.9238	0.8	0.1	0.1	0.8900	0.9100	0.9130
0.3	0.3	0.4	0.9025	0.9060	0.9332		MAX		0.9155	0.9135	0.9518
0.3	0.4	0.3	0.8995	0.9025	0.9279		MIN		0.8880	0.8915	0.7872

TABLE 3.2.2 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I=6,\ J=3$, and $K=4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9000	0.8950	0.8736	0.3	0.5	0.2	0.9000	0.8940	0.9469
0.1	0.2	0.7	0.9090	0.8930	0.8866	0.3	0.6	0.1	0.9090	0.9020	0.9244
0.1	0.3	0.6	0.8985	0.9005	0.9070	0.4	0.1	0.5	0.9015	0.9100	0.9411
0.1	0.4	0.5	0.9060	0.8925	0.9092	0.4	0.2	0.4	0.9085	0.8985	0.9483
0.1	0.5	0.4	0.9045	0.9035	0.9180	0.4	0.3	0.3	0.8960	0.9025	0.9464
0.1	0.6	0.3	0.8990	0.8940	0.9258	0.4	0.4	0.2	0.8930	0.8955	0.9434
0.1	0.7	0.2	0.8965	0.8940	0.9324	0.4	0.5	0.1	0.8990	0.8955	0.9180
0.1	0.8	0.1	0.8915	0.9030	0.9398	0.5	0.1	0.4	0.9055	0.8990	0.9423
0.2	0.1	0.7	0.9010	0.9030	0.9253	0.5	0.2	0.3	0.8990	0.8875	0.9429
0.2	0.2	0.6	0.8995	0.8940	0.9319	0.5	0.3	0.2	0.8965	0.8990	0.9340
0.2	0.3	0.5	0.8985	0.9145	0.9329	0.5	0.4	0.1	0.9180	0.8980	0.9065
0.2	0.4	0.4	0.9090	0.9095	0.9338	0.6	0.1	0.3	0.9050	0.9005	0.9409
0.2	0.5	0.3	0.8955	0.9060	0.9396	0.6	0.2	0.2	0.9065	0.8940	0.9290
0.2	0.6	0.2	0.9015	0.9080	0.9412	0.6	0.3	0.1	0.9110	0.9115	0.9145
0.2	0.7	0.1	0.9040	0.9090	0.9414	0.7	0.1	0.2	0.9055	0.8880	0.9260
0.3	0.1	0.6	0.8975	0.8975	0.9369	0.7	0.2	0.1	0.9060	0.9045	0.8975
0.3	0.2	0.5	0.8945	0.8965	0.9316	0.8	0.1	0.1	0.9045	0.8995	0.8945
0.3	0.3	0.4	0.9005	0.8995	0.9411		MAX		0.9180	0.9145	0.9483
0.3	0.4	0.3	0.9045	0.9070	0.9373		MIN		0.8915	0.8875	0.8736

TABLE 3.2.3 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I=6,\ J=6,$ and $K=2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8975	0.8995	0.8836	0.3	0.5	0.2	0.9070	0.9045	0.9474
0.1	0.2	0.7	0.9005	0.9165	0.8916	0.3	0.6	0.1	0.9070	0.9140	0.9355
0.1	0.3	0.6	0.9020	0.9080	0.8997	0.4	0.1	0.5	0.9030	0.8960	0.9349
0.1	0.4	0.5	0.8925	0.9085	0.9147	0.4	0.2	0.4	0.8925	0.8985	0.9487
0.1	0.5	0.4	0.8960	0.9070	0.9177	0.4	0.3	0.3	0.9075	0.9010	0.9538
0.1	0.6	0.3	0.8890	0.8965	0.9294	0.4	0.4	0.2	0.9070	0.8845	0.9355
0.1	0.7	0.2	0.9150	0.8900	0.9327	0.4	0.5	0.1	0.9005	0.8925	0.9205
0.1	0.8	0.1	0.9015	0.9000	0.9443	0.5	0.1	0.4	0.8970	0.9135	0.9402
0.2	0.1	0.7	0.9010	0.9000	0.9207	0.5	0.2	0.3	0.9125	0.8965	0.9489
0.2	0.2	0.6	0.8860	0.9045	0.9247	0.5	0.3	0.2	0.8995	0.8920	0.9439
0.2	0.3	0.5	0.9065	0.8910	0.9337	0.5	0.4	0.1	0.9035	0.8895	0.9180
0.2	0.4	0.4	0.9000	0.9045	0.9354	0.6	0.1	0.3	0.8995	0.9075	0.9474
0.2	0.5	0.3	0.8885	0.8950	0.9406	0.6	0.2	0.2	0.9145	0.9060	0.9385
0.2	0.6	0.2	0.8980	0.9025	0.9487	0.6	0.3	0.1	0.9020	0.8980	0.9065
0.2	0.7	0.1	0.9045	0.9065	0.9394	0.7	0.1	0.2	0.9000	0.9025	0.9095
0.3	0.1	0.6	0.8985	0.9045	0.9290	0.7	0.2	0.1	0.8995	0.8910	0.9030
0.3	0.2	0.5	0.8910	0.8920	0.9377	0.8	0.1	0.1	0.8980	0.9060	0.9145
0.3	0.3	0.4	0.9015	0.8955	0.9428		MAX		0.9150	0.9165	0.9538
0.3	0.4	0.3	0.8890	0.9045	0.9417		MIN		0.8860	0.8845	0.8836

TABLE 3.2.4 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I = 6,\ J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8920	0.8965	0.9227	0.3	0.5	0.2	0.8900	0.8890	0.9205
0.1	0.2	0.7	0.9095	0.8860	0.9242	0.3	0.6	0.1	0.8965	0.8985	0.9019
0.1	0.3	0.6	0.9065	0.8990	0.9297	0.4	0.1	0.5	0.9015	0.9100	0.9409
0.1	0.4	0.5	0.8975	0.8990	0.9285	0.4	0.2	0.4	0.9040	0.8995	0.9479
0.1	0.5	0.4	0.8970	0.8870	0.9390	0.4	0.3	0.3	0.8995	0.9115	0.9219
0.1	0.6	0.3	0.8985	0.8975	0.9464	0.4	0.4	0.2	0.9010	0.9115	0.9130
0.1	0.7	0.2	0.9010	0.8970	0.9424	0.4	0.5	0.1	0.8980	0.9030	0.9130
0.1	0.8	0.1	0.8980	0.9015	0.9449	0.5	0.1	0.4	0.8965	0.9115	0.9430
0.2	0.1	0.7	0.8900	0.8980	0.9333	0.5	0.2	0.3	0.8985	0.9035	0.9265
0.2	0.2	0.6	0.8925	0.9010	0.9358	0.5	0.3	0.2	0.8920	0.8985	0.9015
0.2	0.3	0.5	0.8975	0.9000	0.9373	0.5	0.4	0.1	0.9205	0.9075	0.9065
0.2	0.4	0.4	0.8915	0.8900	0.9448	0.6	0.1	0.3	0.8965	0.9050	0.9164
0.2	0.5	0.3	0.8995	0.8880	0.9593	0.6	0.2	0.2	0.8960	0.8875	0.9090
0.2	0.6	0.2	0.8995	0.9005	0.9539	0.6	0.3	0.1	0.9075	0.8970	0.9030
0.2	0.7	0.1	0.8995	0.9080	0.9189	0.7	0.1	0.2	0.9105	0.8985	0.8960
0.3	0.1	0.6	0.9060	0.9065	0.9498	0.7	0.2	0.1	0.8885	0.9050	0.8985
0.3	0.2	0.5	0.8975	0.9015	0.9443	0.8	0.1	0.1	0.9005	0.8990	0.9040
0.3	0.3	0.4	0.9065	0.8865	0.9524		MAX		0.9205	0.9115	0.9593
0.3	0.4	0.3	0.9035	0.9055	0.9514		MIN		0.8885	0.8860	0.8960

TABLE 3.2.5 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I = 12$, $J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9010	0.9030	0.8568	0.3	0.5	0.2	0.8995	0.9075	0.9355
0.1	0.2	0.7	0.9050	0.9015	0.8739	0.3	0.6	0.1	0.8900	0.9025	0.9195
0.1	0.3	0.6	0.9000	0.9050	0.8795	0.4	0.1	0.5	0.9075	0.9015	0.9444
0.1	0.4	0.5	0.9005	0.8910	0.8884	0.4	0.2	0.4	0.9005	0.8975	0.9389
0.1	0.5	0.4	0.8975	0.9050	0.9067	0.4	0.3	0.3	0.9020	0.8960	0.9325
0.1	0.6	0.3	0.8935	0.8915	0.9374	0.4	0.4	0.2	0.8920	0.8860	0.9225
0.1	0.7	0.2	0.8995	0.9010	0.9231	0.4	0.5	0.1	0.9205	0.8920	0.9195
0.1	0.8	0.1	0.9030	0.8980	0.9370	0.5	0.1	0.4	0.9030	0.9050	0.9504
0.2	0.1	0.7	0.9110	0.8980	0.9064	0.5	0.2	0.3	0.9070	0.8975	0.9345
0.2	0.2	0.6	0.8970	0.9075	0.9220	0.5	0.3	0.2	0.9075	0.9195	0.9175
0.2	0.3	0.5	0.9010	0.9035	0.9178	0.5	0.4	0.1	0.9005	0.9075	0.9100
0.2	0.4	0.4	0.8960	0.8915	0.9323	0.6	0.1	0.3	0.9040	0.9060	0.9170
0.2	0.5	0.3	0.9020	0.8980	0.9369	0.6	0.2	0.2	0.9060	0.9160	0.9170
0.2	0.6	0.2	0.8870	0.9050	0.9449	0.6	0.3	0.1	0.9040	0.9015	0.9055
0.2	0.7	0.1	0.9030	0.9000	0.9275	0.7	0.1	0.2	0.8985	0.8880	0.8955
0.3	0.1	0.6	0.9060	0.9110	0.9284	0.7	0.2	0.1	0.8930	0.9050	0.9035
0.3	0.2	0.5	0.8940	0.9040	0.9402	0.8	0.1	0.1	0.9080	0.9010	0.9125
0.3	0.3	0.4	0.8940	0.8970	0.9389		MAX		0.9205	0.9195	0.9504
0.3	0.4	0.3	0.9005	0.8920	0.9335		MIN		0.8870	0.8860	0.8568

TABLE 3.2.6 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I = 12$, $J = 3$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8940	0.9010	0.9193	0.3	0.5	0.2	0.8955	0.9025	0.9140
0.1	0.2	0.7	0.9060	0.9060	0.9191	0.3	0.6	0.1	0.8905	0.9000	0.9085
0.1	0.3	0.6	0.8985	0.8865	0.9285	0.4	0.1	0.5	0.9010	0.9005	0.9250
0.1	0.4	0.5	0.9070	0.8880	0.9211	0.4	0.2	0.4	0.9110	0.9030	0.9270
0.1	0.5	0.4	0.9030	0.8900	0.9290	0.4	0.3	0.3	0.9085	0.9040	0.9205
0.1	0.6	0.3	0.8965	0.8985	0.9368	0.4	0.4	0.2	0.8935	0.9045	0.9155
0.1	0.7	0.2	0.9000	0.9020	0.9430	0.4	0.5	0.1	0.9000	0.8985	0.9015
0.1	0.8	0.1	0.9085	0.9035	0.9260	0.5	0.1	0.4	0.8930	0.9005	0.9204
0.2	0.1	0.7	0.8970	0.9090	0.9267	0.5	0.2	0.3	0.8995	0.8940	0.9200
0.2	0.2	0.6	0.9025	0.8990	0.9317	0.5	0.3	0.2	0.8945	0.8990	0.9075
0.2	0.3	0.5	0.8965	0.8945	0.9354	0.5	0.4	0.1	0.8985	0.9070	0.8995
0.2	0.4	0.4	0.9050	0.8990	0.9455	0.6	0.1	0.3	0.9005	0.8980	0.9135
0.2	0.5	0.3	0.9125	0.9060	0.9385	0.6	0.2	0.2	0.9115	0.9015	0.9065
0.2	0.6	0.2	0.8930	0.9080	0.9230	0.6	0.3	0.1	0.8875	0.9020	0.9010
0.2	0.7	0.1	0.9095	0.9080	0.9010	0.7	0.1	0.2	0.8965	0.9030	0.9160
0.3	0.1	0.6	0.8975	0.8990	0.9445	0.7	0.2	0.1	0.9010	0.9000	0.9060
0.3	0.2	0.5	0.8880	0.9010	0.9399	0.8	0.1	0.1	0.8970	0.8900	0.8905
0.3	0.3	0.4	0.8995	0.9060	0.9370		MAX		0.9125	0.9105	0.9455
0.3	0.4	0.3	0.9080	0.9105	0.9130		MIN		0.8875	0.8865	0.8905

TABLE 3.2.7 Range of Simulated Confidence

Coefficients for 90% Two-sided Intervals on σ_P^2

when $I = 12$, $J = 6$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9105	0.9035	0.8940	0.3	0.5	0.2	0.9050	0.8915	0.9245
0.1	0.2	0.7	0.8940	0.8945	0.9116	0.3	0.6	0.1	0.8970	0.9005	0.9110
0.1	0.3	0.6	0.9040	0.9115	0.9256	0.4	0.1	0.5	0.9070	0.9130	0.9360
0.1	0.4	0.5	0.8980	0.8995	0.9253	0.4	0.2	0.4	0.9070	0.9090	0.9185
0.1	0.5	0.4	0.8990	0.8970	0.9331	0.4	0.3	0.3	0.8945	0.8940	0.9115
0.1	0.6	0.3	0.8970	0.9030	0.9433	0.4	0.4	0.2	0.9005	0.9010	0.9065
0.1	0.7	0.2	0.9025	0.8975	0.9419	0.4	0.5	0.1	0.9180	0.9045	0.9030
0.1	0.8	0.1	0.8950	0.8980	0.9154	0.5	0.1	0.4	0.8970	0.9090	0.9250
0.2	0.1	0.7	0.9075	0.9070	0.9403	0.5	0.2	0.3	0.8905	0.9075	0.9105
0.2	0.2	0.6	0.8980	0.8965	0.9333	0.5	0.3	0.2	0.8965	0.9060	0.9060
0.2	0.3	0.5	0.8990	0.9050	0.9479	0.5	0.4	0.1	0.9030	0.8930	0.9035
0.2	0.4	0.4	0.9060	0.9035	0.9489	0.6	0.1	0.3	0.9010	0.8995	0.9130
0.2	0.5	0.3	0.8910	0.9090	0.9395	0.6	0.2	0.2	0.8915	0.9080	0.8985
0.2	0.6	0.2	0.9055	0.9060	0.9105	0.6	0.3	0.1	0.9045	0.9015	0.9110
0.2	0.7	0.1	0.9070	0.8930	0.8995	0.7	0.1	0.2	0.8995	0.9005	0.9065
0.3	0.1	0.6	0.9085	0.8970	0.9419	0.7	0.2	0.1	0.9030	0.8960	0.8985
0.3	0.2	0.5	0.8960	0.9075	0.9474	0.8	0.1	0.1	0.9035	0.8940	0.9100
0.3	0.3	0.4	0.9060	0.9030	0.9279		MAX		0.9180	0.9130	0.9489
0.3	0.4	0.3	0.8935	0.8995	0.9150		MIN		0.8905	0.8915	0.8940

TABLE 3.2.8 Range of Simulated Confidence

Coefficients for 90% Two-sided Intervals on σ_P^2

when $I = 12$, $J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9040	0.9115	0.9305	0.3	0.5	0.2	0.8980	0.9000	0.9100
0.1	0.2	0.7	0.8960	0.8975	0.9362	0.3	0.6	0.1	0.8990	0.9025	0.9005
0.1	0.3	0.6	0.8935	0.9025	0.9278	0.4	0.1	0.5	0.9090	0.8985	0.9125
0.1	0.4	0.5	0.8890	0.8840	0.9478	0.4	0.2	0.4	0.9030	0.8975	0.9015
0.1	0.5	0.4	0.9005	0.8965	0.9370	0.4	0.3	0.3	0.9015	0.9025	0.9095
0.1	0.6	0.3	0.9060	0.9050	0.9464	0.4	0.4	0.2	0.9010	0.8970	0.9055
0.1	0.7	0.2	0.8905	0.9040	0.9210	0.4	0.5	0.1	0.8995	0.9045	0.8725
0.1	0.8	0.1	0.8835	0.9055	0.9000	0.5	0.1	0.4	0.8870	0.9155	0.9130
0.2	0.1	0.7	0.8985	0.8840	0.9404	0.5	0.2	0.3	0.9035	0.9045	0.9045
0.2	0.2	0.6	0.9065	0.9045	0.9335	0.5	0.3	0.2	0.9045	0.8820	0.9045
0.2	0.3	0.5	0.8995	0.8960	0.9445	0.5	0.4	0.1	0.8995	0.9045	0.8990
0.2	0.4	0.4	0.8965	0.8990	0.9524	0.6	0.1	0.3	0.9060	0.8915	0.9120
0.2	0.5	0.3	0.8970	0.8975	0.9485	0.6	0.2	0.2	0.8975	0.9100	0.9150
0.2	0.6	0.2	0.8950	0.8965	0.9045	0.6	0.3	0.1	0.8955	0.8945	0.9085
0.2	0.7	0.1	0.8900	0.8995	0.9110	0.7	0.1	0.2	0.8960	0.8965	0.8985
0.3	0.1	0.6	0.8980	0.9030	0.9205	0.7	0.2	0.1	0.9030	0.8900	0.8825
0.3	0.2	0.5	0.8925	0.8985	0.9380	0.8	0.1	0.1	0.9025	0.8925	0.9060
0.3	0.3	0.4	0.8975	0.9000	0.9150		MAX		0.9090	0.9155	0.9524
0.3	0.4	0.3	0.8975	0.8990	0.9045		MIN		0.8835	0.8820	0.8725

TABLE 3.2.9 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I=24$, $J=3$, and $K=2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8930	0.8905	0.8791	0.3	0.5	0.2	0.8925	0.8985	0.9155
0.1	0.2	0.7	0.9065	0.9005	0.8927	0.3	0.6	0.1	0.8980	0.8825	0.9055
0.1	0.3	0.6	0.9010	0.9030	0.8996	0.4	0.1	0.5	0.8890	0.9020	0.9330
0.1	0.4	0.5	0.8990	0.9050	0.9086	0.4	0.2	0.4	0.8955	0.9060	0.9195
0.1	0.5	0.4	0.8985	0.8890	0.9199	0.4	0.3	0.3	0.9065	0.9090	0.9085
0.1	0.6	0.3	0.8915	0.8995	0.9333	0.4	0.4	0.2	0.8990	0.8940	0.9054
0.1	0.7	0.2	0.8955	0.8995	0.9450	0.4	0.5	0.1	0.9100	0.9135	0.8915
0.1	0.8	0.1	0.9015	0.9050	0.9035	0.5	0.1	0.4	0.8955	0.8995	0.9105
0.2	0.1	0.7	0.9000	0.8920	0.9157	0.5	0.2	0.3	0.9055	0.8985	0.9100
0.2	0.2	0.6	0.9025	0.8960	0.9314	0.5	0.3	0.2	0.9085	0.9100	0.9030
0.2	0.3	0.5	0.8855	0.9055	0.9465	0.5	0.4	0.1	0.9130	0.8950	0.9105
0.2	0.4	0.4	0.9115	0.9000	0.9294	0.6	0.1	0.3	0.8955	0.9035	0.9090
0.2	0.5	0.3	0.8995	0.9000	0.9365	0.6	0.2	0.2	0.8925	0.9140	0.9000
0.2	0.6	0.2	0.9050	0.9085	0.9200	0.6	0.3	0.1	0.9070	0.9075	0.9075
0.2	0.7	0.1	0.8875	0.9080	0.9110	0.7	0.1	0.2	0.8950	0.8935	0.8905
0.3	0.1	0.6	0.8930	0.9170	0.9350	0.7	0.2	0.1	0.8965	0.8905	0.8975
0.3	0.2	0.5	0.8965	0.9090	0.9375	0.8	0.1	0.1	0.9035	0.9140	0.9025
0.3	0.3	0.4	0.8930	0.9040	0.9155		MAX		0.9130	0.9170	0.9465
0.3	0.4	0.3	0.9035	0.9095	0.9290		MIN		0.8855	0.8825	0.8791

TABLE 3.2.10 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I=24$, $J=3$, and $K=4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9100	0.9135	0.9285	0.3	0.5	0.2	0.8925	0.9025	0.9085
0.1	0.2	0.7	0.8960	0.9110	0.9248	0.3	0.6	0.1	0.9030	0.8990	0.9170
0.1	0.3	0.6	0.9050	0.9015	0.9134	0.4	0.1	0.5	0.9000	0.8935	0.9165
0.1	0.4	0.5	0.9015	0.9110	0.9385	0.4	0.2	0.4	0.8980	0.9155	0.9100
0.1	0.5	0.4	0.8925	0.8990	0.9405	0.4	0.3	0.3	0.8980	0.9055	0.9160
0.1	0.6	0.3	0.9000	0.9050	0.9305	0.4	0.4	0.2	0.8935	0.8920	0.9000
0.1	0.7	0.2	0.9035	0.9045	0.9165	0.4	0.5	0.1	0.8910	0.9070	0.9100
0.1	0.8	0.1	0.8915	0.9035	0.9100	0.5	0.1	0.4	0.9110	0.9100	0.9035
0.2	0.1	0.7	0.8925	0.8970	0.9355	0.5	0.2	0.3	0.9055	0.8940	0.9075
0.2	0.2	0.6	0.9085	0.8930	0.9285	0.5	0.3	0.2	0.8940	0.8935	0.9005
0.2	0.3	0.5	0.9010	0.8970	0.9185	0.5	0.4	0.1	0.9005	0.8955	0.9030
0.2	0.4	0.4	0.9025	0.8970	0.9090	0.6	0.1	0.3	0.8990	0.8975	0.8990
0.2	0.5	0.3	0.9010	0.9070	0.9170	0.6	0.2	0.2	0.9065	0.8940	0.9070
0.2	0.6	0.2	0.9065	0.8950	0.9140	0.6	0.3	0.1	0.8985	0.8895	0.9030
0.2	0.7	0.1	0.8950	0.9015	0.8950	0.7	0.1	0.2	0.8875	0.9020	0.9011
0.3	0.1	0.6	0.9175	0.9035	0.9125	0.7	0.2	0.1	0.9115	0.8985	0.9045
0.3	0.2	0.5	0.8990	0.9010	0.9150	0.8	0.1	0.1	0.8965	0.9035	0.8965
0.3	0.3	0.4	0.8865	0.9030	0.9145		MAX		0.9175	0.9155	0.9405
0.3	0.4	0.3	0.9025	0.9050	0.9125		MIN		0.8865	0.8895	0.8950

TABLE 3.2.11 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I=24$, $J=6$, and $K=2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9025	0.9090	0.9215	0.3	0.5	0.2	0.8965	0.8965	0.9135
0.1	0.2	0.7	0.8920	0.8975	0.9216	0.3	0.6	0.1	0.8940	0.8905	0.9110
0.1	0.3	0.6	0.8950	0.8955	0.9335	0.4	0.1	0.5	0.8835	0.8965	0.9075
0.1	0.4	0.5	0.9110	0.9020	0.9294	0.4	0.2	0.4	0.9175	0.8930	0.9175
0.1	0.5	0.4	0.9000	0.8890	0.9319	0.4	0.3	0.3	0.8990	0.8945	0.8975
0.1	0.6	0.3	0.9145	0.9005	0.9320	0.4	0.4	0.2	0.8960	0.8995	0.9115
0.1	0.7	0.2	0.8955	0.9105	0.9170	0.4	0.5	0.1	0.8950	0.9075	0.8905
0.1	0.8	0.1	0.9025	0.8965	0.9030	0.5	0.1	0.4	0.8955	0.8930	0.8945
0.2	0.1	0.7	0.8975	0.8985	0.9354	0.5	0.2	0.3	0.8945	0.9015	0.9105
0.2	0.2	0.6	0.8985	0.9010	0.9340	0.5	0.3	0.2	0.9000	0.9035	0.8925
0.2	0.3	0.5	0.8980	0.8970	0.9250	0.5	0.4	0.1	0.8940	0.8975	0.8995
0.2	0.4	0.4	0.8940	0.8935	0.9190	0.6	0.1	0.3	0.9025	0.8950	0.8985
0.2	0.5	0.3	0.8985	0.8935	0.9045	0.6	0.2	0.2	0.9020	0.8990	0.9020
0.2	0.6	0.2	0.9050	0.8955	0.9100	0.6	0.3	0.1	0.9125	0.9040	0.9015
0.2	0.7	0.1	0.8855	0.9065	0.9105	0.7	0.1	0.2	0.9070	0.8960	0.8950
0.3	0.1	0.6	0.8955	0.9105	0.9120	0.7	0.2	0.1	0.9120	0.8975	0.9005
0.3	0.2	0.5	0.8940	0.9025	0.9070	0.8	0.1	0.1	0.9055	0.9130	0.8895
0.3	0.3	0.4	0.9060	0.9050	0.9210		MAX		0.9175	0.9130	0.9354
0.3	0.4	0.3	0.8995	0.8965	0.9070		MIN		0.8835	0.8890	0.8895

TABLE 3.2.12 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2 when $I=24$, $J=6$, and $K=4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8940	0.8880	0.9385	0.3	0.5	0.2	0.8890	0.9075	0.9030
0.1	0.2	0.7	0.9080	0.8985	0.9344	0.3	0.6	0.1	0.9000	0.9005	0.8940
0.1	0.3	0.6	0.9000	0.8980	0.9385	0.4	0.1	0.5	0.9005	0.8940	0.9130
0.1	0.4	0.5	0.9055	0.8935	0.9135	0.4	0.2	0.4	0.8930	0.9005	0.9050
0.1	0.5	0.4	0.9005	0.9035	0.9230	0.4	0.3	0.3	0.9005	0.8905	0.9015
0.1	0.6	0.3	0.9015	0.9005	0.9055	0.4	0.4	0.2	0.8960	0.9035	0.9095
0.1	0.7	0.2	0.8950	0.9005	0.9020	0.4	0.5	0.1	0.9110	0.9065	0.8985
0.1	0.8	0.1	0.9030	0.8950	0.8975	0.5	0.1	0.4	0.9075	0.9005	0.8940
0.2	0.1	0.7	0.9005	0.8975	0.9165	0.5	0.2	0.3	0.9080	0.9020	0.9000
0.2	0.2	0.6	0.9010	0.9000	0.9220	0.5	0.3	0.2	0.8995	0.8970	0.9000
0.2	0.3	0.5	0.9010	0.9150	0.9045	0.5	0.4	0.1	0.9025	0.8950	0.9050
0.2	0.4	0.4	0.8995	0.8940	0.8990	0.6	0.1	0.3	0.9045	0.8970	0.8910
0.2	0.5	0.3	0.9030	0.8940	0.9015	0.6	0.2	0.2	0.9085	0.9015	0.9015
0.2	0.6	0.2	0.9065	0.8975	0.9085	0.6	0.3	0.1	0.9105	0.9220	0.9110
0.2	0.7	0.1	0.8950	0.9025	0.8940	0.7	0.1	0.2	0.8955	0.9055	0.8945
0.3	0.1	0.6	0.9035	0.9115	0.9050	0.7	0.2	0.1	0.9030	0.8945	0.8985
0.3	0.2	0.5	0.9015	0.8885	0.8980	0.8	0.1	0.1	0.9030	0.8975	0.9045
0.3	0.3	0.4	0.8960	0.9010	0.9045		MAX		0.9110	0.9220	0.9385
0.3	0.4	0.3	0.9010	0.8975	0.9000		MIN		0.8890	0.8880	0.8910

TABLE 3.2.13 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I=6,\ J=3$, and $K=2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9160	0.8965	0.8370	0.3	0.5	0.2	0.8840	0.9115	0.9528
0.1	0.2	0.7	0.8945	0.9055	0.8890	0.3	0.6	0.1	0.9020	0.8975	0.9441
0.1	0.3	0.6	0.9145	0.8975	0.9105	0.4	0.1	0.5	0.9035	0.9055	0.8750
0.1	0.4	0.5	0.9025	0.8990	0.9195	0.4	0.2	0.4	0.9025	0.8955	0.9214
0.1	0.5	0.4	0.8890	0.9115	0.9380	0.4	0.3	0.3	0.8975	0.9025	0.9336
0.1	0.6	0.3	0.9020	0.9000	0.9392	0.4	0.4	0.2	0.8935	0.8960	0.9466
0.1	0.7	0.2	0.8920	0.8940	0.9486	0.4	0.5	0.1	0.9065	0.9070	0.9458
0.1	0.8	0.1	0.9055	0.8820	0.9540	0.5	0.1	0.4	0.9120	0.8955	0.8819
0.2	0.1	0.7	0.8995	0.9135	0.8924	0.5	0.2	0.3	0.9105	0.9070	0.9296
0.2	0.2	0.6	0.8965	0.8975	0.9011	0.5	0.3	0.2	0.9035	0.8985	0.9342
0.2	0.3	0.5	0.8980	0.9000	0.9204	0.5	0.4	0.1	0.9085	0.8850	0.9447
0.2	0.4	0.4	0.9000	0.9090	0.9333	0.6	0.1	0.3	0.8895	0.9125	0.8926
0.2	0.5	0.3	0.8960	0.8990	0.9381	0.6	0.2	0.2	0.8955	0.8935	0.9388
0.2	0.6	0.2	0.8955	0.8905	0.9452	0.6	0.3	0.1	0.9035	0.8885	0.9554
0.2	0.7	0.1	0.9000	0.9025	0.9523	0.7	0.1	0.2	0.9015	0.9020	0.9279
0.3	0.1	0.6	0.9020	0.9080	0.8515	0.7	0.2	0.1	0.9000	0.9045	0.9499
0.3	0.2	0.5	0.8945	0.9025	0.9131	0.8	0.1	0.1	0.9030	0.8995	0.9313
0.3	0.3	0.4	0.9065	0.9020	0.9406		MAX		0.9160	0.9125	0.9554
0.3	0.4	0.3	0.9120	0.9080	0.9369		MIN		0.8840	0.8820	0.8370

TABLE 3.2.14 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I=6$, $J=3$, and $K=4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9000	0.8925	0.8800	0.3	0.5	0.2	0.9040	0.9120	0.9480
0.1	0.2	0.7	0.9070	0.8965	0.9244	0.3	0.6	0.1	0.9085	0.9040	0.9516
0.1	0.3	0.6	0.8960	0.8905	0.9401	0.4	0.1	0.5	0.8980	0.9010	0.9165
0.1	0.4	0.5	0.9020	0.9025	0.9345	0.4	0.2	0.4	0.9060	0.8915	0.9182
0.1	0.5	0.4	0.8930	0.8920	0.9470	0.4	0.3	0.3	0.8940	0.9110	0.9438
0.1	0.6	0.3	0.9035	0.9015	0.9372	0.4	0.4	0.2	0.9000	0.8995	0.9444
0.1	0.7	0.2	0.8935	0.8980	0.9451	0.4	0.5	0.1	0.9015	0.8950	0.9486
0.1	0.8	0.1	0.8990	0.8865	0.9562	0.5	0.1	0.4	0.9010	0.9050	0.9215
0.2	0.1	0.7	0.8980	0.8940	0.8887	0.5	0.2	0.3	0.8940	0.8945	0.9358
0.2	0.2	0.6	0.9030	0.9105	0.9271	0.5	0.3	0.2	0.8935	0.8940	0.9440
0.2	0.3	0.5	0.9110	0.9005	0.9348	0.5	0.4	0.1	0.9155	0.9025	0.9478
0.2	0.4	0.4	0.9035	0.8990	0.9266	0.6	0.1	0.3	0.9085	0.9050	0.9136
0.2	0.5	0.3	0.9080	0.9060	0.9537	0.6	0.2	0.2	0.9000	0.8930	0.9371
0.2	0.6	0.2	0.9085	0.9005	0.9490	0.6	0.3	0.1	0.9005	0.8885	0.9499
0.2	0.7	0.1	0.9075	0.8930	0.9446	0.7	0.1	0.2	0.9055	0.8965	0.9274
0.3	0.1	0.6	0.9000	0.9040	0.9088	0.7	0.2	0.1	0.9080	0.9075	0.9458
0.3	0.2	0.5	0.9115	0.9055	0.9324	0.8	0.1	0.1	0.9110	0.9045	0.9471
0.3	0.3	0.4	0.8940	0.8950	0.9426		MAX		0.9155	0.9120	0.9562
0.3	0.4	0.3	0.9020	0.8960	0.9411		MIN		0.8930	0.8865	0.8800

TABLE 3.2.15 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I=6,\ J=6$, and $K=2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8830	0.8980	0.8838	0.3	0.5	0.2	0.8930	0.9005	0.9385
0.1	0.2	0.7	0.9065	0.9000	0.9231	0.3	0.6	0.1	0.8910	0.9005	0.9200
0.1	0.3	0.6	0.9055	0.9070	0.9372	0.4	0.1	0.5	0.9030	0.9075	0.9045
0.1	0.4	0.5	0.9000	0.8930	0.9441	0.4	0.2	0.4	0.9035	0.8965	0.9427
0.1	0.5	0.4	0.9005	0.9045	0.9468	0.4	0.3	0.3	0.8955	0.8865	0.9427
0.1	0.6	0.3	0.9055	0.9030	0.9424	0.4	0.4	0.2	0.9025	0.9080	0.9455
0.1	0.7	0.2	0.8920	0.9020	0.9254	0.4	0.5	0.1	0.9000	0.9000	0.9180
0.1	0.8	0.1	0.9220	0.8855	0.9025	0.5	0.1	0.4	0.9050	0.8935	0.9229
0.2	0.1	0.7	0.8960	0.9035	0.8940	0.5	0.2	0.3	0.8910	0.8945	0.9449
0.2	0.2	0.6	0.9070	0.9030	0.9368	0.5	0.3	0.2	0.8980	0.9005	0.9564
0.2	0.3	0.5	0.8995	0.9060	0.9392	0.5	0.4	0.1	0.8855	0.9040	0.9174
0.2	0.4	0.4	0.8885	0.9005	0.9427	0.6	0.1	0.3	0.8850	0.9070	0.9264
0.2	0.5	0.3	0.8965	0.9075	0.9473	0.6	0.2	0.2	0.9130	0.8985	0.9393
0.2	0.6	0.2	0.9045	0.9055	0.9245	0.6	0.3	0.1	0.8945	0.8925	0.9309
0.2	0.7	0.1	0.8885	0.8985	0.8990	0.7	0.1	0.2	0.9010	0.9160	0.9411
0.3	0.1	0.6	0.8890	0.9110	0.8892	0.7	0.2	0.1	0.8930	0.8940	0.9458
0.3	0.2	0.5	0.8890	0.8915	0.9291	0.8	0.1	0.1	0.9030	0.8850	0.9443
0.3	0.3	0.4	0.9040	0.9070	0.9408		MAX		0.9220	0.9160	0.9564
0.3	0.4	0.3	0.9080	0.8905	0.9474		MIN		0.8830	0.8850	0.8838

TABLE 3.2.16 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I=6$, $J=6$, and $K=4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9020	0.8975	0.9181	0.3	0.5	0.2	0.9020	0.9000	0.9035
0.1	0.2	0.7	0.9025	0.8985	0.9400	0.3	0.6	0.1	0.9075	0.8995	0.9050
0.1	0.3	0.6	0.8960	0.9105	0.9532	0.4	0.1	0.5	0.8895	0.9045	0.9346
0.1	0.4	0.5	0.9140	0.8975	0.9439	0.4	0.2	0.4	0.8900	0.9235	0.9478
0.1	0.5	0.4	0.8985	0.8990	0.9400	0.4	0.3	0.3	0.9025	0.8945	0.9524
0.1	0.6	0.3	0.8960	0.9035	0.9200	0.4	0.4	0.2	0.9120	0.8965	0.9174
0.1	0.7	0.2	0.8935	0.8975	0.8989	0.4	0.5	0.1	0.8935	0.9010	0.9120
0.1	0.8	0.1	0.9015	0.9020	0.9040	0.5	0.1	0.4	0.9040	0.9025	0.9422
0.2	0.1	0.7	0.9015	0.9005	0.9135	0.5	0.2	0.3	0.9125	0.9035	0.9473
0.2	0.2	0.6	0.8985	0.9060	0.9443	0.5	0.3	0.2	0.9020	0.9010	0.9119
0.2	0.3	0.5	0.9070	0.9000	0.9513	0.5	0.4	0.1	0.9115	0.8990	0.9140
0.2	0.4	0.4	0.9065	0.8945	0.9534	0.6	0.1	0.3	0.9025	0.8990	0.9393
0.2	0.5	0.3	0.8920	0.9110	0.9255	0.6	0.2	0.2	0.9050	0.9110	0.9434
0.2	0.6	0.2	0.8845	0.9065	0.9065	0.6	0.3	0.1	0.8965	0.8985	0.9140
0.2	0.7	0.1	0.9005	0.9030	0.9080	0.7	0.1	0.2	0.8940	0.8980	0.9537
0.3	0.1	0.6	0.8975	0.8955	0.9313	0.7	0.2	0.1	0.8945	0.8945	0.9255
0.3	0.2	0.5	0.9070	0.8920	0.9290	0.8	0.1	0.1	0.8995	0.8915	0.9429
0.3	0.3	0.4	0.9025	0.8955	0.9534		MAX		0.9140	0.9235	0.9537
0.3	0.4	0.3	0.9080	0.8930	0.9282		MIN		0.8845	0.8915	0.8989

TABLE 3.2.17 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I = 12$, $J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8990	0.8920	0.8874	0.3	0.5	0.2	0.9075	0.9075	0.9463
0.1	0.2	0.7	0.9090	0.8995	0.9337	0.3	0.6	0.1	0.8960	0.8930	0.9486
0.1	0.3	0.6	0.9080	0.9030	0.9390	0.4	0.1	0.5	0.8810	0.9060	0.8984
0.1	0.4	0.5	0.8845	0.9065	0.9376	0.4	0.2	0.4	0.8960	0.8940	0.9341
0.1	0.5	0.4	0.8975	0.9065	0.9436	0.4	0.3	0.3	0.9110	0.8965	0.9432
0.1	0.6	0.3	0.9050	0.8990	0.9045	0.4	0.4	0.2	0.8995	0.9060	0.9468
0.1	0.7	0.2	0.8960	0.8975	0.9483	0.4	0.5	0.1	0.8910	0.8875	0.9455
0.1	0.8	0.1	0.9010	0.9040	0.9490	0.5	0.1	0.4	0.8940	0.8920	0.9180
0.2	0.1	0.7	0.9065	0.9040	0.9037	0.5	0.2	0.3	0.9080	0.8940	0.9438
0.2	0.2	0.6	0.9005	0.8965	0.9216	0.5	0.3	0.2	0.9050	0.9115	0.9513
0.2	0.3	0.5	0.9000	0.9005	0.9354	0.5	0.4	0.1	0.9050	0.9000	0.9530
0.2	0.4	0.4	0.9065	0.8985	0.9354	0.6	0.1	0.3	0.9060	0.9050	0.9221
0.2	0.5	0.3	0.9075	0.9055	0.9454	0.6	0.2	0.2	0.8925	0.8985	0.9503
0.2	0.6	0.2	0.8915	0.8955	0.9424	0.6	0.3	0.1	0.9030	0.8980	0.9458
0.2	0.7	0.1	0.8940	0.8960	0.9593	0.7	0.1	0.2	0.8815	0.9065	0.9311
0.3	0.1	0.6	0.8990	0.9030	0.8968	0.7	0.2	0.1	0.9090	0.9050	0.9349
0.3	0.2	0.5	0.9005	0.8960	0.9232	0.8	0.1	0.1	0.8935	0.9110	0.9490
0.3	0.3	0.4	0.9130	0.9075	0.9452		MAX		0.9130	0.9115	0.9593
0.3	0.4	0.3	0.8980	0.8995	0.9446		MIN		0.8810	0.8875	0.8874

TABLE 3.2.18 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I = 12$, $J = 3$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8935	0.9005	0.9137	0.3	0.5	0.2	0.8905	0.8980	0.9441
0.1	0.2	0.7	0.8895	0.8930	0.9289	0.3	0.6	0.1	0.9075	0.8990	0.9418
0.1	0.3	0.6	0.8855	0.9075	0.9377	0.4	0.1	0.5	0.9060	0.9065	0.9366
0.1	0.4	0.5	0.8955	0.8804	0.9376	0.4	0.2	0.4	0.8905	0.9095	0.9529
0.1	0.5	0.4	0.8975	0.9110	0.9558	0.4	0.3	0.3	0.9000	0.9035	0.9510
0.1	0.6	0.3	0.8975	0.9065	0.9505	0.4	0.4	0.2	0.9065	0.9080	0.9490
0.1	0.7	0.2	0.9000	0.8835	0.9467	0.4	0.5	0.1	0.8975	0.9005	0.9518
0.1	0.8	0.1	0.9090	0.9005	0.9172	0.5	0.1	0.4	0.8975	0.9060	0.9372
0.2	0.1	0.7	0.9025	0.9065	0.9146	0.5	0.2	0.3	0.9050	0.8980	0.9491
0.2	0.2	0.6	0.9050	0.8975	0.9332	0.5	0.3	0.2	0.9015	0.8945	0.9492
0.2	0.3	0.5	0.9095	0.9035	0.9431	0.5	0.4	0.1	0.9020	0.8945	0.9472
0.2	0.4	0.4	0.8970	0.9130	0.9349	0.6	0.1	0.3	0.8945	0.8965	0.9298
0.2	0.5	0.3	0.9015	0.8935	0.9523	0.6	0.2	0.2	0.9050	0.8980	0.9508
0.2	0.6	0.2	0.9045	0.9055	0.9496	0.6	0.3	0.1	0.8905	0.9005	0.9477
0.2	0.7	0.1	0.9055	0.8940	0.9376	0.7	0.1	0.2	0.9120	0.9025	0.9383
0.3	0.1	0.6	0.8970	0.8920	0.9272	0.7	0.2	0.1	0.8875	0.8960	0.9495
0.3	0.2	0.5	0.8920	0.8980	0.9399	0.8	0.1	0.1	0.9015	0.9115	0.9489
0.3	0.3	0.4	0.8955	0.9020	0.9453		MAX		0.9120	0.9130	0.9558
0.3	0.4	0.3	0.8955	0.9070	0.9513		MIN		0.8855	0.8804	0.9137

TABLE 3.2.19 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I=12$, $J=6$, and $K=2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8930	0.9030	0.9214	0.3	0.5	0.2	0.8960	0.8980	0.9050
0.1	0.2	0.7	0.9015	0.8890	0.9442	0.3	0.6	0.1	0.8980	0.9055	0.9085
0.1	0.3	0.6	0.8990	0.9045	0.9462	0.4	0.1	0.5	0.8985	0.9020	0.9337
0.1	0.4	0.5	0.9080	0.8815	0.9469	0.4	0.2	0.4	0.8950	0.9045	0.9437
0.1	0.5	0.4	0.9025	0.9030	0.9359	0.4	0.3	0.3	0.9030	0.9010	0.9499
0.1	0.6	0.3	0.8955	0.9020	0.9209	0.4	0.4	0.2	0.8980	0.9000	0.9209
0.1	0.7	0.2	0.9015	0.9085	0.9035	0.4	0.5	0.1	0.9015	0.8925	0.9040
0.1	0.8	0.1	0.9055	0.9015	0.9025	0.5	0.1	0.4	0.9040	0.8920	0.9400
0.2	0.1	0.7	0.8955	0.8960	0.9200	0.5	0.2	0.3	0.9015	0.9005	0.9528
0.2	0.2	0.6	0.8950	0.9015	0.9430	0.5	0.3	0.2	0.9060	0.9015	0.9209
0.2	0.3	0.5	0.9035	0.9045	0.9528	0.5	0.4	0.1	0.9085	0.8930	0.9140
0.2	0.4	0.4	0.8900	0.8950	0.9474	0.6	0.1	0.3	0.8905	0.9075	0.9404
0.2	0.5	0.3	0.8965	0.8975	0.9100	0.6	0.2	0.2	0.9095	0.9020	0.9434
0.2	0.6	0.2	0.8870	0.8970	0.9055	0.6	0.3	0.1	0.9045	0.8930	0.9030
0.2	0.7	0.1	0.9130	0.9040	0.9040	0.7	0.1	0.2	0.9015	0.8930	0.9426
0.3	0.1	0.6	0.8995	0.9005	0.9425	0.7	0.2	0.1	0.9040	0.9015	0.9205
0.3	0.2	0.5	0.9000	0.8970	0.9353	0.8	0.1	0.1	0.9075	0.8970	0.9439
0.3	0.3	0.4	0.9030	0.8875	0.9418		MAX		0.9130	0.9130	0.9528
0.3	0.4	0.3	0.9090	0.9130	0.9354		MIN		0.8870	0.8815	0.9025

TABLE 3.2.20 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I = 12$, $J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8895	0.8920	0.9405	0.3	0.5	0.2	0.8935	0.9095	0.9110
0.1	0.2	0.7	0.9060	0.9065	0.9553	0.3	0.6	0.1	0.8995	0.9030	0.9010
0.1	0.3	0.6	0.8950	0.8925	0.9414	0.4	0.1	0.5	0.9040	0.9055	0.9366
0.1	0.4	0.5	0.8970	0.9075	0.9279	0.4	0.2	0.4	0.8990	0.9035	0.9498
0.1	0.5	0.4	0.9065	0.8915	0.9115	0.4	0.3	0.3	0.9065	0.8855	0.9024
0.1	0.6	0.3	0.9130	0.8950	0.9060	0.4	0.4	0.2	0.9020	0.9010	0.9110
0.1	0.7	0.2	0.9005	0.9080	0.8980	0.4	0.5	0.1	0.8980	0.8960	0.9080
0.1	0.8	0.1	0.8980	0.9050	0.8960	0.5	0.1	0.4	0.8915	0.8980	0.9463
0.2	0.1	0.7	0.8970	0.8865	0.9337	0.5	0.2	0.3	0.9070	0.8935	0.9324
0.2	0.2	0.6	0.9060	0.9065	0.9371	0.5	0.3	0.2	0.8850	0.8970	0.8950
0.2	0.3	0.5	0.8915	0.8970	0.9294	0.5	0.4	0.1	0.9005	0.8935	0.9080
0.2	0.4	0.4	0.9085	0.9095	0.9550	0.6	0.1	0.3	0.9050	0.9055	0.9478
0.2	0.5	0.3	0.9065	0.9040	0.9199	0.6	0.2	0.2	0.9115	0.8970	0.9500
0.2	0.6	0.2	0.8965	0.8975	0.9025	0.6	0.3	0.1	0.9080	0.9070	0.8980
0.2	0.7	0.1	0.8870	0.9005	0.9015	0.7	0.1	0.2	0.8940	0.8890	0.9529
0.3	0.1	0.6	0.8980	0.9025	0.9347	0.7	0.2	0.1	0.8980	0.9005	0.9210
0.3	0.2	0.5	0.8925	0.8995	0.9470	0.8	0.1	0.1	0.8860	0.9015	0.9115
0.3	0.3	0.4	0.9035	0.9075	0.9290		MAX		0.9130	0.9095	0.9553
0.3	0.4	0.3	0.8990	0.8980	0.9200		MIN		0.8850	0.8855	0.8950

TABLE 3.2.21 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I = 24$, $J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9100	0.9040	0.9128	0.3	0.5	0.2	0.8985	0.8950	0.9491
0.1	0.2	0.7	0.8940	0.9115	0.9363	0.3	0.6	0.1	0.9020	0.8985	0.9453
0.1	0.3	0.6	0.8980	0.8970	0.9459	0.4	0.1	0.5	0.8980	0.8960	0.9293
0.1	0.4	0.5	0.8965	0.9050	0.9492	0.4	0.2	0.4	0.8875	0.9070	0.9392
0.1	0.5	0.4	0.9115	0.8925	0.9430	0.4	0.3	0.3	0.9080	0.8990	0.9461
0.1	0.6	0.3	0.8930	0.9080	0.9447	0.4	0.4	0.2	0.9015	0.8945	0.9499
0.1	0.7	0.2	0.9095	0.8875	0.9482	0.4	0.5	0.1	0.8975	0.8940	0.9478
0.1	0.8	0.1	0.8955	0.8895	0.9244	0.5	0.1	0.4	0.9000	0.8980	0.9386
0.2	0.1	0.7	0.9035	0.8980	0.9252	0.5	0.2	0.3	0.8815	0.9020	0.9477
0.2	0.2	0.6	0.8915	0.8945	0.9430	0.5	0.3	0.2	0.9050	0.8990	0.9506
0.2	0.3	0.5	0.8935	0.8940	0.9424	0.5	0.4	0.1	0.8920	0.8980	0.9410
0.2	0.4	0.4	0.9015	0.9025	0.9455	0.6	0.1	0.3	0.9040	0.9055	0.9408
0.2	0.5	0.3	0.8805	0.8905	0.9520	0.6	0.2	0.2	0.9145	0.9010	0.9562
0.2	0.6	0.2	0.8985	0.9065	0.9577	0.6	0.3	0.1	0.9045	0.8970	0.9577
0.2	0.7	0.1	0.8980	0.9050	0.9323	0.7	0.1	0.2	0.8885	0.8875	0.9401
0.3	0.1	0.6	0.8885	0.8995	0.9214	0.7	0.2	0.1	0.8965	0.9080	0.9500
0.3	0.2	0.5	0.9005	0.8970	0.9385	0.8	0.1	0.1	0.9110	0.9115	0.9542
0.3	0.3	0.4	0.8960	0.8920	0.9501		MAX		0.9145	0.9115	0.9577
0.3	0.4	0.3	0.9050	0.9050	0.9421		MIN		0.8805	0.8875	0.9128

TABLE 3.2.22 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I = 24$, $J = 3$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9070	0.9055	0.9355	0.3	0.5	0.2	0.8950	0.8890	0.9396
0.1	0.2	0.7	0.8965	0.9005	0.9445	0.3	0.6	0.1	0.9015	0.9085	0.9168
0.1	0.3	0.6	0.8915	0.8920	0.9576	0.4	0.1	0.5	0.9010	0.9020	0.9338
0.1	0.4	0.5	0.8910	0.9005	0.9502	0.4	0.2	0.4	0.8995	0.8945	0.9408
0.1	0.5	0.4	0.9010	0.8920	0.9471	0.4	0.3	0.3	0.9095	0.8960	0.9508
0.1	0.6	0.3	0.9015	0.9030	0.9527	0.4	0.4	0.2	0.9015	0.9000	0.9400
0.1	0.7	0.2	0.8990	0.8950	0.9283	0.4	0.5	0.1	0.9065	0.9015	0.9193
0.1	0.8	0.1	0.8880	0.9015	0.9189	0.5	0.1	0.4	0.8985	0.9035	0.9308
0.2	0.1	0.7	0.8845	0.9035	0.9476	0.5	0.2	0.3	0.9055	0.8925	0.9442
0.2	0.2	0.6	0.8950	0.8940	0.9408	0.5	0.3	0.2	0.9040	0.8940	0.9535
0.2	0.3	0.5	0.9055	0.8990	0.9482	0.5	0.4	0.1	0.8955	0.9080	0.9253
0.2	0.4	0.4	0.8975	0.9025	0.9489	0.6	0.1	0.3	0.8850	0.8960	0.9399
0.2	0.5	0.3	0.9000	0.9015	0.9460	0.6	0.2	0.2	0.8925	0.9080	0.9494
0.2	0.6	0.2	0.8930	0.8955	0.9502	0.6	0.3	0.1	0.9005	0.9125	0.9462
0.2	0.7	0.1	0.9005	0.9050	0.9164	0.7	0.1	0.2	0.8995	0.8940	0.9402
0.3	0.1	0.6	0.9025	0.9090	0.9323	0.7	0.2	0.1	0.8990	0.9015	0.9581
0.3	0.2	0.5	0.9055	0.8965	0.9492	0.8	0.1	0.1	0.9115	0.9125	0.9499
0.3	0.3	0.4	0.9020	0.8995	0.9511		MAX		0.9115	0.9125	0.9581
0.3	0.4	0.3	0.9065	0.8885	0.9456		MIN		0.8845	0.8885	0.9164

TABLE 3.2.23 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I = 24$, $J = 6$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.9050	0.8975	0.9323	0.3	0.5	0.2	0.9020	0.8745	0.9100
0.1	0.2	0.7	0.9030	0.9070	0.9418	0.3	0.6	0.1	0.9020	0.9075	0.9000
0.1	0.3	0.6	0.8965	0.8975	0.9494	0.4	0.1	0.5	0.8995	0.8980	0.9427
0.1	0.4	0.5	0.9010	0.8985	0.9350	0.4	0.2	0.4	0.9105	0.8970	0.9524
0.1	0.5	0.4	0.9040	0.9070	0.9020	0.4	0.3	0.3	0.9075	0.8940	0.9134
0.1	0.6	0.3	0.9025	0.8925	0.9105	0.4	0.4	0.2	0.8970	0.9015	0.9160
0.1	0.7	0.2	0.9110	0.9125	0.9010	0.4	0.5	0.1	0.8975	0.9025	0.8920
0.1	0.8	0.1	0.9045	0.8975	0.8880	0.5	0.1	0.4	0.9035	0.9060	0.9486
0.2	0.1	0.7	0.8970	0.8905	0.9376	0.5	0.2	0.3	0.9125	0.9040	0.9354
0.2	0.2	0.6	0.8925	0.8995	0.9453	0.5	0.3	0.2	0.8905	0.8795	0.9125
0.2	0.3	0.5	0.8975	0.8975	0.9419	0.5	0.4	0.1	0.9000	0.8875	0.9070
0.2	0.4	0.4	0.9075	0.9045	0.9230	0.6	0.1	0.3	0.9045	0.8950	0.9558
0.2	0.5	0.3	0.8995	0.9095	0.8990	0.6	0.2	0.2	0.8870	0.9085	0.9079
0.2	0.6	0.2	0.9045	0.8970	0.9035	0.6	0.3	0.1	0.9095	0.9025	0.8965
0.2	0.7	0.1	0.9030	0.9125	0.8965	0.7	0.1	0.2	0.9085	0.8990	0.9435
0.3	0.1	0.6	0.9030	0.8995	0.9414	0.7	0.2	0.1	0.8975	0.9075	0.9060
0.3	0.2	0.5	0.8890	0.9045	0.9509	0.8	0.1	0.1	0.8950	0.8855	0.9144
0.3	0.3	0.4	0.9065	0.9050	0.9280		MAX		0.9125	0.9125	0.9558
0.3	0.4	0.3	0.9055	0.8940	0.9509		MIN		0.8870	0.8745	0.8880

TABLE 3.2.24 Range of Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2 when $I = 24$, $J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED	σ_P^2	σ_O^2	σ_E^2	TING	GEN	MIXED
0.1	0.1	0.8	0.8945	0.9025	0.9512	0.3	0.5	0.2	0.9045	0.8865	0.9005
0.1	0.2	0.7	0.8940	0.9030	0.9424	0.3	0.6	0.1	0.9010	0.8945	0.8895
0.1	0.3	0.6	0.8985	0.9080	0.9225	0.4	0.1	0.5	0.9085	0.9025	0.9478
0.1	0.4	0.5	0.8905	0.9060	0.9030	0.4	0.2	0.4	0.9050	0.9080	0.9190
0.1	0.5	0.4	0.9110	0.9085	0.9030	0.4	0.3	0.3	0.9065	0.8980	0.9065
0.1	0.6	0.3	0.8870	0.8875	0.9045	0.4	0.4	0.2	0.9075	0.8945	0.9055
0.1	0.7	0.2	0.9070	0.8920	0.8970	0.4	0.5	0.1	0.9115	0.8965	0.9110
0.1	0.8	0.1	0.8895	0.8945	0.8940	0.5	0.1	0.4	0.9145	0.9080	0.9539
0.2	0.1	0.7	0.8990	0.8835	0.9418	0.5	0.2	0.3	0.9115	0.8870	0.9130
0.2	0.2	0.6	0.8960	0.9030	0.9389	0.5	0.3	0.2	0.9075	0.9065	0.9010
0.2	0.3	0.5	0.9010	0.8970	0.9055	0.5	0.4	0.1	0.9150	0.8970	0.9030
0.2	0.4	0.4	0.8940	0.9060	0.9025	0.6	0.1	0.3	0.8960	0.9070	0.9263
0.2	0.5	0.3	0.9010	0.8975	0.9020	0.6	0.2	0.2	0.8935	0.9005	0.9015
0.2	0.6	0.2	0.9035	0.8950	0.9025	0.6	0.3	0.1	0.8920	0.8950	0.9000
0.2	0.7	0.1	0.9090	0.9015	0.8970	0.7	0.1	0.2	0.8940	0.8990	0.9124
0.3	0.1	0.6	0.9020	0.8995	0.9429	0.7	0.2	0.1	0.9020	0.8960	0.9070
0.3	0.2	0.5	0.8920	0.8920	0.9175	0.8	0.1	0.1	0.8945	0.9055	0.9080
0.3	0.3	0.4	0.8855	0.9060	0.9130		MAX		0.9150	0.9105	0.9539
0.3	0.4	0.3	0.8990	0.9105	0.9100		MIN		0.8855	0.8835	0.8895

TABLE 3.2.25 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 6,\ J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	1.1262	1.1460	0.3	0.5	0.2	1.7132	1.6978
0.1	0.2	0.7	1.0601	1.1089	0.3	0.6	0.1	1.6253	1.6331
0.1	0.3	0.6	1.0163	0.9922	0.4	0.1	0.5	2.5179	2.4996
0.1	0.4	0.5	0.9339	0.9247	0.4	0.2	0.4	2.4328	2.4076
0.1	0.5	0.4	0.8474	0.8290	0.4	0.3	0.3	2.2953	2.3254
0.1	0.6	0.3	0.7598	0.7649	0.4	0.4	0.2	2.2911	2.2928
0.1	0.7	0.2	0.6877	0.6984	0.4	0.5	0.1	2.1744	2.1580
0.1	0.8	0.1	0.6200	0.6064	0.5	0.1	0.4	2.9832	2.9482
0.2	0.1	0.7	1.6787	1.6088	0.5	0.2	0.3	2.8803	2.9150
0.2	0.2	0.6	1.5198	1.5357	0.5	0.3	0.2	2.8137	2.7727
0.2	0.3	0.5	1.4796	1.4629	0.5	0.4	0.1	2.6849	2.6597
0.2	0.4	0.4	1.3743	1.3793	0.6	0.1	0.3	3.2913	3.3660
0.2	0.5	0.3	1.2931	1.3437	0.6	0.2	0.2	3.3513	3.2406
0.2	0.6	0.2	1.2074	1.2236	0.6	0.3	0.1	3.2318	3.2366
0.2	0.7	0.1	1.1204	1.1385	0.7	0.1	0.2	3.9461	3.8537
0.3	0.1	0.6	2.0503	2.0786	0.7	0.2	0.1	3.7367	3.7273
0.3	0.2	0.5	1.9672	1.9690	0.8	0.1	0.1	4.3023	4.2727
0.3	0.3	0.4	1.9139	1.8609		MAX		4.3023	4.2727
0.3	0.4	0.3	1.7874	1.8308		MIN		0.6200	0.6064

TABLE 3.2.26 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 6$, $J = 3$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.8489	0.8715	0.3	0.5	0.2	1.6354	1.6691
0.1	0.2	0.7	0.8012	0.8012	0.3	0.6	0.1	1.5952	1.6152
0.1	0.3	0.6	0.7635	0.7589	0.4	0.1	0.5	2.2548	2.2801
0.1	0.4	0.5	0.9345	0.7224	0.4	0.2	0.4	2.1952	2.2485
0.1	0.5	0.4	0.6867	0.6917	0.4	0.3	0.3	2.1831	2.2602
0.1	0.6	0.3	0.6560	0.6582	0.4	0.4	0.2	2.1780	2.1948
0.1	0.7	0.2	0.6093	0.6217	0.4	0.5	0.1	2.1104	2.1622
0.1	0.8	0.1	0.5752	0.5551	0.5	0.1	0.4	2.7140	2.7444
0.2	0.1	0.7	1.3199	1.3714	0.5	0.2	0.3	2.7360	2.7795
0.2	0.2	0.6	1.3138	1.2801	0.5	0.3	0.2	2.6933	2.6976
0.2	0.3	0.5	1.2663	1.2550	0.5	0.4	0.1	2.5901	2.6117
0.2	0.4	0.4	1.2039	1.2075	0.6	0.1	0.3	3.2218	3.1447
0.2	0.5	0.3	1.2111	1.1407	0.6	0.2	0.2	3.1824	3.2126
0.2	0.6	0.2	1.1123	1.1606	0.6	0.3	0.1	3.1398	3.1563
0.2	0.7	0.1	1.0878	1.0996	0.7	0.1	0.2	3.6908	3.7148
0.3	0.1	0.6	1.7907	1.8806	0.7	0.2	0.1	3.6514	3.6651
0.3	0.2	0.5	1.7740	1.7825	0.8	0.1	0.1	4.2062	4.2487
0.3	0.3	0.4	1.7156	1.7179		MAX		4.2062	4.2487
0.3	0.4	0.3	1.7133	1.6674		MIN		0.5752	0.5551

TABLE 3.2.27 Range of Average Interval Lengths for 90% Two-sided Intervals on σ_P^2 when $I = 6$, $J = 6$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.8508	0.8383	0.3	0.5	0.2	1.6025	1.6570
0.1	0.2	0.7	0.7830	0.8057	0.3	0.6	0.1	1.5881	1.5725
0.1	0.3	0.6	0.7899	0.7624	0.4	0.1	0.5	2.3088	2.2571
0.1	0.4	0.5	0.7277	0.7254	0.4	0.2	0.4	2.2425	2.2332
0.1	0.5	0.4	0.6917	0.7132	0.4	0.3	0.3	2.2592	2.1384
0.1	0.6	0.3	0.6379	0.6631	0.4	0.4	0.2	2.1749	2.1462
0.1	0.7	0.2	0.5940	0.6072	0.4	0.5	0.1	2.1150	2.1448
0.1	0.8	0.1	0.5578	0.5784	0.5	0.1	0.4	2.8589	2.7558
0.2	0.1	0.7	1.3285	1.3379	0.5	0.2	0.3	2.7084	2.7267
0.2	0.2	0.6	1.3196	1.2925	0.5	0.3	0.2	2.6523	2.7369
0.2	0.3	0.5	1.2612	1.2511	0.5	0.4	0.1	2.6438	2.6737
0.2	0.4	0.4	1.1981	1.2074	0.6	0.1	0.3	3.2162	3.1791
0.2	0.5	0.3	1.1836	1.1755	0.6	0.2	0.2	3.1647	3.1806
0.2	0.6	0.2	1.1243	1.1135	0.6	0.3	0.1	3.1363	3.1557
0.2	0.7	0.1	1.0783	1.0714	0.7	0.1	0.2	3.6981	3.7417
0.3	0.1	0.6	1.8027	1.7598	0.7	0.2	0.1	3.7953	3.5948
0.3	0.2	0.5	1.8013	1.7862	0.8	0.1	0.1	4.1315	4.1805
0.3	0.3	0.4	1.7210	1.7837		MAX		4.1315	4.1805
0.3	0.4	0.3	1.7349	1.7014		MIN		0.5578	0.5784

TABLE 3.2.28 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 6$, $J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.6856	0.6994	0.3	0.5	0.2	1.6187	1.6479
0.1	0.2	0.7	0.6577	0.6718	0.3	0.6	0.1	1.5872	1.5873
0.1	0.3	0.6	0.6625	0.6562	0.4	0.1	0.5	2.1623	2.1605
0.1	0.4	0.5	0.6316	0.6106	0.4	0.2	0.4	2.1626	2.1880
0.1	0.5	0.4	0.6176	0.6128	0.4	0.3	0.3	2.1362	2.1148
0.1	0.6	0.3	0.5757	0.5857	0.4	0.4	0.2	2.1480	2.0860
0.1	0.7	0.2	0.5565	0.5621	0.4	0.5	0.1	2.1074	2.0822
0.1	0.8	0.1	0.5539	0.5329	0.5	0.1	0.4	2.6991	2.7021
0.2	0.1	0.7	1.2075	1.2039	0.5	0.2	0.3	2.6475	2.6507
0.2	0.2	0.6	1.1877	1.1883	0.5	0.3	0.2	2.7089	2.6345
0.2	0.3	0.5	1.1588	1.1493	0.5	0.4	0.1	2.5572	2.5863
0.2	0.4	0.4	1.1093	1.1666	0.6	0.1	0.3	3.1689	3.0689
0.2	0.5	0.3	1.0893	1.0911	0.6	0.2	0.2	3.1532	3.1717
0.2	0.6	0.2	1.0752	1.0800	0.6	0.3	0.1	3.1307	3.0856
0.2	0.7	0.1	1.0358	1.0460	0.7	0.1	0.2	3.6902	3.7087
0.3	0.1	0.6	1.7433	1.6839	0.7	0.2	0.1	3.7953	3.6354
0.3	0.2	0.5	1.6505	1.6605	0.8	0.1	0.1	4.1976	4.1821
0.3	0.3	0.4	1.6624	1.6819		MAX		4.1976	4.1821
0.3	0.4	0.3	1.5716	1.6746		MIN		0.5539	0.5329

TABLE 3.2.29 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 12$, $J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.4382	0.4434	0.3	0.5	0.2	0.6465	0.6598
0.1	0.2	0.7	0.4202	0.4170	0.3	0.6	0.1	0.6422	0.6275
0.1	0.3	0.6	0.3831	0.3736	0.4	0.1	0.5	0.9651	0.9847
0.1	0.4	0.5	0.3522	0.3619	0.4	0.2	0.4	0.9289	0.9289
0.1	0.5	0.4	0.3316	0.3303	0.4	0.3	0.3	0.9007	0.9051
0.1	0.6	0.3	0.2970	0.2946	0.4	0.4	0.2	0.8690	0.8595
0.1	0.7	0.2	0.2613	0.2645	0.4	0.5	0.1	0.8162	0.8246
0.1	0.8	0.1	0.2341	0.2332	0.5	0.1	0.4	1.1471	1.1356
0.2	0.1	0.7	0.6279	0.6383	0.5	0.2	0.3	1.1031	1.1030
0.2	0.2	0.6	0.6033	0.5918	0.5	0.3	0.2	1.0619	1.0580
0.2	0.3	0.5	0.5767	0.5650	0.5	0.4	0.1	1.0237	1.0297
0.2	0.4	0.4	0.5302	0.5315	0.6	0.1	0.3	1.3237	1.3202
0.2	0.5	0.3	0.4937	0.5057	0.6	0.2	0.2	1.2601	1.2582
0.2	0.6	0.2	0.4680	0.4634	0.6	0.3	0.1	1.2342	1.2132
0.2	0.7	0.1	0.4293	0.4347	0.7	0.1	0.2	1.4781	1.4720
0.3	0.1	0.6	0.7921	0.7976	0.7	0.2	0.1	1.4455	1.4215
0.3	0.2	0.5	0.7693	0.7435	0.8	0.1	0.1	1.6322	1.6153
0.3	0.3	0.4	0.7342	0.7370		MAX		1.6322	1.6153
0.3	0.4	0.3	0.6993	0.7007		MIN		0.2341	0.2332

TABLE 3.2.30 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 12$, $J = 3$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.3293	0.3287	0.3	0.5	0.2	0.6280	0.6236
0.1	0.2	0.7	0.3095	0.3154	0.3	0.6	0.1	0.6148	0.6095
0.1	0.3	0.6	0.2979	0.2964	0.4	0.1	0.5	0.8790	0.8925
0.1	0.4	0.5	0.2833	0.2807	0.4	0.2	0.4	0.8558	0.8674
0.1	0.5	0.4	0.2639	0.2667	0.4	0.3	0.3	0.8567	0.8452
0.1	0.6	0.3	0.2489	0.2520	0.4	0.4	0.2	0.8259	0.8290
0.1	0.7	0.2	0.2350	0.2298	0.4	0.5	0.1	0.8018	0.8131
0.1	0.8	0.1	0.2148	0.2150	0.5	0.1	0.4	1.0665	1.0734
0.2	0.1	0.7	0.5127	0.5161	0.5	0.2	0.3	1.0581	1.0546
0.2	0.2	0.6	0.5028	0.4995	0.5	0.3	0.2	1.0375	1.0290
0.2	0.3	0.5	0.4753	0.4877	0.5	0.4	0.1	1.0121	1.0072
0.2	0.4	0.4	0.4606	0.4643	0.6	0.1	0.3	1.2315	1.2440
0.2	0.5	0.3	0.4547	0.4598	0.6	0.2	0.2	1.2309	1.2316
0.2	0.6	0.2	0.4354	0.4348	0.6	0.3	0.1	1.2190	1.2114
0.2	0.7	0.1	0.4148	0.4188	0.7	0.1	0.2	1.4114	1.4443
0.3	0.1	0.6	0.7054	0.6892	0.7	0.2	0.1	1.4095	1.4008
0.3	0.2	0.5	0.6909	0.6794	0.8	0.1	0.1	1.6047	1.5720
0.3	0.3	0.4	0.6707	0.6655		MAX		1.6047	1.5720
0.3	0.4	0.3	0.6347	0.6398		MIN		0.2148	0.2150

TABLE 3.2.31 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 12$, $J = 6$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.3248	0.3245	0.3	0.5	0.2	0.6360	0.6222
0.1	0.2	0.7	0.3110	0.3113	0.3	0.6	0.1	0.6136	0.6233
0.1	0.3	0.6	0.2972	0.2975	0.4	0.1	0.5	0.8763	0.8710
0.1	0.4	0.5	0.2744	0.2839	0.4	0.2	0.4	0.8849	0.8488
0.1	0.5	0.4	0.2601	0.2658	0.4	0.3	0.3	0.8562	0.8491
0.1	0.6	0.3	0.2507	0.2448	0.4	0.4	0.2	0.8220	0.8251
0.1	0.7	0.2	0.2314	0.2343	0.4	0.5	0.1	0.8053	0.8124
0.1	0.8	0.1	0.2109	0.2159	0.5	0.1	0.4	1.0709	1.0670
0.2	0.1	0.7	0.5068	0.5200	0.5	0.2	0.3	1.0436	1.0403
0.2	0.2	0.6	0.4917	0.4908	0.5	0.3	0.2	1.0357	1.0344
0.2	0.3	0.5	0.4770	0.4735	0.5	0.4	0.1	1.0170	1.0024
0.2	0.4	0.4	0.4678	0.4659	0.6	0.1	0.3	1.2538	1.2596
0.2	0.5	0.3	0.4483	0.4520	0.6	0.2	0.2	1.2389	1.2171
0.2	0.6	0.2	0.4299	0.4258	0.6	0.3	0.1	1.2082	1.2040
0.2	0.7	0.1	0.4113	0.4174	0.7	0.1	0.2	1.4421	1.4005
0.3	0.1	0.6	0.6939	0.6963	0.7	0.2	0.1	1.4331	1.4412
0.3	0.2	0.5	0.6742	0.6872	0.8	0.1	0.1	1.6483	1.6099
0.3	0.3	0.4	0.6518	0.6693		MAX		1.6483	1.6099
0.3	0.4	0.3	0.6466	0.6466		MIN		0.2109	0.2159

TABLE 3.2.32 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 12$, $J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.2605	0.2635	0.3	0.5	0.2	0.6158	0.6123
0.1	0.2	0.7	0.2584	0.2552	0.3	0.6	0.1	0.6052	0.6076
0.1	0.3	0.6	0.2473	0.2454	0.4	0.1	0.5	0.8452	0.8424
0.1	0.4	0.5	0.2426	0.2388	0.4	0.2	0.4	0.8394	0.8309
0.1	0.5	0.4	0.2308	0.2323	0.4	0.3	0.3	0.8161	0.8263
0.1	0.6	0.3	0.2250	0.2182	0.4	0.4	0.2	0.8112	0.8061
0.1	0.7	0.2	0.2189	0.2177	0.4	0.5	0.1	0.7868	0.7981
0.1	0.8	0.1	0.2103	0.2070	0.5	0.1	0.4	1.0402	1.0267
0.2	0.1	0.7	0.4495	0.4552	0.5	0.2	0.3	1.0266	1.0287
0.2	0.2	0.6	0.4434	0.4510	0.5	0.3	0.2	0.9928	1.0287
0.2	0.3	0.5	0.4402	0.4442	0.5	0.4	0.1	1.0051	0.9920
0.2	0.4	0.4	0.4305	0.4295	0.6	0.1	0.3	1.1955	1.2161
0.2	0.5	0.3	0.4293	0.4261	0.6	0.2	0.2	1.2033	1.2160
0.2	0.6	0.2	0.4140	0.4171	0.6	0.3	0.1	1.2127	1.2295
0.2	0.7	0.1	0.4114	0.4039	0.7	0.1	0.2	1.4049	1.4312
0.3	0.1	0.6	0.6504	0.6397	0.7	0.2	0.1	1.3963	1.3820
0.3	0.2	0.5	0.6440	0.6376	0.8	0.1	0.1	1.5969	1.6171
0.3	0.3	0.4	0.6300	0.6248		MAX		1.5969	1.6171
0.3	0.4	0.3	0.6120	0.6163		MIN		0.2103	0.2070

TABLE 3.2.33 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 24$, $J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.2573	0.2549	0.3	0.5	0.2	0.3769	0.3816
0.1	0.2	0.7	0.2441	0.2438	0.3	0.6	0.1	0.3632	0.3586
0.1	0.3	0.6	0.2257	0.2232	0.4	0.1	0.5	0.5499	0.5516
0.1	0.4	0.5	0.2100	0.2077	0.4	0.2	0.4	0.5266	0.5287
0.1	0.5	0.4	0.1898	0.1916	0.4	0.3	0.3	0.5128	0.5140
0.1	0.6	0.3	0.1732	0.1713	0.4	0.4	0.2	0.4915	0.4859
0.1	0.7	0.2	0.1515	0.1549	0.4	0.5	0.1	0.4676	0.4723
0.1	0.8	0.1	0.1337	0.1324	0.5	0.1	0.4	0.6509	0.6453
0.2	0.1	0.7	0.3592	0.3619	0.5	0.2	0.3	0.6280	0.6278
0.2	0.2	0.6	0.3421	0.3450	0.5	0.3	0.2	0.6024	0.5991
0.2	0.3	0.5	0.3208	0.3208	0.5	0.4	0.1	0.5889	0.5862
0.2	0.4	0.4	0.3066	0.3065	0.6	0.1	0.3	0.7437	0.7283
0.2	0.5	0.3	0.2848	0.2895	0.6	0.2	0.2	0.7192	0.7176
0.2	0.6	0.2	0.2630	0.2644	0.6	0.3	0.1	0.6997	0.6996
0.2	0.7	0.1	0.2472	0.2458	0.7	0.1	0.2	0.8452	0.8361
0.3	0.1	0.6	0.4520	0.4533	0.7	0.2	0.1	0.8084	0.8150
0.3	0.2	0.5	0.4358	0.4387	0.8	0.1	0.1	0.9246	0.9261
0.3	0.3	0.4	0.4204	0.4200		MAX		0.9246	0.9261
0.3	0.4	0.3	0.3981	0.3963		MIN		0.1337	0.1324

TABLE 3.2.34 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 24$, $J = 3$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.1901	0.1905	0.3	0.5	0.2	0.3578	0.3559
0.1	0.2	0.7	0.1803	0.1818	0.3	0.6	0.1	0.3499	0.3560
0.1	0.3	0.6	0.1707	0.1707	0.4	0.1	0.5	0.5006	0.5032
0.1	0.4	0.5	0.1596	0.1633	0.4	0.2	0.4	0.4969	0.4898
0.1	0.5	0.4	0.1522	0.1536	0.4	0.3	0.3	0.4832	0.4834
0.1	0.6	0.3	0.1411	0.1435	0.4	0.4	0.2	0.4706	0.4781
0.1	0.7	0.2	0.1316	0.1324	0.4	0.5	0.1	0.4651	0.4644
0.1	0.8	0.1	0.1232	0.1225	0.5	0.1	0.4	0.6064	0.6020
0.2	0.1	0.7	0.2940	0.2946	0.5	0.2	0.3	0.5978	0.5980
0.2	0.2	0.6	0.2829	0.2858	0.5	0.3	0.2	0.5920	0.5755
0.2	0.3	0.5	0.2752	0.2737	0.5	0.4	0.1	0.5738	0.5778
0.2	0.4	0.4	0.2634	0.2660	0.6	0.1	0.3	0.7047	0.7095
0.2	0.5	0.3	0.2569	0.2540	0.6	0.2	0.2	0.7041	0.7059
0.2	0.6	0.2	0.2411	0.2469	0.6	0.3	0.1	0.6895	0.6912
0.2	0.7	0.1	0.2352	0.2398	0.7	0.1	0.2	0.8116	0.8194
0.3	0.1	0.6	0.3906	0.4007	0.7	0.2	0.1	0.8004	0.8101
0.3	0.2	0.5	0.3859	0.3894	0.8	0.1	0.1	0.9224	0.9135
0.3	0.3	0.4	0.3772	0.3816		MAX		0.9224	0.9135
0.3	0.4	0.3	0.3711	0.3652		MIN		0.1232	0.1225

TABLE 3.2.35 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 24$, $J = 6$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.1896	0.1898	0.3	0.5	0.2	0.3598	0.3617
0.1	0.2	0.7	0.1793	0.1818	0.3	0.6	0.1	0.3517	0.3497
0.1	0.3	0.6	0.1680	0.1713	0.4	0.1	0.5	0.5064	0.5005
0.1	0.4	0.5	0.1624	0.1603	0.4	0.2	0.4	0.4918	0.4981
0.1	0.5	0.4	0.1508	0.1522	0.4	0.3	0.3	0.4855	0.4833
0.1	0.6	0.3	0.1393	0.1418	0.4	0.4	0.2	0.4673	0.4744
0.1	0.7	0.2	0.1313	0.1324	0.4	0.5	0.1	0.4584	0.4643
0.1	0.8	0.1	0.1224	0.1249	0.5	0.1	0.4	0.6106	0.6012
0.2	0.1	0.7	0.2977	0.2964	0.5	0.2	0.3	0.5957	0.5901
0.2	0.2	0.6	0.2819	0.2810	0.5	0.3	0.2	0.5901	0.5830
0.2	0.3	0.5	0.2747	0.2751	0.5	0.4	0.1	0.5800	0.5808
0.2	0.4	0.4	0.2665	0.2652	0.6	0.1	0.3	0.7104	0.7125
0.2	0.5	0.3	0.2534	0.2571	0.6	0.2	0.2	0.7045	0.6836
0.2	0.6	0.2	0.2467	0.2454	0.6	0.3	0.1	0.6855	0.6794
0.2	0.7	0.1	0.2382	0.2379	0.7	0.1	0.2	0.8183	0.8114
0.3	0.1	0.6	0.3944	0.3991	0.7	0.2	0.1	0.8055	0.8010
0.3	0.2	0.5	0.3900	0.3844	0.8	0.1	0.1	0.9131	0.9106
0.3	0.3	0.4	0.3745	0.3800		MAX		0.9131	0.9106
0.3	0.4	0.3	0.3675	0.3720		MIN		0.1224	0.1249

TABLE 3.2.36 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_P^2

when $I = 24$, $J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.1518	0.1504	0.3	0.5	0.2	0.3500	0.3474
0.1	0.2	0.7	0.1469	0.1461	0.3	0.6	0.1	0.3422	0.3471
0.1	0.3	0.6	0.1410	0.1420	0.4	0.1	0.5	0.4760	0.4741
0.1	0.4	0.5	0.1365	0.1370	0.4	0.2	0.4	0.4760	0.4733
0.1	0.5	0.4	0.1326	0.1326	0.4	0.3	0.3	0.4709	0.4665
0.1	0.6	0.3	0.1262	0.1295	0.4	0.4	0.2	0.4638	0.4633
0.1	0.7	0.2	0.1231	0.1217	0.4	0.5	0.1	0.4579	0.4571
0.1	0.8	0.1	0.1180	0.1157	0.5	0.1	0.4	0.5828	0.5875
0.2	0.1	0.7	0.2590	0.2589	0.5	0.2	0.3	0.5824	0.5818
0.2	0.2	0.6	0.2567	0.2545	0.5	0.3	0.2	0.5730	0.5698
0.2	0.3	0.5	0.2534	0.2500	0.5	0.4	0.1	0.5711	0.5683
0.2	0.4	0.4	0.2483	0.2449	0.6	0.1	0.3	0.7023	0.6885
0.2	0.5	0.3	0.2449	0.2441	0.6	0.2	0.2	0.6893	0.6867
0.2	0.6	0.2	0.2363	0.2357	0.6	0.3	0.1	0.6855	0.6840
0.2	0.7	0.1	0.2314	0.2307	0.7	0.1	0.2	0.8007	0.8087
0.3	0.1	0.6	0.3698	0.3695	0.7	0.2	0.1	0.8007	0.7879
0.3	0.2	0.5	0.3586	0.3617	0.8	0.1	0.1	0.9117	0.9074
0.3	0.3	0.4	0.3632	0.3590		MAX		0.9117	0.9074
0.3	0.4	0.3	0.3526	0.3541		MIN		0.1180	0.1157

TABLE 3.2.37 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 6,\ J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	41.2280	39.9970	0.3	0.5	0.2	128.6260	133.6180
0.1	0.2	0.7	66.7740	64.4340	0.3	0.6	0.1	160.3300	166.4220
0.1	0.3	0.6	84.7690	89.1080	0.4	0.1	0.5	35.0400	36.2870
0.1	0.4	0.5	118.6260	112.5690	0.4	0.2	0.4	56.2780	60.5430
0.1	0.5	0.4	138.6160	138.7830	0.4	0.3	0.3	84.4010	81.8600
0.1	0.6	0.3	155.6330	161.2230	0.4	0.4	0.2	104.0160	107.6760
0.1	0.7	0.2	175.0060	173.9400	0.4	0.5	0.1	124.7860	125.1090
0.1	0.8	0.1	206.3010	199.6560	0.5	0.1	0.4	34.8120	34.5300
0.2	0.1	0.7	39.8220	42.1850	0.5	0.2	0.3	54.6980	58.0090
0.2	0.2	0.6	64.6690	61.2710	0.5	0.3	0.2	81.5130	80.2250
0.2	0.3	0.5	86.6470	87.5220	0.5	0.4	0.1	100.0820	108.1530
0.2	0.4	0.4	109.6170	112.6760	0.6	0.1	0.3	31.5960	29.5770
0.2	0.5	0.3	129.9920	144.2040	0.6	0.2	0.2	58.9650	56.8200
0.2	0.6	0.2	159.8200	175.6120	0.6	0.3	0.1	77.4030	79.3600
0.2	0.7	0.1	178.1340	172.2840	0.7	0.1	0.2	29.0740	27.9340
0.3	0.1	0.6	39.4330	38.5810	0.7	0.2	0.1	52.4770	52.1490
0.3	0.2	0.5	63.2580	58.7850	0.8	0.1	0.1	26.9850	25.8890
0.3	0.3	0.4	87.0310	85.6460		MAX		206.3010	199.6560
0.3	0.4	0.3	108.3340	109.2060		MIN		26.9850	25.8890

TABLE 3.2.38 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 6$, $J = 3$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	34.9660	34.1150	0.3	0.5	0.2	128.7660	124.3280
0.1	0.2	0.7	61.4870	58.3610	0.3	0.6	0.1	157.3990	154.1850
0.1	0.3	0.6	82.6490	82.9950	0.4	0.1	0.5	31.9690	30.3390
0.1	0.4	0.5	111.0430	105.0830	0.4	0.2	0.4	56.1160	55.3820
0.1	0.5	0.4	137.3810	133.8720	0.4	0.3	0.3	83.1180	80.9270
0.1	0.6	0.3	156.9460	158.7980	0.4	0.4	0.2	102.1030	104.1390
0.1	0.7	0.2	183.3230	184.5570	0.4	0.5	0.1	128.3220	128.1520
0.1	0.8	0.1	190.2220	205.0110	0.5	0.1	0.4	29.8150	30.7960
0.2	0.1	0.7	33.9000	32.1030	0.5	0.2	0.3	56.2670	56.1730
0.2	0.2	0.6	59.0870	56.6090	0.5	0.3	0.2	80.4930	78.2860
0.2	0.3	0.5	79.2270	80.5710	0.5	0.4	0.1	96.5560	108.3320
0.2	0.4	0.4	99.8950	106.7420	0.6	0.1	0.3	28.1470	27.4130
0.2	0.5	0.3	126.6320	131.0580	0.6	0.2	0.2	51.1990	56.1010
0.2	0.6	0.2	149.0080	152.7570	0.6	0.3	0.1	77.6700	77.9640
0.2	0.7	0.1	181.0060	181.8180	0.7	0.1	0.2	26.6460	28.3690
0.3	0.1	0.6	31.5170	30.4120	0.7	0.2	0.1	48.2520	51.4130
0.3	0.2	0.5	54.6210	52.5150	0.8	0.1	0.1	25.3970	26.1900
0.3	0.3	0.4	80.5730	82.4390		MAX		190.2220	205.0110
0.3	0.4	0.3	103.7140	107.9850		MIN		25.3970	26.1900

TABLE 3.2.39 Range of Average Interval Lengths for 90% Two-sided Intervals on σ_O^2 when $I=6,\ J=6$, and $K=2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.8802	0.8496	0.3	0.5	0.2	2.7586	2.6532
0.1	0.2	0.7	1.3039	1.3295	0.3	0.6	0.1	3.2640	3.1628
0.1	0.3	0.6	1.8046	1.8823	0.4	0.1	0.5	0.7252	0.7344
0.1	0.4	0.5	2.2831	2.3246	0.4	0.2	0.4	1.2052	1.2277
0.1	0.5	0.4	2.7097	2.8030	0.4	0.3	0.3	1.7190	1.6729
0.1	0.6	0.3	3.1840	3.2328	0.4	0.4	0.2	2.1499	2.1697
0.1	0.7	0.2	3.8538	3.7468	0.4	0.5	0.1	2.6532	2.6220
0.1	0.8	0.1	4.2047	4.2895	0.5	0.1	0.4	0.6933	0.7029
0.2	0.1	0.7	0.8337	0.8072	0.5	0.2	0.3	1.1735	1.1775
0.2	0.2	0.6	1.2829	1.3029	0.5	0.3	0.2	1.6598	1.6898
0.2	0.3	0.5	1.7607	1.7249	0.5	0.4	0.1	2.1970	2.0730
0.2	0.4	0.4	2.2660	2.2629	0.6	0.1	0.3	0.6486	0.6517
0.2	0.5	0.3	2.6810	2.7209	0.6	0.2	0.2	1.1241	1.0951
0.2	0.6	0.2	3.1897	3.2032	0.6	0.3	0.1	1.5885	1.5944
0.2	0.7	0.1	3.6999	3.6668	0.7	0.1	0.2	0.6122	0.5902
0.3	0.1	0.6	0.7683	0.7628	0.7	0.2	0.1	1.0755	1.1022
0.3	0.2	0.5	1.2476	1.2453	0.8	0.1	0.1	0.5826	0.5604
0.3	0.3	0.4	1.6864	1.7232		MAX		4.2047	4.2895
0.3	0.4	0.3	2.2280	2.2085		MIN		0.5826	0.5604

TABLE 3.2.40 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 6$, $J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.6912	0.6968	0.3	0.5	0.2	2.6518	2.6503
0.1	0.2	0.7	1.1958	1.2137	0.3	0.6	0.1	3.1121	3.1406
0.1	0.3	0.6	1.6794	1.6995	0.4	0.1	0.5	0.6163	0.6271
0.1	0.4	0.5	2.1703	2.1986	0.4	0.2	0.4	1.1347	1.1003
0.1	0.5	0.4	2.7144	2.6531	0.4	0.3	0.3	1.6010	1.6362
0.1	0.6	0.3	3.2157	3.1804	0.4	0.4	0.2	2.1057	2.1140
0.1	0.7	0.2	3.5585	3.7412	0.4	0.5	0.1	2.5862	2.5754
0.1	0.8	0.1	4.1661	4.2859	0.5	0.1	0.4	0.5888	0.6199
0.2	0.1	0.7	0.6831	0.6654	0.5	0.2	0.3	1.1027	1.1020
0.2	0.2	0.6	1.1782	1.1966	0.5	0.3	0.2	1.5560	1.6127
0.2	0.3	0.5	1.6276	1.7187	0.5	0.4	0.1	2.0594	2.1038
0.2	0.4	0.4	2.1932	2.1579	0.6	0.1	0.3	0.5916	0.5796
0.2	0.5	0.3	2.7086	2.6640	0.6	0.2	0.2	1.0926	1.1068
0.2	0.6	0.2	3.1815	3.1943	0.6	0.3	0.1	1.6288	1.5678
0.2	0.7	0.1	3.5845	3.6426	0.7	0.1	0.2	0.5778	0.5540
0.3	0.1	0.6	0.6441	0.6501	0.7	0.2	0.1	1.0818	1.0655
0.3	0.2	0.5	1.1612	1.1510	0.8	0.1	0.1	0.5574	0.5398
0.3	0.3	0.4	1.6360	1.6926		MAX		4.1661	4.2859
0.3	0.4	0.3	2.0746	2.1283		MIN		0.5574	0.5398

TABLE 3.2.41 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 12$, $J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	34.2360	34.6380	0.3	0.5	0.2	128.7850	129.3130
0.1	0.2	0.7	57.6980	58.2530	0.3	0.6	0.1	157.9580	152.1130
0.1	0.3	0.6	83.5680	81.0600	0.4	0.1	0.5	33.0630	29.0530
0.1	0.4	0.5	111.9440	102.0030	0.4	0.2	0.4	56.1230	57.8670
0.1	0.5	0.4	128.0870	130.3090	0.4	0.3	0.3	78.9920	79.4960
0.1	0.6	0.3	151.8230	157.8610	0.4	0.4	0.2	103.5410	99.2340
0.1	0.7	0.2	174.5010	181.8950	0.4	0.5	0.1	125.8790	134.9990
0.1	0.8	0.1	207.3970	211.8480	0.5	0.1	0.4	31.0470	28.2320
0.2	0.1	0.7	32.5940	33.1180	0.5	0.2	0.3	51.9080	56.6400
0.2	0.2	0.6	58.8100	59.0010	0.5	0.3	0.2	76.9310	74.8750
0.2	0.3	0.5	80.6940	81.8940	0.5	0.4	0.1	104.0590	98.2220
0.2	0.4	0.4	108.9360	106.5810	0.6	0.1	0.3	27.4060	27.4840
0.2	0.5	0.3	126.3520	123.8040	0.6	0.2	0.2	51.7690	56.0890
0.2	0.6	0.2	156.5070	153.0520	0.6	0.3	0.1	77.3460	75.0640
0.2	0.7	0.1	180.9660	179.8620	0.7	0.1	0.2	28.8710	25.7870
0.3	0.1	0.6	32.5660	31.7720	0.7	0.2	0.1	51.5540	50.7930
0.3	0.2	0.5	58.2670	56.7880	0.8	0.1	0.1	26.2830	24.9890
0.3	0.3	0.4	81.8020	81.9000		MAX		207.3970	211.8480
0.3	0.4	0.3	104.2830	103.8270		MIN		26.2830	24.9890

TABLE 3.2.42 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 12$, $J = 3$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	29.5520	29.7570	0.3	0.5	0.2	139.3480	130.4570
0.1	0.2	0.7	53.1010	57.8540	0.3	0.6	0.1	145.1240	148.0940
0.1	0.3	0.6	79.3340	76.0030	0.4	0.1	0.5	28.6110	26.9420
0.1	0.4	0.5	102.5410	106.9470	0.4	0.2	0.4	53.9970	52.7470
0.1	0.5	0.4	135.1940	126.8820	0.4	0.3	0.3	79.7700	78.8660
0.1	0.6	0.3	156.9340	159.6790	0.4	0.4	0.2	99.4980	102.3800
0.1	0.7	0.2	178.0260	191.2700	0.4	0.5	0.1	127.6320	126.5050
0.1	0.8	0.1	197.8040	203.7820	0.5	0.1	0.4	27.9790	26.8280
0.2	0.1	0.7	28.2740	30.0160	0.5	0.2	0.3	53.0480	51.4480
0.2	0.2	0.6	52.0030	55.1680	0.5	0.3	0.2	76.7250	77.7970
0.2	0.3	0.5	77.5070	77.4770	0.5	0.4	0.1	98.9830	96.0550
0.2	0.4	0.4	106.1150	97.6990	0.6	0.1	0.3	26.6620	26.1740
0.2	0.5	0.3	127.8760	132.9160	0.6	0.2	0.2	50.2850	53.4620
0.2	0.6	0.2	146.9840	150.3130	0.6	0.3	0.1	78.2460	75.2920
0.2	0.7	0.1	170.1280	181.4810	0.7	0.1	0.2	25.6230	26.3460
0.3	0.1	0.6	27.8920	28.3180	0.7	0.2	0.1	51.8640	53.4990
0.3	0.2	0.5	52.8650	54.3430	0.8	0.1	0.1	26.0170	25.7650
0.3	0.3	0.4	76.9570	78.5890		MAX		197.8040	203.7820
0.3	0.4	0.3	102.4960	105.0220		MIN		25.6230	25.7650

TABLE 3.2.43 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 12$, $J = 6$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.6971	0.6895	0.3	0.5	0.2	2.6554	2.6170
0.1	0.2	0.7	1.1915	1.1863	0.3	0.6	0.1	3.1686	3.1372
0.1	0.3	0.6	1.6620	1.7220	0.4	0.1	0.5	0.6253	0.6349
0.1	0.4	0.5	2.1783	2.2143	0.4	0.2	0.4	1.1412	1.1442
0.1	0.5	0.4	2.6066	2.6390	0.4	0.3	0.3	1.6307	1.6405
0.1	0.6	0.3	3.1918	3.2009	0.4	0.4	0.2	2.1313	2.1109
0.1	0.7	0.2	3.5913	3.7158	0.4	0.5	0.1	2.6183	2.5723
0.1	0.8	0.1	4.1242	4.2819	0.5	0.1	0.4	0.6037	0.6039
0.2	0.1	0.7	0.6773	0.6729	0.5	0.2	0.3	1.1116	1.1070
0.2	0.2	0.6	1.1897	1.1577	0.5	0.3	0.2	1.5919	1.6073
0.2	0.3	0.5	1.6679	1.6712	0.5	0.4	0.1	2.1348	2.0885
0.2	0.4	0.4	2.1859	2.1870	0.6	0.1	0.3	0.5832	0.5762
0.2	0.5	0.3	2.6830	2.6749	0.6	0.2	0.2	1.0764	1.0940
0.2	0.6	0.2	3.0723	3.1910	0.6	0.3	0.1	1.5967	1.6170
0.2	0.7	0.1	3.5987	3.6025	0.7	0.1	0.2	0.5636	0.5601
0.3	0.1	0.6	0.6379	0.6738	0.7	0.2	0.1	1.0784	1.0433
0.3	0.2	0.5	1.1530	1.1579	0.8	0.1	0.1	0.5403	0.5517
0.3	0.3	0.4	1.6816	1.6852		MAX		4.1242	4.2819
0.3	0.4	0.3	2.1269	2.1440		MIN		0.5403	0.5517

TABLE 3.2.44 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 12$, $J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.5996	0.6150	0.3	0.5	0.2	2.6491	2.6087
0.1	0.2	0.7	1.0942	1.1161	0.3	0.6	0.1	3.0902	3.1345
0.1	0.3	0.6	1.6222	1.6620	0.4	0.1	0.5	0.5782	0.5733
0.1	0.4	0.5	2.1383	2.1092	0.4	0.2	0.4	1.0971	1.0932
0.1	0.5	0.4	2.5889	2.6310	0.4	0.3	0.3	1.5684	1.6301
0.1	0.6	0.3	3.1887	3.1113	0.4	0.4	0.2	2.0939	2.0955
0.1	0.7	0.2	3.5777	3.5740	0.4	0.5	0.1	2.5857	2.6041
0.1	0.8	0.1	4.1022	4.0877	0.5	0.1	0.4	0.5625	0.5772
0.2	0.1	0.7	0.5989	0.6047	0.5	0.2	0.3	1.0819	1.1155
0.2	0.2	0.6	1.1216	1.1243	0.5	0.3	0.2	1.6372	1.6048
0.2	0.3	0.5	1.6409	1.6360	0.5	0.4	0.1	2.0774	2.1342
0.2	0.4	0.4	2.1032	2.1265	0.6	0.1	0.3	0.5412	0.5653
0.2	0.5	0.3	2.6589	2.6416	0.6	0.2	0.2	1.0562	1.0817
0.2	0.6	0.2	3.1740	3.1414	0.6	0.3	0.1	1.5593	1.5667
0.2	0.7	0.1	3.6724	3.6737	0.7	0.1	0.2	0.5500	0.5448
0.3	0.1	0.6	0.5755	0.5880	0.7	0.2	0.1	1.0966	1.0558
0.3	0.2	0.5	1.0959	1.1118	0.8	0.1	0.1	0.5352	0.5277
0.3	0.3	0.4	1.6297	1.5976		MAX		4.1022	4.0877
0.3	0.4	0.3	2.1644	2.0909		MIN		0.5352	0.5277

TABLE 3.2.45 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 24$, $J = 3$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	27.4860	28.6610	0.3	0.5	0.2	133.5450	126.3010
0.1	0.2	0.7	53.4370	51.4770	0.3	0.6	0.1	160.3030	149.6080
0.1	0.3	0.6	75.2240	77.1490	0.4	0.1	0.5	27.8870	29.7940
0.1	0.4	0.5	105.6260	109.9170	0.4	0.2	0.4	54.9520	50.4320
0.1	0.5	0.4	131.0810	132.3230	0.4	0.3	0.3	74.1450	76.0780
0.1	0.6	0.3	164.1510	148.4710	0.4	0.4	0.2	103.3260	102.8240
0.1	0.7	0.2	172.6020	181.5040	0.4	0.5	0.1	132.0460	129.3730
0.1	0.8	0.1	205.1960	211.2450	0.5	0.1	0.4	28.2570	28.5490
0.2	0.1	0.7	28.8070	30.3300	0.5	0.2	0.3	52.5110	55.6220
0.2	0.2	0.6	53.6660	55.6890	0.5	0.3	0.2	80.7900	77.0920
0.2	0.3	0.5	78.8980	84.8860	0.5	0.4	0.1	105.9270	100.3460
0.2	0.4	0.4	100.8070	103.4990	0.6	0.1	0.3	25.4230	25.9830
0.2	0.5	0.3	134.9150	130.6110	0.6	0.2	0.2	49.2190	51.6510
0.2	0.6	0.2	159.4260	147.3140	0.6	0.3	0.1	78.7470	76.5410
0.2	0.7	0.1	179.0820	181.5500	0.7	0.1	0.2	27.6110	25.2810
0.3	0.1	0.6	29.0230	28.9950	0.7	0.2	0.1	51.1270	48.4810
0.3	0.2	0.5	54.1230	52.3180	0.8	0.1	0.1	25.4910	24.7330
0.3	0.3	0.4	82.7330	82.0180		MAX		205.1960	211.2450
0.3	0.4	0.3	98.3360	104.4720		MIN		25.4230	24.7330

TABLE 3.2.46 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I=24$, $J=3$, and $K=4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	27.0070	27.3250	0.3	0.5	0.2	126.6620	130.0570
0.1	0.2	0.7	53.1470	52.9450	0.3	0.6	0.1	159.4130	155.5530
0.1	0.3	0.6	76.5400	80.8150	0.4	0.1	0.5	26.8090	26.7110
0.1	0.4	0.5	102.7540	105.1570	0.4	0.2	0.4	54.0760	52.3040
0.1	0.5	0.4	127.6640	123.2050	0.4	0.3	0.3	77.6300	79.6960
0.1	0.6	0.3	165.6880	150.4870	0.4	0.4	0.2	105.0850	106.0740
0.1	0.7	0.2	182.5380	170.0140	0.4	0.5	0.1	134.0360	130.9680
0.1	0.8	0.1	216.4460	206.6890	0.5	0.1	0.4	27.1090	26.5270
0.2	0.1	0.7	28.0110	27.4400	0.5	0.2	0.3	52.2250	53.2230
0.2	0.2	0.6	53.9890	51.3830	0.5	0.3	0.2	77.9930	76.7690
0.2	0.3	0.5	77.0940	78.5240	0.5	0.4	0.1	103.2420	102.5400
0.2	0.4	0.4	109.4740	103.6290	0.6	0.1	0.3	26.4440	26.1880
0.2	0.5	0.3	128.5670	129.1990	0.6	0.2	0.2	52.4730	48.8490
0.2	0.6	0.2	161.9280	151.7190	0.6	0.3	0.1	76.9280	74.7030
0.2	0.7	0.1	171.8740	173.4900	0.7	0.1	0.2	24.9920	25.7320
0.3	0.1	0.6	27.0030	27.4190	0.7	0.2	0.1	47.2500	51.1660
0.3	0.2	0.5	51.6950	51.9850	0.8	0.1	0.1	24.9220	25.2500
0.3	0.3	0.4	71.9060	77.6490		MAX		216.4460	206.6890
0.3	0.4	0.3	104.1280	108.8730		MIN		24.9220	25.2500

TABLE 3.2.47 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 24$, $J = 6$, and $K = 2$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.5953	0.6034	0.3	0.5	0.2	2.6400	2.6372
0.1	0.2	0.7	1.1105	1.1176	0.3	0.6	0.1	3.1089	3.1461
0.1	0.3	0.6	1.6202	1.6091	0.4	0.1	0.5	0.5622	0.5755
0.1	0.4	0.5	2.0854	2.0567	0.4	0.2	0.4	1.0435	1.0855
0.1	0.5	0.4	2.5841	2.6898	0.4	0.3	0.3	1.5793	1.6253
0.1	0.6	0.3	3.1539	3.2110	0.4	0.4	0.2	2.1097	2.0743
0.1	0.7	0.2	3.5694	3.6434	0.4	0.5	0.1	2.5776	2.6866
0.1	0.8	0.1	4.1620	4.0718	0.5	0.1	0.4	0.5722	0.5548
0.2	0.1	0.7	0.5894	0.6050	0.5	0.2	0.3	1.0477	1.0787
0.2	0.2	0.6	1.1204	1.1141	0.5	0.3	0.2	1.6052	1.5245
0.2	0.3	0.5	1.6110	1.6034	0.5	0.4	0.1	2.1582	2.0721
0.2	0.4	0.4	2.1201	2.0978	0.6	0.1	0.3	0.5596	0.5604
0.2	0.5	0.3	2.6830	2.5645	0.6	0.2	0.2	1.0789	1.0479
0.2	0.6	0.2	3.1731	3.1560	0.6	0.3	0.1	1.5856	1.5559
0.2	0.7	0.1	3.6258	3.5766	0.7	0.1	0.2	0.5509	0.5492
0.3	0.1	0.6	0.5828	0.5925	0.7	0.2	0.1	1.0427	1.0873
0.3	0.2	0.5	1.1061	1.0700	0.8	0.1	0.1	0.5321	0.5339
0.3	0.3	0.4	1.5929	1.5846		MAX		4.1620	4.0718
0.3	0.4	0.3	2.1730	2.1615		MIN		0.5321	0.5339

TABLE 3.2.48 Range of Average Interval Lengths

for 90% Two-sided Intervals on σ_O^2

when $I = 24$, $J = 6$, and $K = 4$

σ_P^2	σ_O^2	σ_E^2	TING	GEN	σ_P^2	σ_O^2	σ_E^2	TING	GEN
0.1	0.1	0.8	0.5559	0.5664	0.3	0.5	0.2	2.6311	2.6410
0.1	0.2	0.7	1.0805	1.1029	0.3	0.6	0.1	3.2010	3.1442
0.1	0.3	0.6	1.5343	1.6080	0.4	0.1	0.5	0.5584	0.5482
0.1	0.4	0.5	2.0939	2.1470	0.4	0.2	0.4	1.0550	1.0675
0.1	0.5	0.4	2.6083	2.5855	0.4	0.3	0.3	1.5761	1.6048
0.1	0.6	0.3	3.1946	3.0974	0.4	0.4	0.2	2.1019	2.0531
0.1	0.7	0.2	3.8067	3.6484	0.4	0.5	0.1	2.6268	2.5773
0.1	0.8	0.1	4.1850	4.1526	0.5	0.1	0.4	0.5421	0.5326
0.2	0.1	0.7	0.5557	0.5688	0.5	0.2	0.3	1.0709	1.0758
0.2	0.2	0.6	1.0520	1.0727	0.5	0.3	0.2	1.5399	1.5848
0.2	0.3	0.5	1.5754	1.5669	0.5	0.4	0.1	2.0340	2.0714
0.2	0.4	0.4	2.0792	2.0928	0.6	0.1	0.3	0.5318	0.5469
0.2	0.5	0.3	2.5827	2.7061	0.6	0.2	0.2	1.0676	1.0864
0.2	0.6	0.2	3.1827	3.2555	0.6	0.3	0.1	1.5508	1.6257
0.2	0.7	0.1	3.6921	3.7204	0.7	0.1	0.2	0.5363	0.5255
0.3	0.1	0.6	0.5703	0.5451	0.7	0.2	0.1	1.0587	1.0498
0.3	0.2	0.5	1.0725	1.0493	0.8	0.1	0.1	0.5264	0.5277
0.3	0.3	0.4	1.6222	1.5900		MAX		4.1850	4.1526
0.3	0.4	0.3	2.0683	2.1199		MIN		0.5264	0.5255

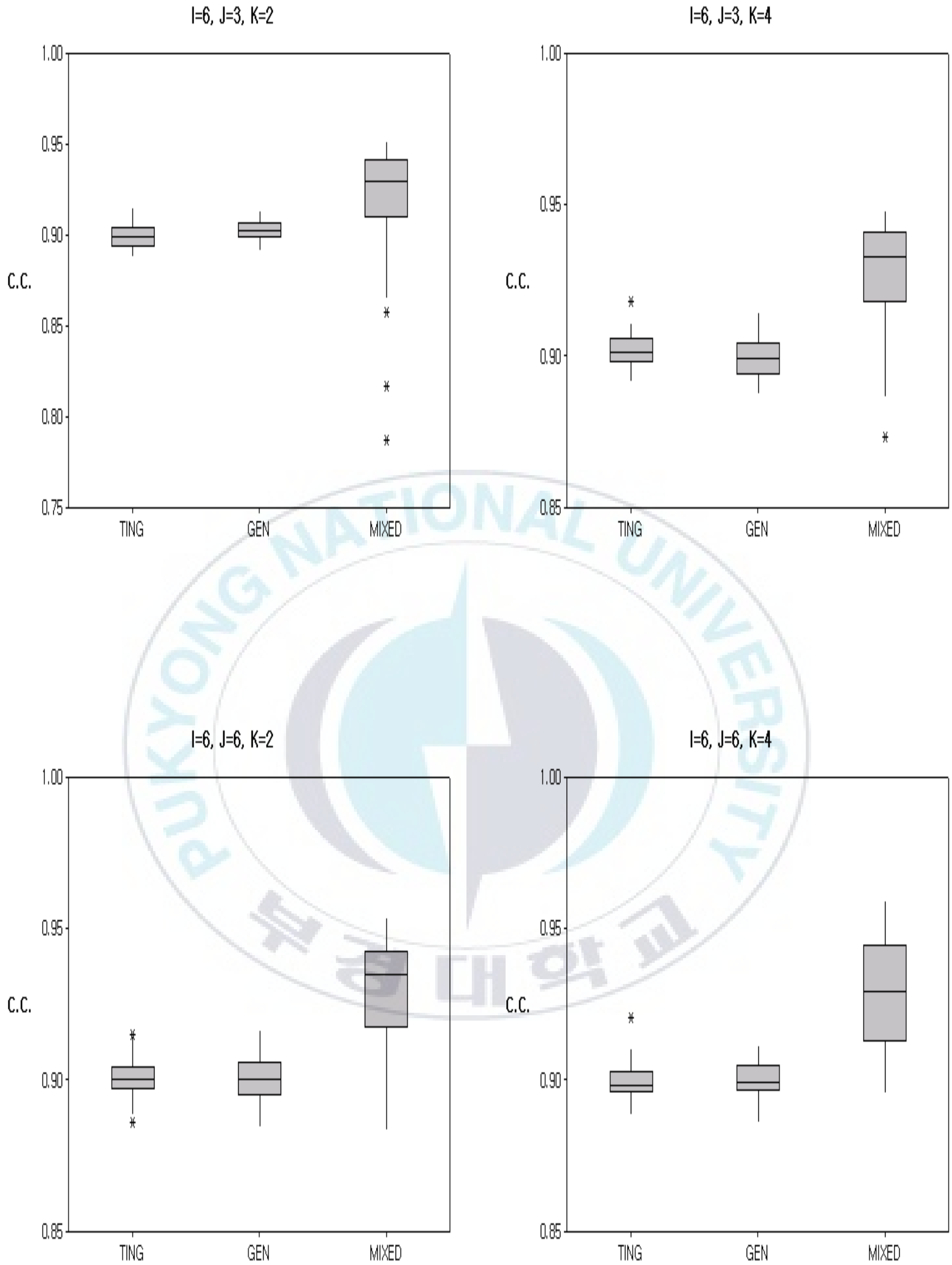

Figure 3.2.1.a Boxplots for Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2

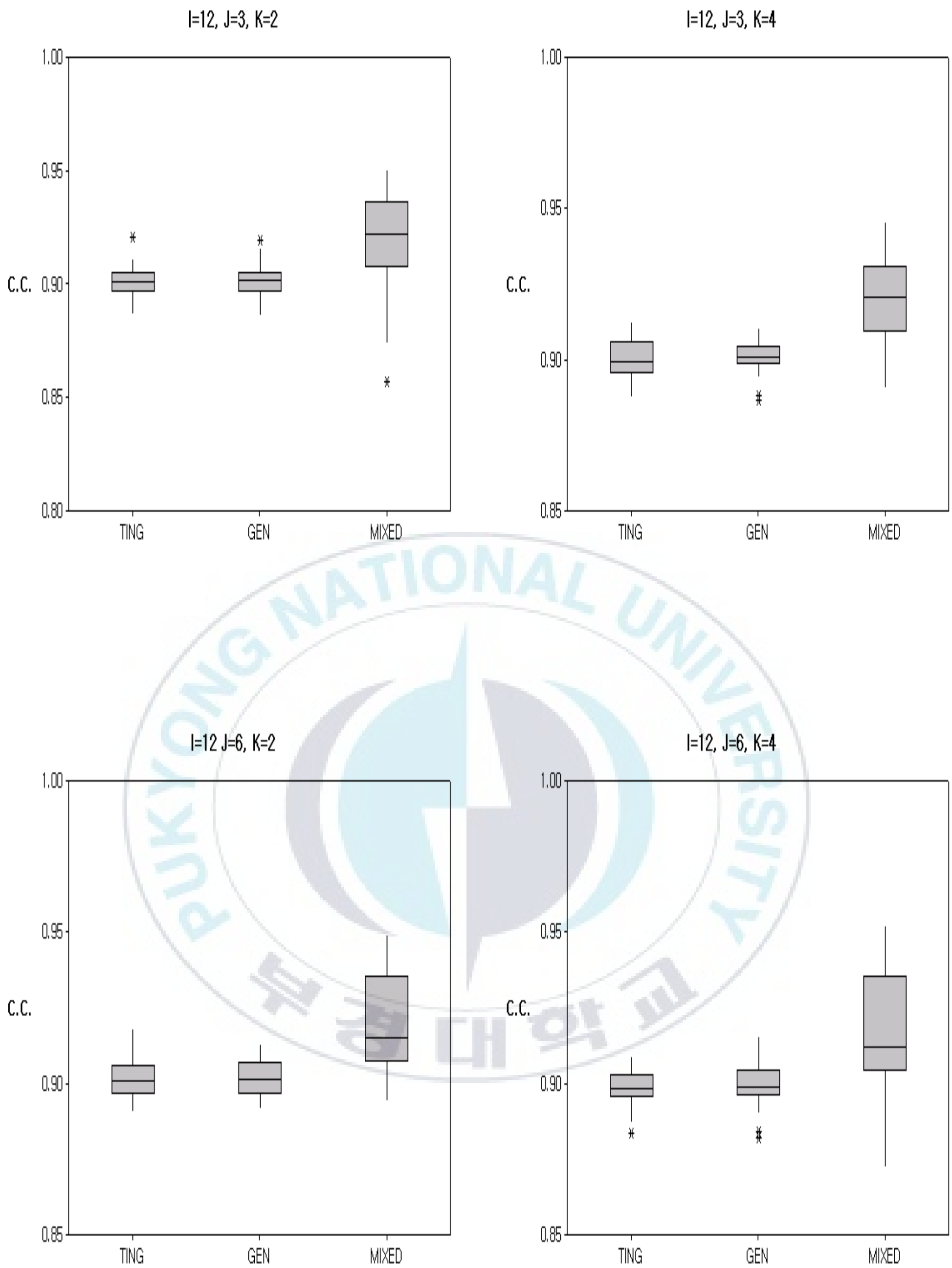

Figure 3.2.1.b Boxplots for Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2

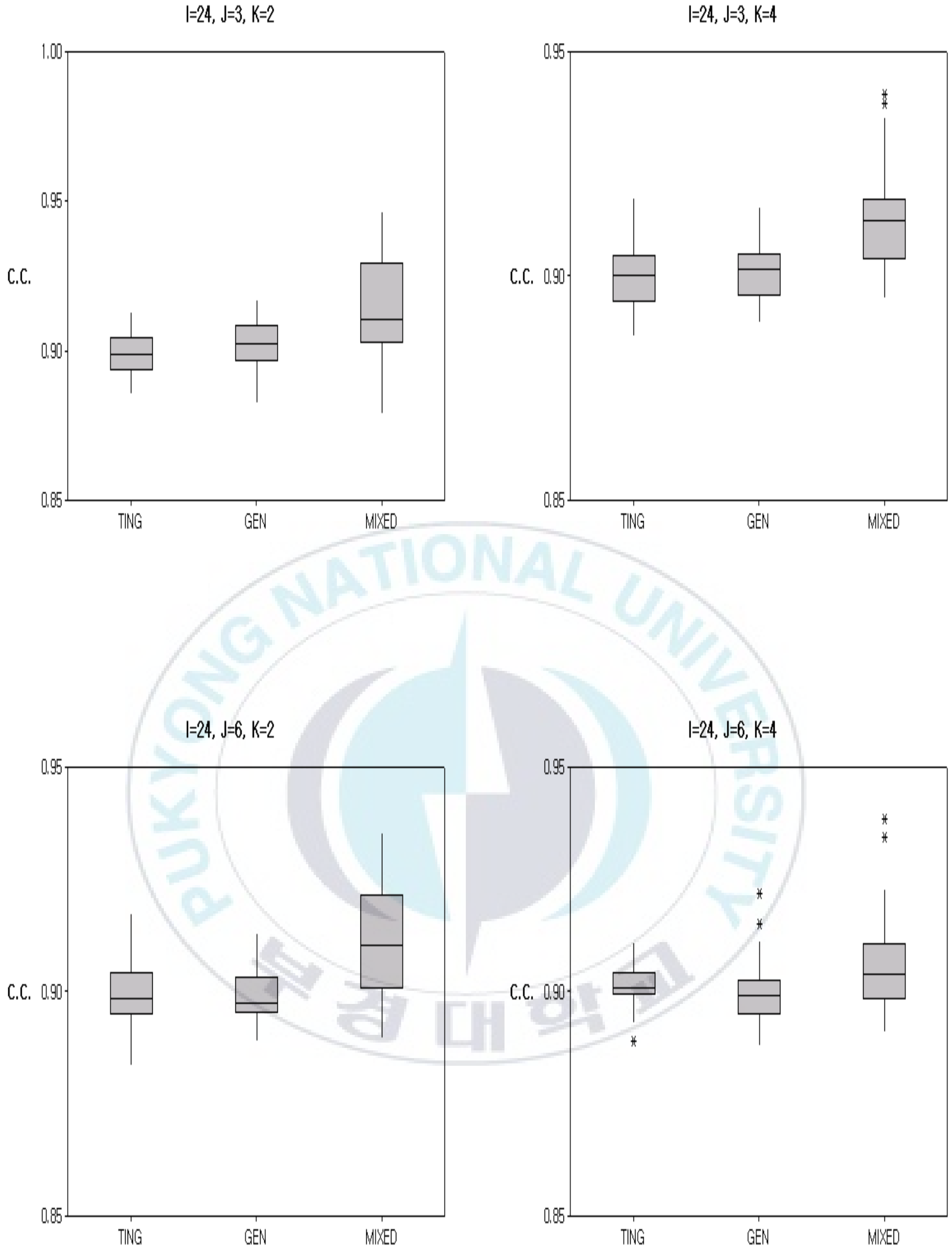

Figure 3.2.1.c Boxplots for Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_P^2

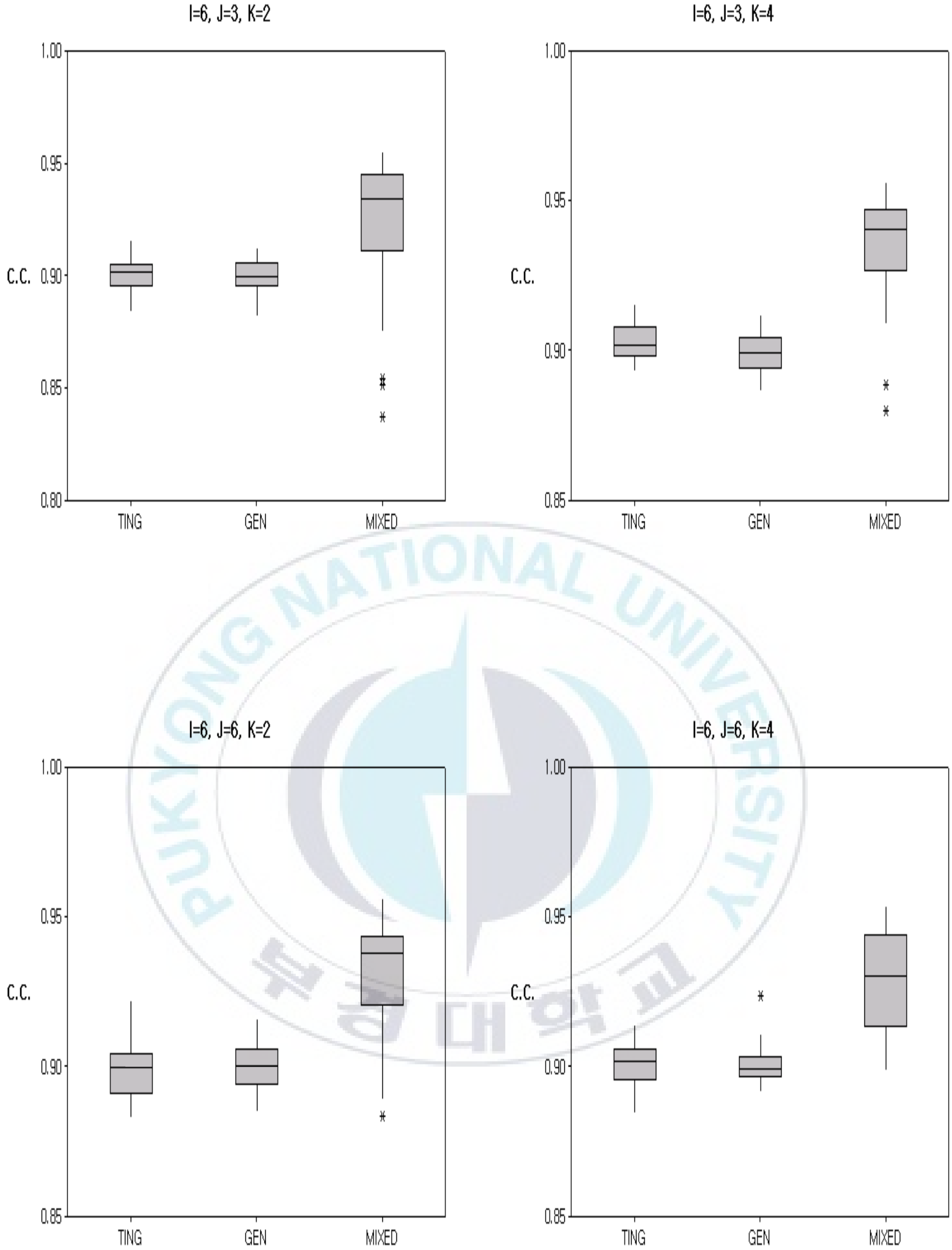

Figure 3.2.2.a Boxplots for Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2

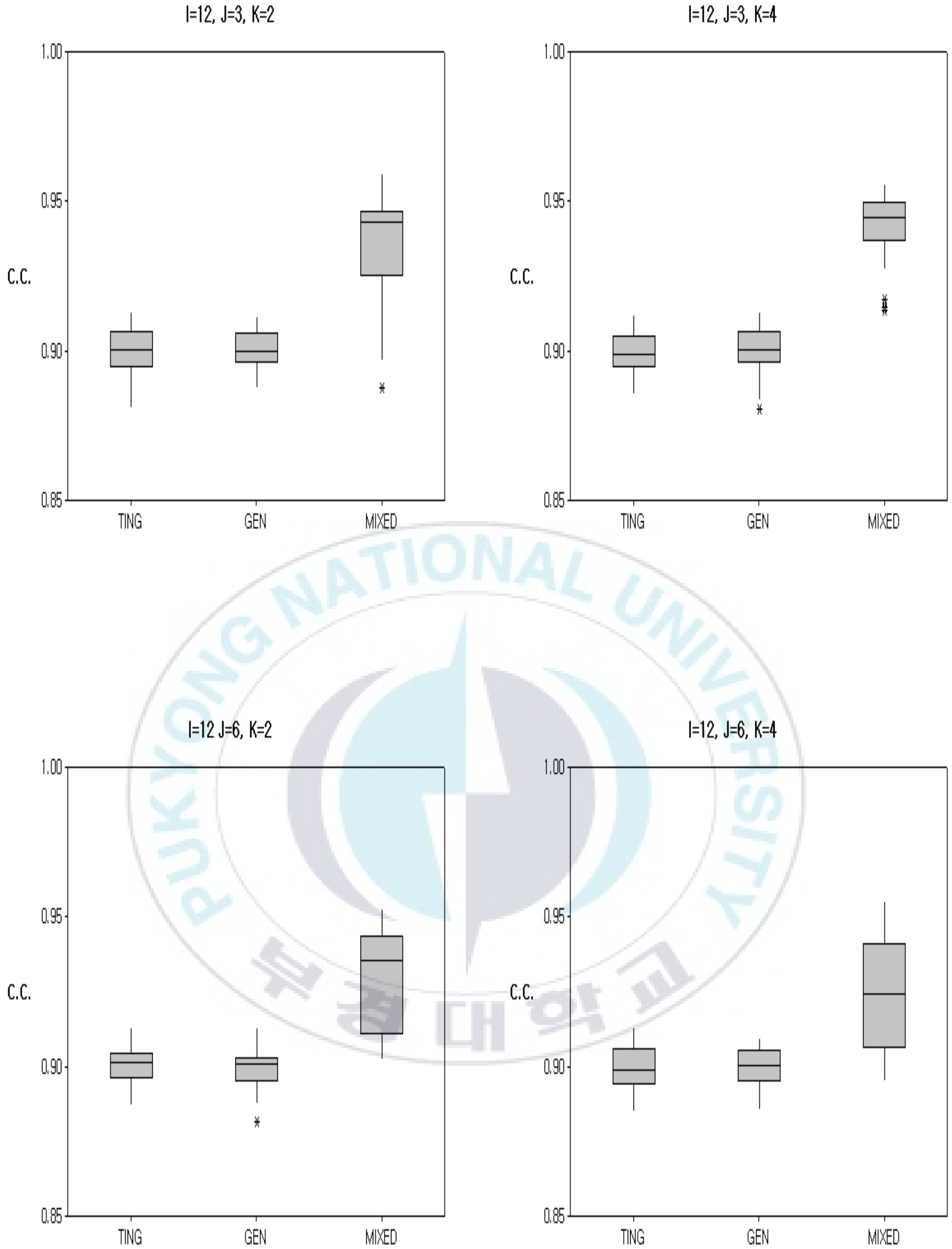

Figure 3.2.2.b Boxplots for Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2

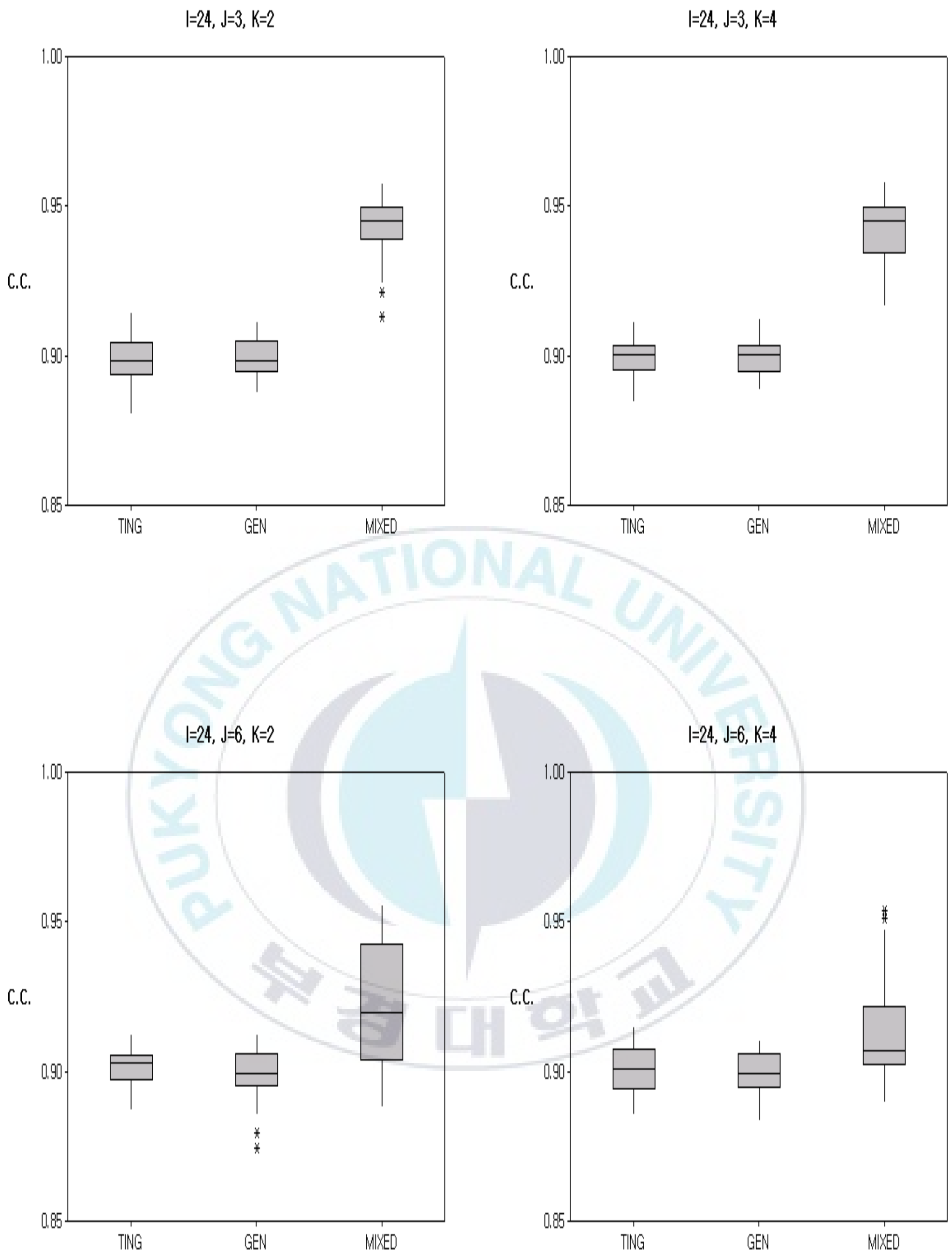

Figure 3.2.2.c Boxplots for Simulated Confidence Coefficients for 90% Two-sided Intervals on σ_O^2

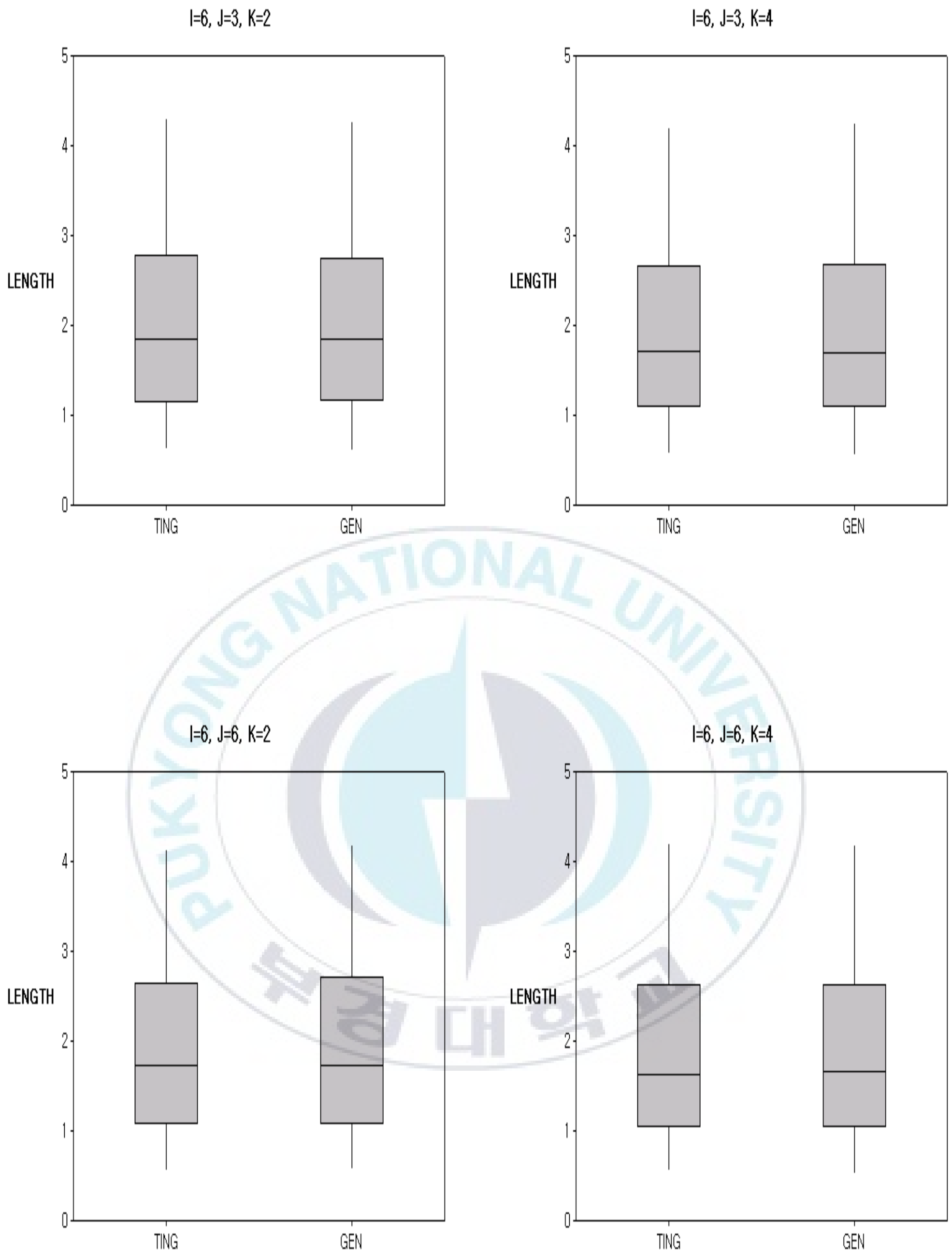

Figure 3.2.3.a Boxplots for Average Interval Lengths for 90% Two-sided Intervals on σ_P^2

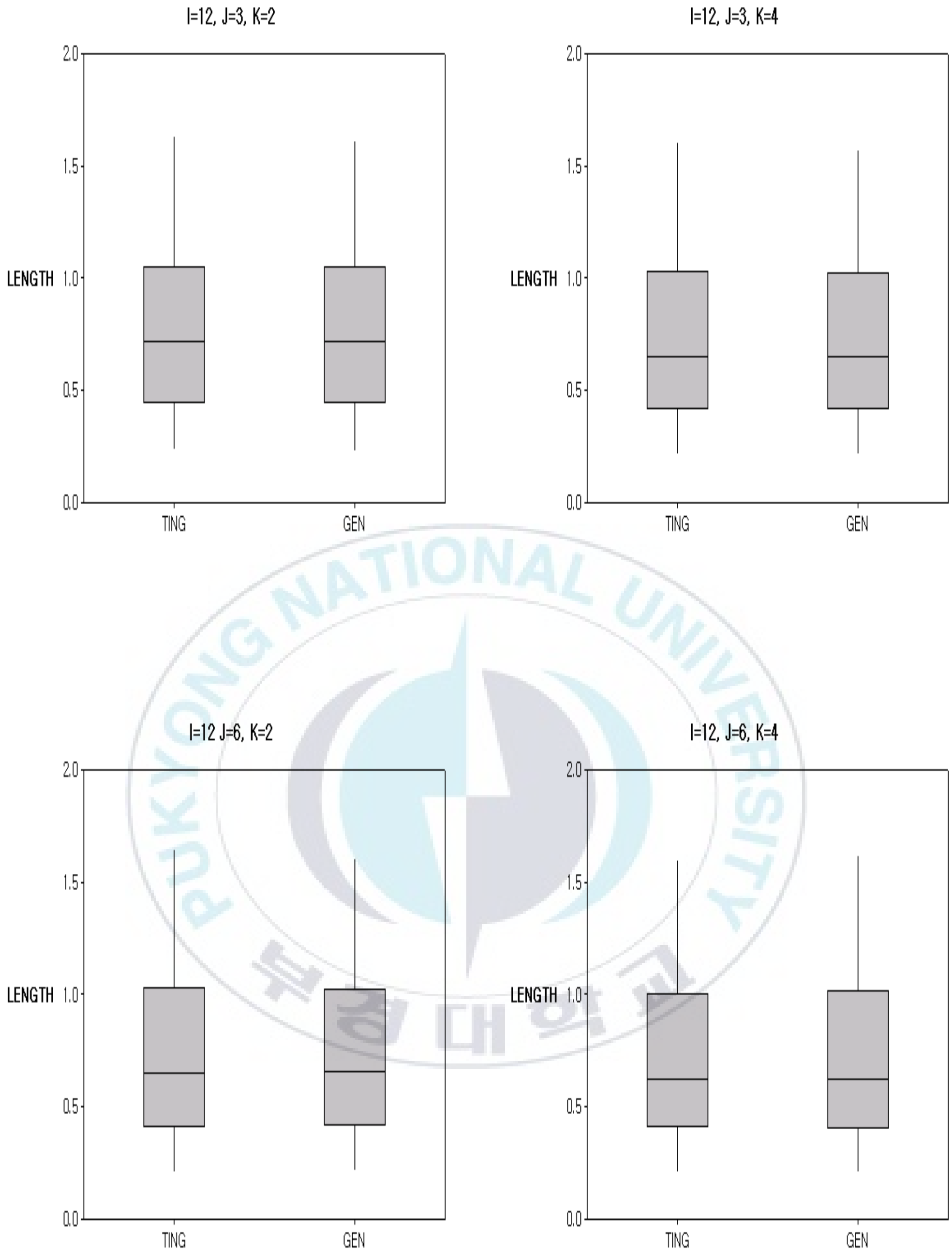

Figure 3.2.3.b Boxplots for Average Interval Lengths for 90% Two-sided Intervals on σ_P^2

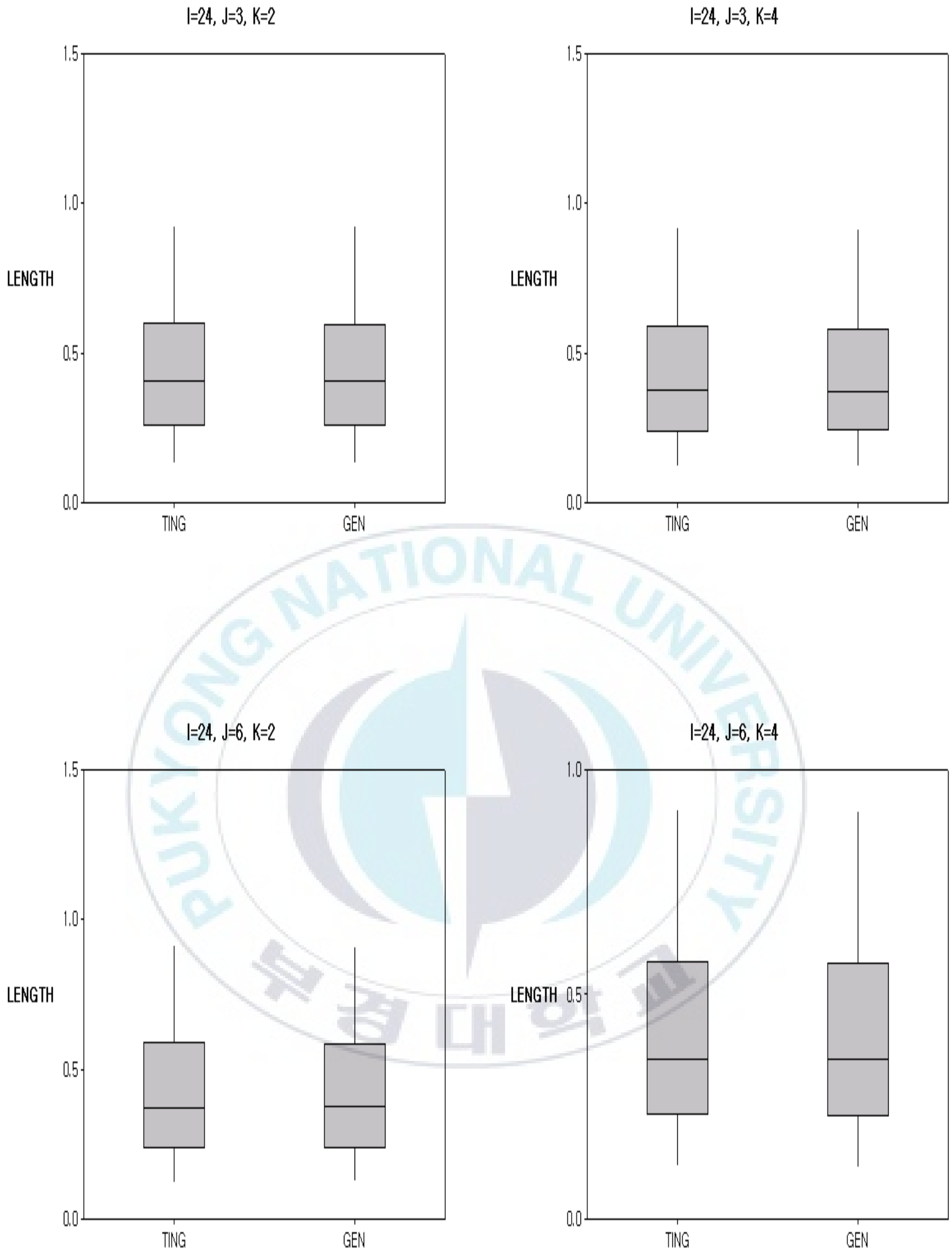

Figure 3.2.3.c Boxplots for Average Interval Lengths for 90% Two-sided Intervals on σ_P^2

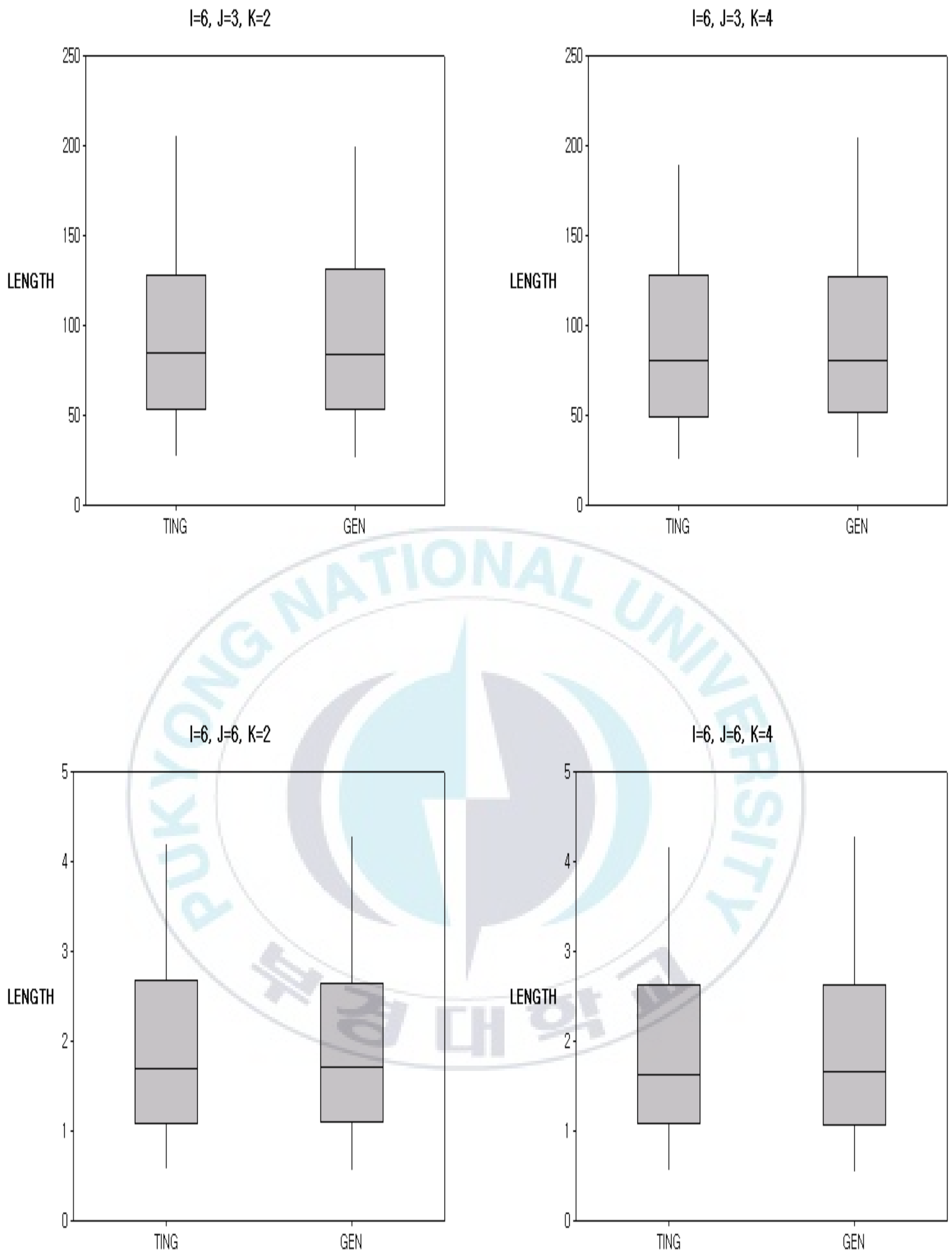

Figure 3.2.4.a Boxplots for Average Interval Lengths for 90% Two-sided Intervals on σ_O^2

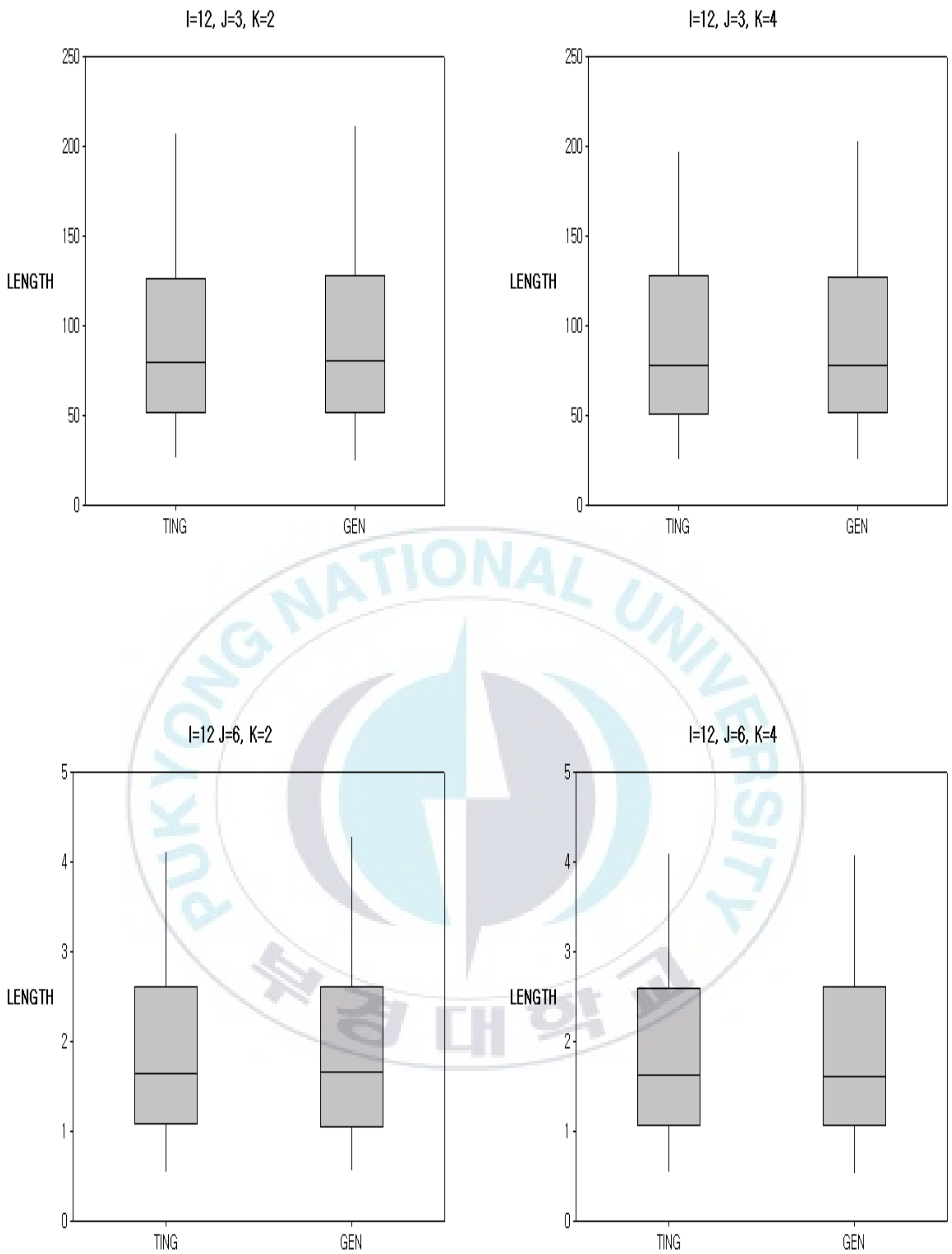

Figure 3.2.4.b Boxplots for Average Interval Lengths for 90% Two-sided Intervals on σ_O^2

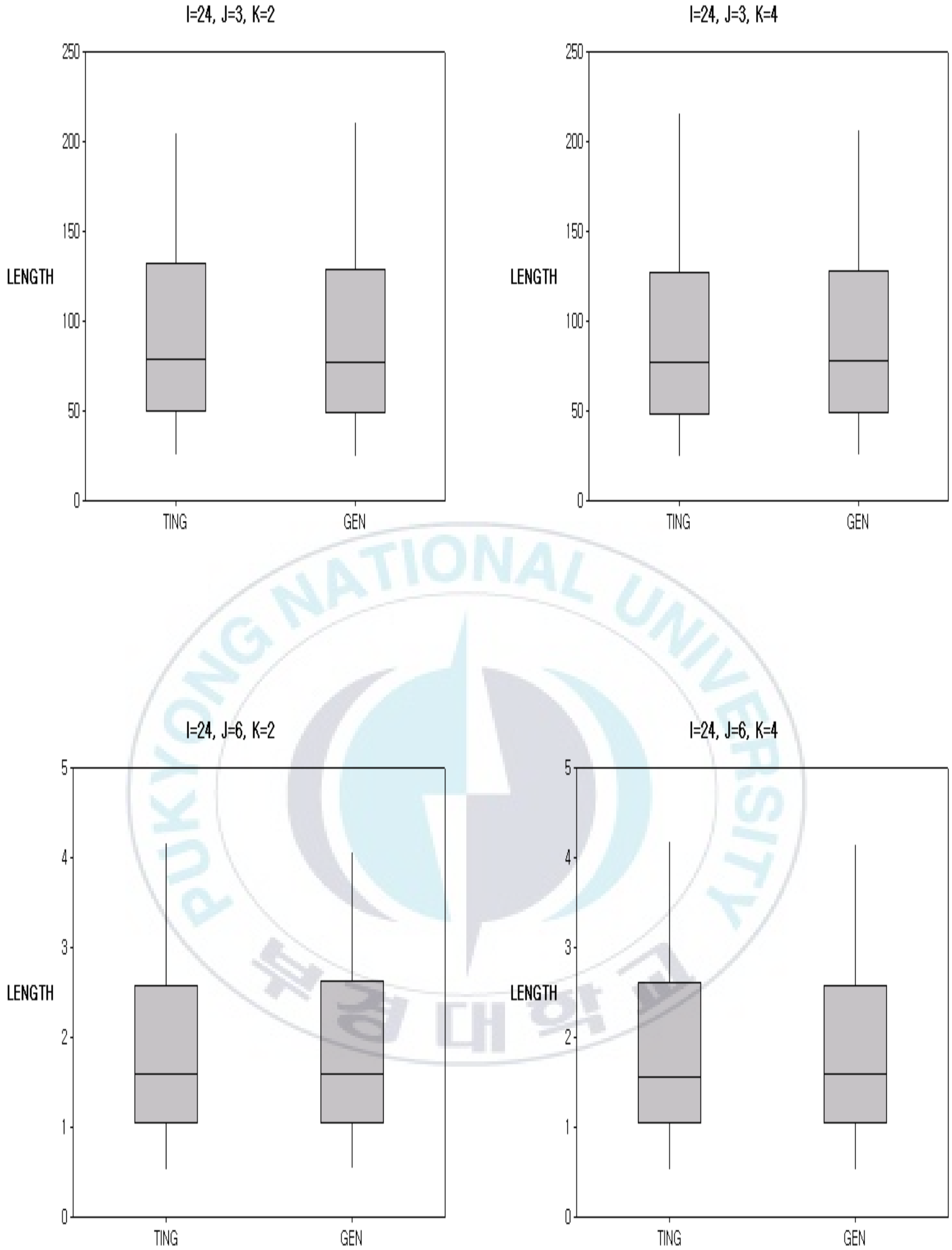

Figure 3.2.4.c Boxplots for Average Interval Lengths for 90% Two-sided Intervals on σ_O^2

3.3 A Numerical Example

The confidence intervals on variance components are now applied to a data set. Milliken and Johnson(2002 p.370) published a data set from an experiment designed to evaluate the effect of the speed in rpm of a circular bar of steel and the feed rate into a cutting tool on the roughness of the surface finish of the final turned product, where the depth of cut was set to 0.02 in. The treatment structure is a two-way with four levels of speed(rpm) and four levels of feed rate. Eight replications were performed for each combination of speed and feed rate. There is variation in the hardness of the bar stock used in the experiment; thus the hardness of each piece of bar was measured to be used as a possible covariate. There is a linear relationship between roughness and hardness.

In order to apply to this data set, the speed and feed rate are assumed to be operator effects and part effects, respectively. The data set was constructed by selecting first three speed rates(operators) and four feed rate(parts) with first two observations of each combination of speed and feed rate to represent replicated measurements. Roughness(Y) is linearly related to hardness(X). The selected data set is listed in Table 3.3.1. The SAS code is written to calculate confidence intervals on the variance for parts, operators, and measurement error. The SAS code used is listed in the Appendix. The mean squares in Table 2.2.2, $S_P^2/(JK)$, $S_O^2/(IK)$, and S_E^2 can be obtained in MEAN SQUARE ERRORS of ANOVA Tables by executing following PROC GLM statements.

TABLE 3.3.1 Data Set Used

	Operators					
	1		2		3	
Parts	Y	X	Y	X	Y	X
1	50	61	65	59	84	64
	53	65	55	44	104	70
2	64	54	81	65	108	41
	61	58	81	53	118	67
3	97	62	103	61	123	41
	79	48	105	53	137	41
4	141	66	158	56	192	69
	142	61	154	49	195	57

TABLE3.3.2 An Example Statement of PROC GLM for Calculating Mean Squares

```
.
PROC GLM DATA = TWO;
MODEL YMEAN = XMEAN;
.
.
PROC GLM DATA = THREE;
MODEL YMEAN2 = XMEAN2;
.
PROC GLM DATE = ONE;
CLASS PART OPERATOR;
MODEL Y = X PART OPERATOR;
.
```

The results of PROC GLM are shown in Table 3.3.3.

TABLE 3.3.3 Results of PROC GLM

SV	DF	MS	EMS
Parts	2	15417.25	$\sigma_E^2 + JK\sigma_P^2$
Operators	1	6688.86	$\sigma_E^2 + IK\sigma_O^2$
Replicates	17	28.89	σ_E^2

The ANOVA estimators of σ_P^2, σ_O^2, and σ_E^2 are computed using the results of PROC GLM and (2.2.8). In particular,

$$\begin{aligned}\hat{\sigma}_P^2 &= \frac{S_P^2 - S_E^2}{JK} = \frac{15417.25 - 28.89}{3 \times 2} = 2564.7 \\ \hat{\sigma}_O^2 &= \frac{S_O^2 - S_E^2}{IK} = \frac{6688.86 - 28.89}{4 \times 2} = 832.49 \\ \hat{\sigma}_E^2 &= 28.89\end{aligned} \tag{3.3.1}$$

The calculated values of S_P^2, S_O^2, and S_E^2 are substituted in (2.3.3) and (2.3.5) for computing TING method. The values are used as observed values to construct the distribution in (2.4.2) and (2.4.4) for GEN method . PROC MIXED automatically generates confidence intervals based on MIXED method. The resulting confidence intervals are reported in Table 3.3.4.

Since

$$\begin{aligned}\frac{\hat{\sigma}_P^2}{\hat{\sigma}_P^2 + \hat{\sigma}_O^2 + \hat{\sigma}_E^2} &= \frac{2564.7}{3425.1} = 0.7, \\ \frac{\hat{\sigma}_O^2}{\hat{\sigma}_P^2 + \hat{\sigma}_O^2 + \hat{\sigma}_E^2} &= \frac{832.49}{3425.1} = 0.2,\end{aligned} \tag{3.3.2}$$

the estimates are assumed to be $\hat{\sigma}_P^2 = 0.7$, $\hat{\sigma}_O^2 = 0.2$, and $\hat{\sigma}_E^2 = 0.1$ without loss of generality. By simulation study it is shown that the confidence interval based on PROC MIXED does not keep the stated confidence coefficients when $\hat{\sigma}_P^2$ or $\hat{\sigma}_O^2$ is approximately below 0.3. Thus the confidence interval based on $\hat{\sigma}_O^2$ is not recommended in this example although it is shorter than ones based on TING and GEN methods.

TABLE 3.3.4 90% Confidence Intervals on Variance Components

Effects	Method	Lower Bound	Upper Bound	Interval Length
Part	TING	852.8	50089.9	49237.1
	GEN	865.1	50362.0	49496.9
	MIXED	654.8	14705.0	14050.2
Operator	TING	214.0	212630.3	212416.3
	GEN	217.2	213030.4	212813.2
	MIXED	197.1	11923.0	11725.9
Measurement Error	TING	17.8	56.6	38.8
	GEN	17.8	56.7	38.9
	MIXED	17.8	56.6	38.8

3.4 Concluding Remarks

In this thesis two-factor crossed mixed model with one covariate for gauge R & R study is considered. This model employs J operators randomly chosen to conduct measurements on I randomly selected parts from a manufacturing process. In this R&R study each operator measures each part k times. The measurement is assumed to be linearly related to one covariate in this R& R study. The variabilities of the parts, operators, and measurements can be measured in terms of confidence intervals. Confidence intervals of the variance of part effects, operator effects, and measurement errors are useful tools to determine whether the variability is appropriate in a manufacturing process.

Three confidence intervals proposed are based on Ting et al.(1990) method (TING), generalized p-values concept (GEN), and SAS PROC MIXED (MIXED). Eight different designs of I, J, and K are considered for simulation study to see if the proposed intervals maintain the stated confidence coefficients and to compare the average interval lengths of three methods. Two-sided 90% confidence intervals are computed 2000 times for eight designs with 36 combinations of the values of σ_P^2, σ_O^2, and σ_E^2. Based on the simulation study TING and GEN methods generally maintain the stated confidence coefficients for all combinations of I, J, and K. Their average interval lengths are similar and comparable. However, MIXED method does not keep the stated confidence coefficient especially for small values of σ_P^2 and σ_O^2. In particular, when the values of σ_P^2 and σ_O^2 are small, the simulated confidence coefficients dramatically fall below the stated confidence coefficient 0.9. Thus TING and GEN methods are recommended for calculating confidence intervals on the variance of part and operator effects for all combinations of I, J, and K. MIXED method is an alternative for computing confidence intervals if the values of I, J, and K are large enough.

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Appendix

SAS Code for constructing confidence intervals on variance components

```
OPTIONS LS=75 PS=55;
data one;
input y x part operator replicate @@;
datalines;
50 61 1 1 1 53 65 1 1 2 65 59 1 2 1 55 44 1 2 2
84 64 1 3 1 104 70 1 3 2 64 54 2 1 1 61 58 2 1 2
81 65 2 2 1 81 53 2 2 2 108 41 2 3 1 118 67 2 3 2
97 62 3 1 1 79 48 3 1 2 103 61 3 2 1 105 53 3 2 2
123 41 3 3 1 137 41 3 3 2 141 66 4 1 1 142 61 4 1 2
158 56 4 2 1 154 49 4 2 2 192 69 4 3 1 195 57 4 3 2
;
proc sort data=one;
  by part;
proc means;
  by part;
  var y x;
  output out=two mean= ymean xmean;
PROC GLM DATA = two;
  MODEL ymean = xmean /solution ;
proc sort data=one;
  by operator;
proc means;
  by operator;
  var y x;
  output out=three mean= ymean2 xmean2;
PROC GLM DATA = three;
  MODEL ymean2 = xmean2 /solution;
PROC GLM data=one ;
  class part operator ;
  MODEL y = x part operator /solution;
proc mixed data=one asycov cl alpha=0.1 method=reml;
  classes part operator;
  model y = x;
  random part operator;
*---F values to calculate confidence intervals
```

```
using TING method;
proc iml;
SP2 = 15417.25483;
SO2 = 6688.860576;
SE2 = 28.89106;
ALPHA=.10;
print SP2 SO2 SE2 alpha;
I = 4;
J = 3;
K= 2;
N1 = I - 2;
N2 = J - 2;
N3 = I*J*K - I - J;
print I J K N1 N2 N3;
F1 = FINV( 1 - alpha/2, N1, N3);
F2 = FINV( alpha/2, N1, N3);
G1 = 1 - 1/(CINV(1 - ALPHA/2, N1)/N1);
H2 = 1/(CINV(ALPHA/2, N3)/N3) - 1;
H1 = 1/(CINV(ALPHA/2, N1)/N1) - 1;
G2 = 1 - 1/(CINV(1 - ALPHA/2, N3)/N3);
G12 = ((F1 - 1)**2 - (G1 * F1)**2 - H2**2)/F1;
H12 = ((1 - F2)**2 - (H1 * F2)**2 - G2**2)/F2;
PRINT ALPHA F1 F2;
PRINT G1 H2 H1 G2 G12 H12;
*---Calculation of Confidence Intervals on SigPsq
using TING method;
SigPlb = ( SP2 - SE2 - SQRT(G1**2 * SP2**2
         + H2**2 * SE2**2 + G12*SP2*SE2))/(J*K);
SigPub = ( SP2 - SE2 + SQRT(H1**2 * SP2**2
         + G2**2 * SE2**2 + H12*SP2*SE2))/(J*K);
SigPint = max(0, SigPub) -  max(0, SigPlb);
PRINT SigPlb SigPub SigPint;
F3 = FINV( 1 - alpha/2, N2, N3);
F4 = FINV( alpha/2, N2, N3);
G3 = 1 - 1/(CINV(1 - ALPHA/2, N2)/N2);
H4 = 1/(CINV(ALPHA/2, N3)/N3) - 1;
H3 = 1/(CINV(ALPHA/2, N2)/N2) - 1;
G4 = 1 - 1/(CINV(1 - ALPHA/2, N3) /N3);
G34 = ((F3 - 1)**2 - (G3 * F3)**2 - H4**2)/F3;
```

```
H34 = ((1 - F4)**2 - (H3 * F4)**2 - G4**2)/F4;
PRINT ALPHA F3 F4;
PRINT G3 H4 H3 G4 G34 H34;
*---Calculation of Confidence Intervals on SigOsq
using TING method;
SigOlb = ( SO2 - SE2 - SQRT(G3**2 * SO2**2
         + H4**2 * SE2**2 + G34*SO2*SE2))/(I*K);
SigOub = ( SO2 - SE2 + SQRT(H3**2 * SO2**2
         + G4**2 * SE2**2 + H34*SO2*SE2))/(I*K);
SigOint = max(0, SigOub) -  max(0, SigOlb);
PRINT SigOlb SigOub SigOint;
*---Calculation of Confidence Intervals on SigEsq
using TING method;
SigElb = SE2/(CINV(1 - ALPHA/2, N3)/N3);
SigEub = SE2/(CINV(ALPHA/2, N3)/N3);
SigEint = max(0, SigEub) - max(0, SigElb);
PRINT SigElb SigEub SigEint;
proc iml;
y = {50, 53, 65, 55, 84, 104, 64, 61, 81, 81,
     108, 118, 97, 79, 103, 105, 123, 137, 141,
     142, 158, 154, 192, 195};
x = {61, 65, 59, 44, 64, 70, 54, 58, 65, 53,
     41, 67, 62, 48, 61, 53,41, 41, 66, 61, 56,
     49, 69, 57};
one = j(24, 1, 1);
x = one||x;
print y x;
z1 = {1 0 0 0, 1 0 0 0, 1 0 0 0, 1 0 0 0, 1 0 0 0, 1 0 0 0,
      0 1 0 0, 0 1 0 0, 0 1 0 0, 0 1 0 0, 0 1 0 0, 0 1 0 0,
      0 0 1 0, 0 0 1 0, 0 0 1 0, 0 0 1 0, 0 0 1 0, 0 0 1 0,
      0 0 0 1, 0 0 0 1, 0 0 0 1, 0 0 0 1, 0 0 0 1, 0 0 0 1};
z2 = {1 0 0, 1 0 0, 0 1 0, 0 1 0, 0 0 1, 0 0 1,
      1 0 0, 1 0 0, 0 1 0, 0 1 0, 0 0 1, 0 0 1,
      1 0 0, 1 0 0, 0 1 0, 0 1 0, 0 0 1, 0 0 1,
      1 0 0, 1 0 0, 0 1 0, 0 1 0, 0 0 1, 0 0 1};
I = ncol(z1);
J = ncol(z2);
K = 2;
print I J K;
```

```
z3 = I(I*J*K);
w1 = (1/(J*K))*z1`;
w2 = (1/(I*K))*z2`;
w3 = z3`;
x1 = w1* x;
x2 = w2* x;
x3 = x||z1||z2;
print x1 x2;
h1 = x1*inv(x1`*x1)*x1`;
h2 = x2*inv(x2`*x2)*x2`;
h3 = x3*ginv(x3`*x3)*x3`;
i4 = i(I);
i3 = i(J);
i24 = i(I*J*K);
r1 = J*K * y`*w1`*(i4 - h1)*w1*y;
r2 = I*K * y`*w2`*(i3 - h2)*w2*y;
r3 = y`*w3`*(i24 - h3)*w3*y;
print r1 r2 r3;
*--- Observed values-----;
sp2 = r1 / (I - 2);
so2 = r2 / (J - 2);
se2 = r3 / (I*J*K - I - J);
print sp2 so2 se2;
DF3 = I*J*K - I - J;
DF1 = I - 2;
DF2 = J - 2;
ALPHA = 0.10;
SEED1 = 239480;
SEED2 = 189790;
SEED3 = 3344393;
SEED4 = 9007889;
IT2 = 10000;
IT1 = 1;
KKK = IT1;
GPQ=J(IT2, IT1);
DO LL = 1 TO IT2 ;
  USS = 2 * RANGAM(SEED3, DF3/2);
  UTS = 2 * RANGAM(SEED4, DF1/2);
  ESTIMATEP = (1/(J*K)) *
```

```
  ( (DF1 / UTS) * SP2 - (DF3 / USS) * SE2);
  GPQ(|LL, KKK|) = ESTIMATEP ;
END;
*------Sort  GPQ and find PERCENTILES;
FINAL=J(IT2, IT1, 0); DO MM = 1 TO IT1;
     X = GPQ(|, MM|);
     B = X;
     X[RANK(X),] = B;
     FINAL(|,MM|) = X;
END;
LOWERBDP = FINAL(|IT2 * ALPHA/2,|);
UPPERBDP = FINAL(|IT2*(1-ALPHA/2),|);
PRINT LOWERBDP UPPERBDP;
DO LL = 1 TO IT2 ;
  USS = 2 * RANGAM(SEED3, DF3/2);
  UTS = 2 * RANGAM(SEED4, DF2/2);
  ESTIMATEO = (1/(I*K)) *
  ( (DF2 / UTS) * SO2 - (DF3 / USS) * SE2);
  GPQ(|LL, KKK|) = ESTIMATEO ;
END;
*------Sort  GPQ and find PERCENTILES;
FINAL=J(IT2, IT1, 0);
DO MM = 1 TO IT1;
     X = GPQ(|, MM|);
     B = X;
     X[RANK(X),] = B;
     FINAL(|,MM|) = X;
END;
LOWERBDO = FINAL(|IT2 * ALPHA/2,|);
UPPERBDO = FINAL(|IT2*(1-ALPHA/2),|);
PRINT LOWERBDO UPPERBDO;
DO LL = 1 TO IT2 ;
  USS = 2 * RANGAM(SEED3, DF3/2);
  ESTIMATEE =  (DF3 / USS) * SE2  ;
  GPQ(|LL, KKK|) = ESTIMATEE ;
END;
*------Sort  GPQ and find PERCENTILES;
FINAL=J(IT2, IT1, 0);
DO MM = 1 TO IT1;
```

```
  X = GPQ(|, MM|);
  B = X;
  X[RANK(X),] = B;
  FINAL(|,MM|) = X;
END;
LOWERBDE = FINAL(|IT2 * ALPHA/2,|);
UPPERBDE = FINAL(|IT2*(1-ALPHA/2),|);
PRINT LOWERBDE UPPERBDE;
RUN;
```